Symmetric pairs with finite-multiplicity property for branching laws of admissible representations

By Toshiyuki Kobayashi

Kavli IPMU (WPI) and Graduate School of Mathematical Sciences, the University of Tokyo, Meguro-ku, Tokyo, 153-8914, Japan

E-mail address: toshi@ms.u-tokyo.ac.jp

Abstract: We accomplish the classification of the reductive symmetric pairs (G,H) for which the dimension of the space $\mathrm{Hom}_H(\pi|_H,\tau)$ of H-intertwining operators is finite for any irreducible smooth representation π of G and for any irreducible smooth representation τ of H.

Key words: branching law; restriction of representation; reductive group; real spherical variety; symmetric pair.

1. Finite-multiplicity in Induction and Restriction One of the basic problems in representation theory is to understand how a given representation is decomposed into irreducible representations. Given a pair of groups $G \supset H$, there are two important settings for this problem:

I) (Induction) For a simple H-module τ, understand $\mathrm{Ind}_H^G(\tau)$ as a G-module.

II) (Restriction) For a simple G-module π, understand $\pi|_H$ as an H-module.

We shall highlight the case where G is a real reductive linear Lie group.

Concerning Induction Problem (I), a special case is the unitary induction $\mathrm{Ind}_H^G(\tau)$ from the trivial one-dimensional representation $\tau = \mathbf{1}$ of H, which is unitarily equivalent to the regular representation of G on $L^2(G/H)$ if G/H admits a G-invariant Radon measure. Its irreducible decomposition is called the *Plancherel-type theorem* for G/H, and the theory has been developed extensively for reductive symmetric pairs (G,H) over several decades since the pioneering work of the Gelfand school and Harish-Chandra. Such a successful analysis is built on the following finiteness property [1]: For any reductive symmetric pair (G,H) and for any irreducible admissible representation π:

$$\dim \mathrm{Hom}_G(\pi, C^\infty(G/H)) < \infty. \tag{1.1}$$

We note that the finite-multiplicity property (1.1) holds not only for irreducible unitary representations but also for non-unitary representations π. More strongly, there exists a constant $C \equiv C(G,H) < \infty$ such that

$$\dim \mathrm{Hom}_G(\pi, C^\infty(G/H)) \le C,$$

for any irreducible smooth representation π of G, as far as $G_{\mathbb{C}}/H_{\mathbb{C}}$ is spherical, see [13, Theorem A].

Concerning Restriction Problem (II), the H-irreducible decomposition of the restriction $\pi|_H$ is called the *branching law*.

If H is a maximal compact subgroup K of the reductive group G, then for any irreducible unitary representation π of G, we have the following admissibility theorem of Harish-Chandra [4]:

$$\dim \mathrm{Hom}_K(\tau, \pi|_K) < \infty \tag{1.2}$$

for any irreducible (finite-dimensional) representation τ of K. Equivalently, the condition (1.2) can

2010 Mathematics Subject Classification. Primary 22E46; Secondary 14M15, 53C35.

be replaced by

$$\dim \mathrm{Hom}_K(\pi|_K, \tau) < \infty \tag{1.3}$$

because K is compact. Harish-Chandra's admissibility theorem has led to the concept of $(\mathfrak{g}, K)$-modules, providing us with an algebraic powerful tool in studying irreducible unitary representations of reductive Lie groups.

A continuous representation π of a real reductive group G of finite length on a complete, locally convex topological vector space is called *admissible* if (1.2) is satisfied. We say π is an *admissible smooth representation* (sometimes referred to as a smooth Fréchet representation of moderate growth [20, Chapter 11]) if π is realized in the space of smooth vectors of a Banach representation of finite length. An irreducible admissible smooth representation will be called an *irreducible smooth representation* in this article for simplicity. By the Casselman–Wallach globalization theory, there is a canonical equivalence of categories between the category of $(\mathfrak{g}, K)$-modules of finite length and the category of admissible smooth representations of G.

In contrast to the Riemannian symmetric pair (G, K), it is notorious that a finite-multiplicity theorem for the restriction (see (1.3)) may fail for reductive symmetric pairs (G, H), namely, it may well happen that

$$\dim \mathrm{Hom}_H(\pi|_H, \tau) = \infty$$

for some irreducible smooth representation π of G and some irreducible smooth representation τ of H. Here $\mathrm{Hom}_H(\ ,\ )$ denotes the space of continuous H-homomorphisms.

