

教授 (Professor)

小林 俊行 (KOBAYASHI Toshiyuki)

A. 研究概要

2018–2022 の 5 年間に於いては、主に以下の 3 テーマの理論構築を行い、総計で約 1,200 ページの論文を著した。また数学の未解決問題の Special Volume の分担執筆を行った [11]。

1. 緩増加等質空間

リー群 G が多様体 X に作用しているとき、 X 上の half density bundle の L^2 切断のなすヒルベルト空間 $L^2(X)$ 上に定まるユニタリ表現の基本問題として、 $L^2(X)$ がいつ緩増加表現になるかという問題を提起し、数学の異分野との新しい繋がりを追求する研究を開始した (Y. Benoist と共同)。まず、 X が簡約型等質空間 $X = G/H$ に対して、力学系のアイデアを用いて、 $L^2(X)$ の行列要素の L^p 評価を与え、特に、 $L^2(X)$ が緩増加となるための幾何的な必要十分条件を決定した [J. Euro. Math. 2015]。さらに、 X が簡約型とは限らない一般の場合に、群作用をもつ測度空間にある半順序を導入し、力学系の手法と、ユニタリ表現論の手法を合わせることで、緩増加性の判定法を証明した (第 2 論文 [1])。第 3 論文 [2] で非緩増加な実簡約型等質空間の完全な分類を与え、第 4 論文 [3] では、緩増加性という解析的な性質、リー代数の極限に関する位相的性質、古典極限としての余随伴軌道の幾何的性質、凸多面体の組合せ論的性質の 4 つが同値であることを証明した。また fusion rule への応用を [4] で著した。

2. 対称性破れ作用素の構成と分類問題

簡約リー群の無限次元表現の「分岐則」に関して、定性的理論から定量的理論に深化させるプログラムを提起した [2]。

2.A. (定性的理論) 離散的な分岐則の理論の要となる K' -admissibility の十分条件 ([Ann. Math., 1998] の主定理の 1 つ) が、実は必要十分条件であることをシンプレクティック幾何の手法を用いて証明を与えた [Kostant 追悼論文, 2021]。

2.B. (定性的理論 2) 無限次元表現を部分群に制限したときの重複度が有限・一様有界になるための判定条件を無限次元表現の“大きさ”の言葉を用いて与え [5, 6, 9, 10]、特に、対称対に周期をもつ表現の別の対称対への表現が一様重複度をもつケースを可視的作用の理論を援用して分類した。

2.C. (定量的理論 1—対称性破れ作用素) 無限次元表現に対する対称性破れ作用素を、非局所作用素まで含めて構成し、分類するプログラムを提唱し [7]、最初の重要な応用例として、2 つの平坦な多様体の微分形式に関する対称性破れ作用素の分類理論の国際研究を主導し、総計 650 頁の長編論文で完成させた (スカラー値は Memoirs of AMS, 2015; 行列値の対称性破れ微分作用素の分類は Lect. Notes. Math. **2170** (2016)、これらを基盤とした完全な分類は著書 [2])。

2.D. (定量的理論 2—ホログラフィック変換) 対称性破れ作用素の族の双対として“ホログラフィック変換”の概念を導入した [Ann. Inst. Fourier, 2020]。また擬リーマン空間形の離散系列表現の分岐則における離散スペクトラムをホログラフィック変換を用いて構成し、完全に決定した [単著, Adv. Math., 2021]。

3. 不連続群

筆者の長年のモチーフである「リーマン幾何学の枠組を越えた不連続群」について、新しい幾何に対するスペクトル理論の構築に初めて踏み込んだ。幾何学的な準備として、離散群の作用の不連続性を量的に評価する sharpness という概念を導入し、高次元タイヒミュラー空間上で安定な離散スペクトラムを構成し、長編の論文 [Adv. Math.] を出版した。さらに、長編の第 2 論文 [JLT2019] および [Progr. Math. 2017] で隠れた対称性を用いた微分作用素環の構造定理を証明し、それを標準的な擬リーマン局所対称空間のスペクトル解析に活用した [8]。

For the last five years (–2023), I have been working on the following research topics.

