Tensor products of Operator Spaces and Grothendieck' s Theorem

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Abstract

Probably the most famous of Grothendieck's contributions to Analysis is the result that he himself described as "the fundamental theorem in the metric theory of tensor products". That is now commonly referred to as "Grothendieck's theorem" (GT in short), or sometimes as "Grothendieck's inequality". This had a major impact first in Banach space theory (roughly after 1968), then, later on, in C^*-algebra theory, (roughly after 1978). More recently, in this millennium, a new version of GT has been successfully developed in the framework of "operator spaces" or non-commutative Banach spaces. In addition, GT independently surfaced in several quite unrelated fields: in connection with Bell's inequality in quantum mechanics, in graph theory where the Grothendieck constant of a graph has been introduced and in computer science where the Grothendieck inequality is invoked to replace certain NP hard problems by others that can be treated by "semidefinite programming' and hence solved in polynomial time. This minicourse will present a review of some of these topics, starting from the original GT. We will concentrate on the more recent developments in operator space theory and the connections with exactness, random matrices, the Haagerup tensor product and C^*-tensor products.
The audience can use as notes our recent survey [1].

References

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