An opposite extremal case is that the restriction $\pi|_H$ is still irreducible as an H-module. This is rare but still happens for (infinite-dimensional) irreducible representations π and for reductive symmetric pairs (G, H), see [9].

A special case of a symmetric pair is the group case

$$(G, H) = (G' \times G', \mathrm{diag}G'),$$

for which the branching problem (II) deals with the decomposition of the tensor product of two irreducible representations of G'. Even in this case, the branching laws do not always behave nicely. For example, the tensor product of two irreducible unitary principal series representations of a simple group such as $SL(n, \mathbb{R})$ $(n \geq 3)$ involves infinite multiplicities in the irreducible decomposition. See [7, 9] for more details about "bad behaviours" and "good behaviours" of the restriction with respect to symmetric pairs.

These observations suggest that the condition that H is a maximal reductive subgroup of G would be too general to develop a concrete analysis of branching laws of irreducible unitary representations of G. In other words, one could expect detailed analysis on branching laws only if we were able to discover "very nice frameworks." Indeed, the analysis of branching laws has been developed extensively in the following nice settings:

(1) (Theta correspondence, Howe's dual pair) π is the metaplectic representation of $G = Mp(n, \mathbb{R})$ and $H = H_1 \cdot H_2$ is a dual pair in G [5].
(2) (Admissible restriction) The restriction $\pi|_H$ is *H-admissible*, i.e., it decomposes discretely into a direct sum of irreducible representations of H with finite multiplicities [7].

These examples impose strong constraints on the representation π of G. For instance, in the theta correspondence (1), the representation π attains its minimum Gelfand–Kirillov dimension among all infinite-dimensional representations of G. The recent papers [14, 15] gave a classification of the triples (G, H, π) for which the admissibility of the restriction (2) holds in the setting that (G, H) is a reductive symmetric pair and π is relatively "small" (*e.g.*, Zuckerman's derived functor modules, minimal representations, *etc.*).

In this article, we consider a more general framework, and try to relax any assumption on π such as "small" representations. Thus, we wish to understand clearly for which pairs (G, H) of reductive groups we could expect that the branching laws $\pi|_H$ behave reasonably for *arbitrary* irreducible representations π. To be more precise, we ask whether a given pair (G, H) satisfies the following finite-multiplicity property for the restriction of admissible representations:

(FM) (Finite-multiplicity restriction) $\dim \operatorname{Hom}_H(\pi|_H, \tau) < \infty$, for any admissible smooth representation π of G and for any admissible smooth representation τ of H.

2. Statement of Main Results Here is the complete classification of the reductive symmetric pairs (G, H) having the property (FM).

Theorem 1. *Suppose (G, H) is a reductive symmetric pair. Then the following two conditions are equivalent:*

(i) *(G, H) satisfies the finite-multiplicity property* (FM) *for restriction of admissible smooth representations.*

(ii) *The pair of the Lie algebras $(\mathfrak{g}, \mathfrak{h})$ is isomorphic (up to outer automorphisms) to the direct sum of the following pairs:*

A) *Trivial case: $\mathfrak{g} = \mathfrak{h}$.*

B) *Abelian case: $\mathfrak{g} = \mathbb{R}$, $\mathfrak{h} = \{0\}$.*

C) *Compact case: $\mathfrak{g}$ is the Lie algebra of a compact simple Lie group.*

D) *Riemannian symmetric pair: $\mathfrak{h}$ is the Lie algebra of a maximal compact subgroup K of a non-compact simple Lie group G.*

E) *Split rank one case* ($\operatorname{rank}_{\mathbb{R}} G = 1$)*:*

E1) $(\mathfrak{o}(p+q,1), \mathfrak{o}(p)+\mathfrak{o}(q,1))$ $(p+q \geq 2)$.

E2) $(\mathfrak{su}(p+q,1), \mathfrak{s}(\mathfrak{u}(p)+\mathfrak{u}(q,1)))$ $(p+q \geq 1)$.

E3) $(\mathfrak{sp}(p+q,1), \mathfrak{sp}(p)+\mathfrak{sp}(q,1))$ $(p+q \geq 1)$.

E4) $(\mathfrak{f}_{4(-20)}, \mathfrak{so}(8,1))$.