1. Tempered homogeneous spaces

This is a challenge to the global analysis on homogeneous spaces beyond symmetric spaces. Jointly with Y. Benoist [J. Euro. Math. '15], we proved a criterion for L^p -temperedness of the regular representation on G/H in the generality that $G \supset H$ are pair of reductive groups, and in [1] for general H . A complete description of nontempered homogeneous spaces G/H with $H \subset G$ reductive has been accomplished in [2], and a further connection with other dis-

ciplines of mathematics has been explored in [3]. Further references include [4].

2. Restriction of representations: symmetry breaking operators

Branching problems ask the behavior of the restriction of irreducible representations to subgroups. I proposed in [Progr. Math., 2015] a program to advance branching problems for reductive groups, see [11] for further perspectives.

2.A Concerning the discretely decomposability of the restriction of representations, I proved in [Kostant Memorial Vol., 2021] by using symplectic geometry, the converse of one of the main theorems in my earlier paper [Ann. Math., 1998] based on microlocal analysis.

2.B I formulated and proved a criterion for finite multiplicity/bounded multiplicity of the restriction of ‘small’ infinite-dimensional representations to reductive subgroups in [5, 6, 9, 10]. In particular, I classified the triples (G, H, G') such that (G, H) is a symmetric pair and that any irreducible H -distinguished representations have bounded multiplicity when restricted to another symmetric pair (G, G') .

2.C With B. Speh, I classified *symmetry breaking operators* (SBOs) of principal series for a pair of Lorentz groups (Memoirs of AMS 2015 and book [2]), which give the first successful for the complete classification of SBOs. A part of this work is extended to higher rank case.

2.D As an “inversion” of the symmetry breaking, I introduced the concept of **holographic transform** in [Adv. Math. 2021] and in a joint paper with Pevzner ([Ann. Inst. Fourier 2020]).

3. Analysis on locally symmetric spaces—beyond the Riemannian case

Developing my long motif on discontinuous groups beyond the Riemannian case, I initiated the study on global analysis on locally non-Riemannian symmetric spaces with F. Kassel in [Adv. Math. 2016] and proved the existence of “stable spectrum” under small deformation of discontinuous groups. Further new ideas are proposed in [Progr. Math. 2017], [JLT2019],

and [8].

B. 発表論文

(論文は 2022 年以降のものを記載する． 2018 年～2021 年の論文は，過去の Annual Report の各年度に記載．)

1. Y. Benoist and T. Kobayashi, Tempered homogeneous spaces II, In: Dynamics, Geometry, Number Theory: The Impact of Margulis on Modern Mathematics (eds. D. Fisher, D. Kleinbock, and G. Soifer), pp. 213–245, The University of Chicago Press, 2022.
2. Y. Benoist and T. Kobayashi. Tempered homogeneous spaces III. Journal of Lie Theory, **31**, (2021), pp. 833–869.
3. Y. Benoist and T. Kobayashi, Tempered homogeneous spaces IV. Journal of the Institute of Mathematics of Jussieu, pages 1–28, 2022. DOI: 10.1017/S1474748022000287. Published online, First View: 07 June, 2022.
4. Y. Benoist, Y. Inoue, and T. Kobayashi, Temperedness criterion of the tensor product of parabolic induction for GL_n , Journal of Algebra, **617** 1–16, 2023. DOI: 10.1016/j.jalgebra.2022.10.029.
5. T. Kobayashi. Bounded Multiplicity Theorems for Induction and Restriction. Journal of Lie Theory, **32**, (2022), pp. 197–238.
6. T. Kobayashi, Multiplicity in restricting small representations, preprint. Proc. Japan Acad. **98**, Ser. A Math. Sci., (2022), pp. 19–24.
7. T. Kobayashi, Recent advances in branching problems of representations, To appear in Sugaku Expositions, Amer. Math. Soc. arXiv: 2112.00642.
8. F. Kassel and T. Kobayashi. Spectral analysis on standard locally homogeneous spaces, preprint, 69 pages, ArXiv: 1912.12601.
9. T. Kobayashi, Multiplicity in Restricting Minimal Representations, In: Lie Theory

and Its Applications in Physics (eds. V. Dobrev). Springer Proceedings in Mathematics & Statistics, vol **396**. Springer, Singapore, 2023, pp. 3–20.

