Laguerre semigroup and Dunkl operators.

(Ben Saïd–Kobayashi–Ørsted [*]) The authors constructed a deformation $\mathcal{I}_{k,a}(z)$ of the classical Fourier transform with three deformation parameter (a, k, z), and developed a theory blending classical analysis and modern harmonic analysis, inspired by the recent progress in representation theory, *i.e.* discretely decomposable branching laws and minimal representations of reductive groups,

The underlying setting is a weight function $\mu_{k,a}$ on $\mathbb{R}^N$ whose radial part is a power of the radius and whose spherical part is a product of powers of linear functions vanishing on the mirrors of a Coxeter group. The operators $\mathcal{I}_{k,a}(z)$ forms a holomorphic semigroup on $L^2(\mathbb{R}^N, \mu_{k,a}dx)$ with respect to a complex parameter z for $\operatorname{Re} z > 0$, and k is a multiplicity function for the Dunkl operators. The last parameter $a > 0$ arises from the $\mathfrak{sl}(2, \mathbb{R})$-interpolation of the Schrödinger model of the Segal–Shale–Weil representation of the double covering of the symplectic group $Sp(N, \mathbb{R})$ and the minimal representation of the conformal group $O(N + 1, 2)$.

The boundary value of the semigroup $\mathcal{I}_{k,a}(z)$ for $\operatorname{Re} z = 0$ gives a family of unitary operators, in particular, $\mathcal{F}_{k,a} := \mathcal{I}_{k,a}(\sqrt{-1}\pi/2)$ includes the Dunkl transform $(a = 2)$ and the Fourier transform $k \equiv 0$ and $a = 2$, and Hankel-type transform $k \equiv 0$ and $a = 1$. In the $k \equiv 0$ case, the semigroup $\mathcal{I}_{k,a}(t)$ generalizes the Hermite semigroup for $a = 2$ and the Laguerre semigroup for $a = 1$, studied by R. Howe (1987) and by Kobayashi–Mano (2007), respectively. The authors established the inversion formula, the Plancherel theorem, the Hecke identity, the Bochner identity, and a Heisenberg uncertainty relation for the unitary operators $\mathcal{F}_{k,a}$ generalizing classical results for Fourier transforms and Dunkl transforms. Among others, a special case gives a representation-theoretic proof of two classical integral formulae for Bessel functions, and also another special case relates to the rational Cherednik algebra.