F) *Strong Gelfand pairs and their real forms:*

F1) $(\mathfrak{sl}(n+1,\mathbb{C}), \mathfrak{gl}(n,\mathbb{C}))$ $(n \geq 2)$.

F2) $(\mathfrak{o}(n+1,\mathbb{C}), \mathfrak{o}(n,\mathbb{C}))$ $(n \geq 2)$.

F3) $(\mathfrak{sl}(n+1,\mathbb{R}), \mathfrak{gl}(n,\mathbb{R}))$ $(n \geq 1)$.

F4) $(\mathfrak{su}(p+1,q), \mathfrak{u}(p,q))$ $(p+q \geq 1)$.

F5) $(\mathfrak{o}(p+1,q), \mathfrak{o}(p,q))$ $(p+q \geq 2)$.

G) *Group case: $(\mathfrak{g}, \mathfrak{h}) = (\mathfrak{g}' + \mathfrak{g}', \operatorname{diag} \mathfrak{g}')$*

G1) *$\mathfrak{g}'$ is the Lie algebra of a compact simple Lie group.*

G2) $\mathfrak{g}' \simeq \mathfrak{o}(n,1)$ $(n \geq 2)$.

H) *Other cases:*

H1) $(\mathfrak{o}(2n,2), \mathfrak{u}(n,1))$.

H2) $(\mathfrak{su}^*(2n+2), \mathfrak{su}(2)+\mathfrak{su}^*(2n)+\mathbb{R})$ $(n \geq 1)$.

H3) $(\mathfrak{o}^*(2n+2), \mathfrak{o}(2)+\mathfrak{o}^*(2n))$ $(n \geq 1)$.

H4) $(\mathfrak{sp}(p+1,q), \mathfrak{sp}(p,q)+\mathfrak{sp}(1))$.

H5) $(\mathfrak{e}_{6(-26)}, \mathfrak{so}(9,1)+\mathbb{R})$.

For the "group case" (G), Theorem 1 implies the following:

Corollary 2. *Suppose G is a simple Lie group. Then the following three conditions on G are equivalent:*

(i) *For any triple of admissible smooth representations π_1, π_2, and π_3 of G,*

$$\dim \operatorname{Hom}_G(\pi_1 \otimes \pi_2, \pi_3) < \infty.$$

(ii) *For any triple of admissible smooth representations π_1, π_2 and π_3 of G, invariant trilinear forms are finite-dimensional:*

$$\dim \operatorname{Hom}_G(\pi_1 \otimes \pi_2 \otimes \pi_3, \mathbb{C}) < \infty.$$

(iii) *Either G is compact or $\mathfrak{g}$ is isomorphic to $\mathfrak{o}(n,1)$ $(n \geq 2)$.*

3. Uniformly Bounded Multiplicities In addition to the aforementioned finite-multiplicity property **(FM)**, we consider the following two properties on a pair of reductive groups (G, H):

(BM) (*Bounded-multiplicity restriction*) There exists a constant $C \equiv C(G, H) < \infty$ such that

$$\dim \operatorname{Hom}_H(\pi|_H, \tau) \leq C,$$

for any irreducible admissible representations π and τ of G and H, respectively.

(MF) (*Multiplicity-free restriction*) One can take C to be 1 in (BM), namely,

$$\dim \operatorname{Hom}_H(\pi|_H, \tau) \leq 1$$

for any irreducible admissible representations π and τ of G and H, respectively.

Clearly, we have

$$\text{(MF)} \Rightarrow \text{(BM)} \Rightarrow \text{(FM)}.$$

We note that the properties (FM) and (BM) depend only on the Lie algebra $(\mathfrak{g}, \mathfrak{h})$. Moreover, we have discovered in [13, Theorem D] that the bounded-multiplicity property (BM) depends only on the complexified Lie algebra $(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}}) \equiv (\mathfrak{g} \otimes_{\mathbb{R}} \mathbb{C}, \mathfrak{h} \otimes_{\mathbb{R}} \mathbb{C})$. On the other hand, the multiplicity-free property (MF) is not determined by the pair of Lie algebras $(\mathfrak{g}, \mathfrak{h})$, but depends on the groups G and H (*e.g.*, the disconnectedness of the groups may affect the best constant C in (BM)).