10. T. Kobayashi, Bounded multiplicity branching for symmetric pairs, Preprint. 32 pages. Journal of Lie Theory, **33** (1) (2023), 305–328, Special Volume for Karl Heinrich Hofmann. arXiv: 2210.13146.
11. T. Kobayashi, Conjectures on reductive homogeneous spaces, 19 pages. To appear in “Mathematics Going Forward”, Lecture Notes in Mathematics, vol.2313, Springer-Nature, 2023.

著書:

1. T. Kobayashi and B. Speh. Symmetry Breaking for Representations of Rank One Orthogonal Groups II, Lecture Notes in Mathematics. **2234** Springer, 2018, xv+342 pages. ISBN:978-981-13-2900-5.
2. T. Kobayashi. 分担執筆, 解説:リー群の表現論における最近の進展, In: 杉浦光夫『ユニタリ表現入門』, pages 214–242. 東京図書, 2018.

C. 口頭発表

1. “Visible actions” and “only one” — Geometric structure that produces multiplicity-free representations. 大学院数理科学研究科設立 30 周年記念講演 (The 30th Anniversary Ceremony of the Foundation of the Graduate School of Mathematical Sciences). The University of Tokyo, 15 October 2022.
2. On the Crossroads of Global Analysis and Representation Theory. Geometry, Analysis, and Representation Theory of Lie Groups (organized by Y. Oshima, H. Sekiguchi, T. Kubo, T. Okuda, Y. Tanaka, and M. Kitagawa). The University of Tokyo, 5–9 September 2022.
3. Harish-Chandra’s admissibility theorem and beyond. (**3.A.** と **3.B.** では講演タ

イトル, 内容は異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.)

3.A. Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil — A satellite conference of the virtual ICM 2022 (organized by Hung Yean Loke, Tomasz Przebinda, Angela Pasquale, and Binyong Sun). the Institute for Mathematical Sciences, National University of Singapore, Singapore, 9 July 2022. **3.B.** Bounded multiplicity in the branching problems of “small” infinite-dimensional representations, 5 October 2021. リー群論・表現論セミナー (オンライン), 東京大学.

4. Proper Actions and Representation Theory. (**4.A.**–**4.G.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので 1 つにまとめる.) **4.A.** Properness criterion. **4.B.** Discontinuous group, Weil’s local rigidity, and deformation. **4.C.** Tempered Subgroups and tempered homogeneous spaces. (**4.A.**, **4.B.**, **4.C.** は Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil — A satellite conference of the virtual ICM 2022 の 4 連続講演のうちの 3 つ). The Institute for Mathematical Sciences, National University of Singapore, Singapore, 1–15 July 2022. organized by Hung Yean Loke, Tomasz Przebinda, Angela Pasquale, and Binyong Sun. **4.D.** Discontinuous dual and properness criterion (25 April, 2022) **4.E.** The Mackey analogy and proper actions (2 May, 2022) **4.F.** Tempered subgroups a la Margulis (9 May, 2022) **4.G.** Tempered homogeneous spaces (16 May 2022) (**4.D.**–**4.G.** は Proper Actions and Representation Theory. Mini-courses of Mini-lectures (Organizers: Pierre Clare, Nigel Higson and Birgit Speh) における 4 つのテーマの講演, AIM Research Community: Representation Theory & Noncommutative Geome-

