The 2006 Osaka Science Prize Citation

The Osaka Science Committee has decided to award the 2006 Osaka Science Prize to

Professor Toshiyuki Kobayashi
(RIMS, Kyoto University)

for his discovery of the theory of discontinuous groups beyond Riemannian geometry, and discovery of the theory of discretely decomposable branching laws for infinite-dimensional representations.

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In the late 1980s, the winner pioneered the theory of discontinuous groups for non-Riemannian geometry, the first systematic study in this direction. He discovered and analyzed non-Riemannian phenomena concerning the global nature of locally homogeneous spaces of high dimension, and established the foundations of the theory single-handed. His achievements opened up a new area of research at the interface between geometry and Lie theory.

A major trend in 20th century geometry has been from local to global study of geometries, with some remarkable achievements, particularly in Riemannian geometry. In contrast, in areas such as Lorentz geometry, familiar to us as the spacetime of relativity theory, and more generally in pseudo-Riemannian geometries, as well as in various other kinds of geometry (symplectic geometry, complex geometry, etc.) going beyond the Riemannian case, difficulties were encountered in the investigation of global properties, even when a locally homogeneous structure was imposed.

The prize-winner first provided a theoretical solution to the mystery of the Calabi-Markus phenomenon by giving a necessary and sufficient condition that used the notion of ``rank". He then developed a method in indefinite Riemannian homogeneous manifolds to decide whether the isometric action of a discrete group is properly discontinuous or not. Further, building on this method, the prize-winner made fundamental contributions to the global theory of locally homogeneous spaces in the non-Riemannian case. These include his discovery of a criterion for the finiteness of the fundamental group, a breakthrough on the existence problem for compact/finite-volume space forms, his theory of rigidity/continuous deformation of locally non-Riemannian symmetric spaces, and his discovery of the discontinuous duality theorem.

Since the mid 1990s, Kobayashi's pioneering work on the global theory of locally non-Riemannian homogeneous spaces has been closely followed by the world's top experts in various areas such as representation theory, Lie theory, ergodic theory, harmonic maps, topology, and differential geometry. As a result, his work has been significantly developed by taking the methods and ideas from these various areas of mathematics. Despite his young age, the prize-winner has become a world leader and an influential contributor to those developments.

The second important achievement of the prize-winner concerns the restriction theory of unitary representations. Induction and restriction are two major themes in representation theory. Branching problems ask how an irreducible representation of a group decomposes when restricted to a subgroup. Its description is called the branching law. While there has been extensive study of the induction of representations (e.g., analysis on homogeneous spaces) for more than 50 years now, very little was known about branching problems for the restriction, in particular, the fundamental case of branching problems for reductive Lie groups, except for compact groups. In fact, at that time, many experts were pessimistic about the prospects for a substantial understanding of branching problems for unitary representations.

By building an analogue of his theory for discontinuous groups and transferring it to infinite-dimensional representations by geometric construction, the prize-winner discovered a strange phenomenon concerning branching laws, namely, that they do not contain continuous spectra. This phenomenon became an epoch-making prototype for discrete branching laws in a general setting, where the restriction was considered for non-highest-weight representations with respect to non-compact subgroups.

The prize-winner analysed this discovery in full generality by using micro-local analysis and algebraic representation theory, and established the theory of discrete branching laws. Kobayashi's achievement brought about a substantial breakthrough in the restriction theory of unitary representations, and opened up various directions of research on branching problems. His theory of discretely decomposable restrictions has also found applications, not only to representation theory itself, but also to other areas such as modular varieties in locally symmetric spaces and non-commutative harmonic analysis (e.g. construction of new discrete series representations).