Here is the classification of symmetric pairs $(\mathfrak{g}, \mathfrak{h})$ satisfying the property (BM) as a subclass of (FM):

Proposition 3. *Suppose* $(\mathfrak{g}, \mathfrak{h})$ *is a real reductive symmetric pair. Then the following three conditions are equivalent:*

(i) *For any real reductive Lie groups* $G \supset H$ *with Lie algebras* $\mathfrak{g} \supset \mathfrak{h}$*, respectively, the pair* (G, H) *satisfies the bounded multiplicity property* (BM) *for restriction.*

(ii) *There exists a pair of (possibly disconnected) real reductive Lie groups* $G \supset H$ *such that* (G, H) *satisfies the multiplicity-free property* (MF) *for restriction.*

(iii) *The pair of the Lie algebras* $(\mathfrak{g}, \mathfrak{h})$ *is isomorphic (up to outer automorphisms) to the direct sum of pairs* (A), (B) *and* (F1) – (F5).

The implication (ii) $\Rightarrow$ (i) is obvious as mentioned. The equivalence (i) $\Leftrightarrow$ (iii) was proved in [13, Theorem D]. The implication (iii) $\Rightarrow$ (ii) was proved in Sun–Zhu [19]. (Thus there are two different proofs for the implication (iii) $\Rightarrow$ (i).) As a more refined form of the implication (iii) $\Rightarrow$ (ii), Gross and Prasad formulated a conjecture about the restriction of an irreducible admissible tempered representation of an inner form G of the group $O(n)$ over a local field to a subgroup which is an inner form $O(n-1)$ (*cf.* (F2) and (F4) for the Archimedean field), [3].

Similarly to Corollary 2, we apply Proposition 3 to the group case and get the following (see [8], [11, Corollary 4.2] for further equivalence, *e.g.* the finite-dimensionality of the space of *Shintani functions*):

Corollary 4. *Suppose* G *is a simple Lie group. Then the following three conditions on* G *are equivalent:*

(i) *There exists a constant* $C < \infty$ *such that*

$$\dim \operatorname{Hom}_G(\pi_1 \otimes \pi_2, \pi_3) \leq C,$$

for any irreducible smooth representations π_1, π_2, *and* π_3 *of* G.

(ii) *There exists a constant* $C < \infty$ *such that*

$$\dim \operatorname{Hom}_G(\pi_1 \otimes \pi_2 \otimes \pi_3, \mathbb{C}) \leq C,$$

for any irreducible smooth representations π_1, π_2, *and* π_3 *of* G.

(iii) *The Lie algebra* $\mathfrak{g}$ *is isomorphic to one of* $\mathfrak{su}(2) \simeq \mathfrak{o}(3)$, $\mathfrak{su}(1,1) \simeq \mathfrak{sl}(2,\mathbb{R}) \simeq \mathfrak{o}(2,1)$ *or* $\mathfrak{sl}(2,\mathbb{C}) \simeq \mathfrak{o}(3,1)$.

4. Strategy of Proof A complex manifold $X_{\mathbb{C}}$ with action of a complex reductive group $G_{\mathbb{C}}$ is called *spherical* if a Borel subgroup of $G_{\mathbb{C}}$ has an open orbit in $X_{\mathbb{C}}$, and there is a vast literature on spherical varieties. In the real setting, in search of a good framework for global analysis on homogeneous spaces which are broader than the usual (*e.g.* symmetric spaces), we emphasised in [8] the importance of the following notion and proposed to call:

Definition 5. A smooth manifold X with action of a real reductive group G is *real spherical* if a minimal parabolic subgroup P of G has an open orbit in X.

In the case where G acts transitively on X, a minimal parabolic subgroup P has finitely many orbits in X if and only if P has an open orbit in X by the works of Kimelfeld [6] and Matsuki [18], see also [13, Remark 2.5] and references therein.