- try, online), 25 April–16 May 2022.
5. Tempered Homogeneous Spaces. (**5.A.**–**5.F.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので1つにまとめる.) **5.A.** Symmetry in Geometry and Analysis (organized by M. Pevzner and H. Sekiguchi). Reims University, France, 6–10 June 2022. **5.B.** 緩増加な等質空間 (Tempered Homogeneous Spaces). 日本数学会年会函数解析学分科会特別講演 (慶応大学, オンライン), 16 March 2021. **5.C.** Limit Algebras and Tempered Representations. (opening lecture). RIMS Workshop: Lie Theory, Representation Theory and Related Areas. (online), 10 August 2021. **5.D.** Limit Algebras and Tempered Representations. (plenary opening lecture). XIV. International Workshop: Lie Theory and Its Applications in Physics. Bulgaria (online), 20–26 June 2021. **5.E.** Limit algebras and tempered representation. Lie Groups and Representation Theory Seminar. The University of Tokyo, 15 June 2021. **5.F.** This is What I do: Limit algebras and tempered representations. Representation Theory & Noncommutative Geometry. AIM Research Community (online), 8 April 2021.
 6. Schrödinger model of minimal representations and branching problems. Minimal Representations and Theta Correspondence: (Gordan Savin 教授還暦記念研究集会). (online), The Erwin Schrödinger International Institute for Mathematics and Physics (ESI), 11–15 April 2022.
 7. Regular Representations on Homogeneous Spaces, (**7.A.**–**7.J.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっている所以1つにまとめる.) **7.A.** Is representation theory useful for global analysis on a manifold? — Multiplicity: Approach from PDEs, **7.B.** Tempered homogeneous spaces and tempered subgroups — Dynamical approach, **7.C.** Classification theory of non-tempered G/H — Combinatorics of convex polyhedra, **7.D.** Tempered homogeneous spaces — Interaction with topology and geometry, Analysis on Homogeneous Spaces (4 lectures). Noncommutative Geometry and Analysis on Homogeneous Spaces. Williamsburg, USA, 16–20 January 2023. **7.E.** Basic Questions in Group-Theoretic Analysis on Manifolds. MATH-IMS Joint Pure Mathematics Colloquium Series. The Chinese University of Hong Kong, 25 November 2022. **7.F.** A Foundation of Group-theoretic Analysis on Manifolds. Colloquium di dipartimento. Dipartimento di Matematica, Università di Roma “Tor Vergata” (online), 18 February 2021. **7.G.** Representation Theory of Reductive Groups from Geometric and Analytic Methods (in honour of Simon Gindikin). Kavli IPMU, Japan, 27–28 January 2020; **7.H.** Regular Representations on Homogeneous Spaces. (plenary lecture). International Workshop: Lie Theory and Its Applications in Physics (LT-13). Varna, Bulgaria, 17–23 June 2019; **7.I.** Regular Representations on Homogeneous Spaces. (opening lecture). RIMS Workshop: Developments in Representation Theory and Related Topics (organizer: Yoshiki Oshima). RIMS, Kyoto University, 9–12 July 2019; **7.J.** Regular Representations on Homogeneous Spaces. Dynamics of Group Actions (Yves Benoist 教授還暦記念研究集会). Cetraro, Italy, 27–31 May 2019.
 8. F-method for Constructing Symmetry Breaking Operators. (**8.A.** と **8.B.** は講演タイトルも内容は異なるが, 関連したテーマなので1つにまとめる.) **8.A.** Finite Multiplicity Theorems and Real Spherical Varieties. 松本久義氏還暦記念研究集会, (opening lecture) Tokyo, March 27–29, 2019. **8.B.** Holographic Transform, 20 August,

- 2021, Workshop on "Actions of Reductive Groups and Global Analysis (Online Tambara), August 17-21, 2021.
9. Global Geometry and Analysis on Locally Symmetric Spaces—Beyond the Riemannian Case. (**9.A.**–**9.K.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので1つにまとめる.) **9.A.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium, National University of Singapore. (online), 13 August 2021. **9.B.** Sound of an anti-de Sitter manifold. (opening lecture). Inaugural Day of the French–Kazakhstan school of Mathematics. (online), 25 June 2021. **9.C.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Seminar. University of Padova, Italy, 3 June 2019. **9.D.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium. Oklahoma State University, 3 May 2019. **9.E.** Global Analysis of Locally Symmetric Spaces with Indefinite-metric. Colloquium. Yale University, USA, 17 April 2019. **9.F.** Semisimple Symmetric Spaces and Discontinuous Groups: What I Learned from Professor Toshio Oshima. 大島利雄先生古希記念研究集会. Josai University, Tokyo, Japan, 26-27 December 2018. **9.G.** “Geometric Quantization and Applications” M. Vergne 教授記念集会. Luminy, France, 8-12 October 2018. **9.H.** Symposium on Representation Theory 2018. Tottori, Japan, 13-16 November 2018. **9.I.** Colloquium. Hiroshima University, Japan, 6 November 2018. **9.J.** (plenary lecture). The 65th Geometry Symposium. Tohoku University, Sendai, Japan, 28-31 August 2018. **9.K.** Glances at Manifolds: Aleksy Tralle 教授還暦記念研究集会. the Jagiellonian University, Krakow, Poland, 2-6 July 2018.
10. Branching Laws for Infinite Dimensional Representations of Real Lie Groups; Symmetry Breaking Operators. (**10.A.**–**10.F.** では講演タイトル, 内容は個々に異なるが, 大きなテーマとしては繋がっているので1つにまとめる.) **10.A.** Branching Problems and Symmetry Breaking Operators. Geometry, Symmetry and Physics. Yale University, USA, 23 April 2019. **10.B.** Journées SL2R de théorie des représentations et analyse harmonique (Hubert Rubenthaler 教授退官記念研究集会). I.R.M.A., University of Strasbourg, France, 22-23 March 2018. **10.C.** Joachim Hilgert 教授還暦記念研究集会. Paderborn, Germany, 23-27 July 2018. **10.D.** (plenary lecture). the 32nd International Colloquium on Group Theoretical Methods in Physics (Group32). Czech Technical University, Prague, Czech Republic, 9-13 July 2018. **10.E.** Representation theory, geometry, and quantization: the mathematical legacy of Bertram Kostant (Kostant の追悼国際研究集会). MIT, USA, 28 May-1 June 2018. **10.F.** A Program for Branching Problems in the Representation Theory of Real Reductive Groups: Classification Problem of Symmetry Breaking Operators. Representation Theory inspired by the Langlands Conjectures, in connection with the AMS-AWM Noether lecture by Birgit Speh. Denver, USA, 17 January 2020.
- D. 講義
1. 数理科学概論 I (文科学部) (教養学部文科 1,2 年生, S セメスター, 対面): フェルミ推定, 微積分, Taylor 展開, 偏微分, Lagrange の未定乗数法, 近似と概算, 微分方程式の初歩, 多変数関数の積分を講義した.
 2. リー環論/幾何学 XH (数理大学院・4 年生 共通講義, A セメスター, 対面): I started basic theory of Lie algebras such as Levi decomposition, Cartan decomposition, root

space decomposition, and a more recent advanced topic such as “limit algebra”. After explaining the corresponding global structures of Lie groups and geometries of homogeneous spaces, I discussed the state of art on the unitary representations of reductive groups G on $L^2(X)$ on homogeneous spaces X with emphasis on the limit algebras and the tempered criterion developed by Benoist–Kobayashi (Tempered homogeneous spaces I–IV, 2015–2022).

3. 数学講究 XB (数理科学概説)「不連続群の幾何学と大域解析」, (理学部数学科 4 年生), 2022 年 6 月 22 日.
4. 数学講究 XA, S セメスター: テキスト “Heat Kernels and Dirac Operators” (Berline, Getzler, Vergne) (理学部数学科 4 年生)
5. Proper Actions and Representation Theory. Mini-courses of Mini-lectures, AIM Research Community: Representation Theory & Noncommutative Geometry (online), 25 April–16 May 2022, USA:
In the series of four lectures, I introduced some topics on proper actions with emphasis on their relation to representation theory. No special background knowledge will be required. The lectures will be short without details, but with elementary examples so that they fit into teatime. The lectures are loosely related but are mostly independent of one another. The first two topics are of more geometric nature, and the last two are of more analytic nature. Here are the subtitles of the lectures.
(1) Discontinuous dual and properness criterion.
(2) The Mackey analogy and proper actions.
(3) Tempered subgroups a la Margulis.
(4) Tempered homogeneous spaces.
6. Proper Actions and Representation Theory. Distinguished Lecture Series. The Institute for Mathematical Sciences, National University of Singapore, Singapore, 1–15

July 2022.