Representation theoretic properties (FM) or (BM) are characterised by the geometric conditions on real or complex flag varieties, respectively, as follows:

Fact 6 ([13, Theorems C and D]). *Suppose G is a real reductive Lie group, and H a reductive subgroup defined algebraically over $\mathbb{R}$.*

1) *The finite-multiplicity property* (FM) *holds if and only if* $(G \times H)/\operatorname{diag} H$ *is real spherical.*
2) *The bounded-multiplicity property* (BM) *holds if and only if* $(G_{\mathbb{C}} \times H_{\mathbb{C}})/\operatorname{diag} H_{\mathbb{C}}$ *is spherical.*

Here $G_{\mathbb{C}}$ is a complexification of G, and $H_{\mathbb{C}}$ a subgroup of $G_{\mathbb{C}}$ with complexified Lie algebra $\mathfrak{h}_{\mathbb{C}} = \mathfrak{h} \otimes_{\mathbb{R}} \mathbb{C}$.

Therefore, we can reduce the proof of Theorem 1 to a purely algebraic question, namely, the classification of real spherical variety of the form $(G \times H)/\operatorname{diag} H$.

For this, it is sufficient to deal with the case where $(\mathfrak{g}, \mathfrak{h})$ is an irreducible symmetric pair, which consists of two families:

1) (group case) $(\mathfrak{g}' + \mathfrak{g}', \operatorname{diag} \mathfrak{g}')$ with $\mathfrak{g}'$ simple,
2) $(\mathfrak{g}, \mathfrak{h})$ with $\mathfrak{g}$ simple.

In the sequel, we say (G, H) satisfies (PP) if $(G \times H)/\operatorname{diag} H$ is real spherical, and (BB) if $(G_{\mathbb{C}} \times H_{\mathbb{C}})/\operatorname{diag} H_{\mathbb{C}}$ is spherical.

The classification of real spherical homogeneous spaces of the form $(G \times H)/\operatorname{diag} H$ with (G, H) irreducible symmetric pairs was accomplished as follows:

Theorem 7 ([12]). *For irreducible symmetric pairs $(\mathfrak{g}, \mathfrak{h})$, the following two conditions are equivalent:*

(i) $(G \times H)/\operatorname{diag} H$ *is real spherical.*
(ii) $(\mathfrak{g}, \mathfrak{h})$ *is isomorphic to one of* (C)–(H) *up to outer automorphisms.*

Remark 8. *In connection with branching problems, the classification in Theorem 7 was established earlier in the following special cases:*

1) $(\mathfrak{g}, \mathfrak{h})$*: complex pairs* (PP) $\Leftrightarrow$ (BB) $\Leftrightarrow$ (F1) *or* (F2) ([17]).
2) $(\mathfrak{g}, \mathfrak{h}) = (\mathfrak{g}' + \mathfrak{g}', \operatorname{diag} \mathfrak{g}')$ (*group case*) (PP) $\Leftrightarrow$ (G) ([8]).

The case (1) was studied in connection with *finite-dimensional* representations of compact Lie groups, and the case (2) with the tensor product of two representations as we saw in Corollary 2. We also notice that $(\mathfrak{g}' + \mathfrak{g}', \mathfrak{g}')$ satisfies (PP) if and only if the homogeneous space $(G' \times G' \times G')/\operatorname{diag} G'$ is a real spherical variety in view of the following isomorphism:

$$(P_{G'} \times P_{G'} \times P_{G'})\backslash(G' \times G' \times G')/\operatorname{diag} G' \simeq (P_{G'} \times P_{G'})\backslash(G' \times G')/P_{G'}.$$

5. Concluding Remarks As mentioned at the beginning of this article, the original motivation of this work is to single out *good pairs* (G, H) of reductive groups, with which we hope to open a new theory of geometric analysis of the branching laws $\pi|_H$ of arbitrary irreducible smooth representations π of G. We mention here some few examples of the recent progress in this direction for some specific pairs (G, H) that appear in the list of Theorem 1:

- Analysis on invariant trilinear forms [2]
  - $(G, H) = (G' \times G', \operatorname{diag} G')$ with $G' = O(n, 1)$, see Corollary 2.
- Classification and explicit construction of conformally covariant (integral, differential, ...) operators [10, 16].
  - $(G, H) = (O(n+1, 1), O(n, 1))$, see (E1) or (F5) in Theorem 1.

The main results of this paper were announced in the conferences “Group Actions with Applications in Geometry and Analysis” at Reims University (France) in June, 2013 and in “Representations of Reductive Groups” at the University of Utah (USA) in July, 2013.

Detailed proofs are given in [11, 12, 13].

References

[1] E. P. van den Ban, *Invariant differential operators on a semisimple symmetric space and finite multiplicities in a Plancherel formula*, Ark. Mat. **25** (1987), 175–187.

[2] J.-L. Clerc, T. Kobayashi, B. Ørsted, and M. Pevzner, *Generalized Bernstein–Reznikov integrals*, Math. Ann. **349** (2011), 395–431.

[3] B. Gross, D. Prasad, On the decomposition of a representations of SO_n when restricted to SO_{n-1}, Canad. J. Math. **44**, (1992), 974–1002.

[4] Harish-Chandra, Representations of semisimple Lie groups on a Banach space, Proc. Nat. Acad. Sci. U. S. A. **37**, (1951), 170–173.

[5] R. Howe, θ-series and invariant theory. In: Automorphic forms, representations and L-functions. Proc. Symp. Pure Math., **33**, pp. 275–285, (eds. A. Borel, W. Casselman), Providence, RI, Amer. Math. Soc. 1979

[6] B. Kimelfeld, Homogeneous domains in flag manifolds of rank 1, J. Math. Anal. & Appl. **121**, (1987), pp. 506–588.

[7] T. Kobayashi, *Discrete decomposability of the restriction of $A_{\mathfrak{q}}(\lambda)$ with respect to reductive subgroups and its applications*, Invent. Math. **117** (1994), 181–205; Part II, Ann. of Math. (2) **147** (1998), 709–729; Part III, Invent. Math. **131** (1998), 229–256.

[8] ———, Introduction to harmonic analysis on real spherical homogeneous spaces, Proceedings of the 3rd Summer School on Number Theory "Homogeneous Spaces and Automorphic Forms" in Nagano (F. Sato, ed.), 1995, 22–41 (in Japanese).

[9] ———, Branching problems of Zuckerman derived functor modules, In: Representation Theory and Mathematical Physics (in honor of Gregg Zuckerman), eds. J. Adams, B. Lian, and S. Sahi, Contemporary Mathematics, **557**, pages 23–40. Amer. Math. Soc., Providence, RI, 2011.

[10] ———, F-method for symmetry breaking operators, Differential Geom. Appl. **33** (2014), 272–289, Special issue in honour of M. Eastwood, DOI:10.1016/j.difgeo.2013.10.003. Published online 20 November 2013, (available at arXiv:1303.3545).

[11] ———, Shintani functions, real spherical manifolds, and symmetry breaking operators, to appear in Progr. Math. Birkhäuser, (available at arXiv:1401.0117).

[12] T. Kobayashi, T. Matsuki, Classification of finite-multiplicity symmetric pairs, Transform. Groups **19** (2014), 37 pp. Special issue in honour of Professor Dynkin for his 90th birthday, doi:10.1007/s00031-014-9265-x (available at arXiv:1312.4246).

[13] T. Kobayashi, T. Oshima, Finite multiplicity theorems for induction and restriction, Adv. Math. **248** (2013), 921–944.

[14] T. Kobayashi, Y. Oshima, Classification of discretely decomposable $A_{\mathfrak{q}}(\lambda)$ with respect to reductive symmetric pairs, Adv. Math., **231** (2012), 2013–2047.

[15] ———, Classification of symmetric pairs with discretely decomposable restrictions of $(\mathfrak{g}, K)$-modules, Crelles Journal. published on line 2013 July 13. 19 pp. doi:10.1515/crelle-2013-0045.

[16] T. Kobayashi, B. Speh, Symmetry breaking for representations of rank one orthogonal groups, 131 pages, arXiv:1310.3213, (announcement: Intertwining operators and the restriction of representations of rank one orthogonal groups, C. R. Acad. Sci. Paris, Ser. I, **352** (2014), 89–94).

[17] M. Krämer, *Multiplicity free subgroups of compact connected Lie groups*, Arch. Math. (Basel) **27** (1976), 28–36.

[18] T. Matsuki, Orbits on flag manifolds, Proceedings of the International Congress of Mathematicians, Kyoto 1990, Vol. II (1991), Springer-Verlag, pp. 807–813.

[19] B. Sun and C.-B. Zhu, *Multiplicity one theorems: the Archimedean case*, Ann. of Math. **175** (2012), 23–44.

[20] N. R. Wallach, Real reductive groups. I., Pure and Applied Mathematics, **132** Academic Press, Inc., Boston, MA, 1988. xx+412 pp.