In the series of lecture, I explained some recent topics on proper actions with emphasis on their relation to representation theory. I begin with some geometric problems of group actions including properness criterion for reductive homogeneous spaces. In turn, I introduce a “quantification” of proper actions and bring geometric ideas to analytic representation theory such as temperedness criterion. Basic notion will be illustrated by examples. No special background knowledge will be required.

7. Analysis on Homogeneous Spaces at mini-course. Winter School: Noncommutative Geometry and Analysis on Homogeneous Spaces. Williamsburg, USA, 16–20 January 2023.

I delivered four lectures on some foundational progress in recent years about “Analysis on homogeneous spaces” to graduate students, posdoc researchers, and experts. We discuss what kind of geometry X guarantees a good control of the transformation group on the function space, and the answer brings us naturally to the notion of spherical varieties/real spherical manifolds in f Lecture 1 by using PDEs. In Lectures 2–4 we discussed spectrum of the unitary representation $L^2(X)$ with emphasis on the temperedness criterion. We employ “dynamical approach” in Lecture 2, combinatorics approach in Lecture 3, and “limit algebras” and viewpoints from “geometric quantization” in Lecture 4.

E. 修士・博士論文

1. (課程博士) 里見貴志 (SATOMI Takashi): Refinement of Young’s convolution inequality on locally compact groups and generalizations of related inequalities (局所コンパクト群上の Young の畳み込み不等式の精密化と関連の不等式の拡張).
2. (修士) 樋川達郎 (HIKAWA Tatsuro): Lie

- 代数 $\mathfrak{sl}(2, \mathbb{R})$ の微分作用素による表現の変形パラメータの反転, および関連するスペクトル分解の収束公式.
3. (修士) 西畑みなみ (NISHIHATA Minami): $GL(n)$ の有限次元表現に対する微分対称性破れ作用素の構成と複素射影空間上のベクトル束値の調和形式の具体的記述への応用.
- F. 対外研究サービス
1. Kavli IPMU(数物宇宙連携機構), 上席科学研究員併任 (2009.8–2011.5); 主任研究員 (Principal Investigator) 併任 (2011.6–2022.3); 連携研究員 (2022.4–).
- [ジャーナルのエディター]
2. Editor in Chief, Japanese Journal of Mathematics (日本数学会, Springer-Nature) (2006–)
 3. Editor, International Mathematics Research Notices (Oxford 大学出版) (2002–2021)
 4. Editor in Chief, Takagi Booklet, vol. 1–22 (日本数学会) (2006–)
 5. Editor, Geometriae Dedicata (Springer) (2000–)
 6. Editor, Advances in Pure and Applied Mathematics (de Gruyter) (2008–)
 7. Editor, International Journal of Mathematics (World Scientific) (2004–)
 8. Editor, Journal of Mathematical Sciences, The University of Tokyo (2007–)
 9. Editor, Kyoto Journal of Mathematics (2010–)
 10. Editor, Representation Theory (アメリカ数学会) (2015–2019)
 11. Editor, AMS Translation Series (アメリカ数学会) (2016–)
 12. Editor, Tunijian Journal of Mathematics (2017–)
 13. Editor, Special Issue in commemoration of Professor Kunihiko Kodaira's centennial birthday (J. Math. Sciences, The University of Tokyo).
 14. Editor, Special Issue in honor of Professor Masaki Kashiwara's 70th birthday (Publ. RIMS) 2017–2021.
 15. Chief Editor, Mikio Sato's Collected Papers, (Springer-Nature).
 16. 共立出版, 『共立講座 数学探検 (全 18 巻)』, 『共立講座 数学の魅力 (全 14 巻 + 別巻 1)』, 『共立講座 数学の輝き (全 40 巻予定)』の 3 シリーズ編集委員
 17. 編集委員, 数学の現在 i, e, π , (with 斎藤毅, 河東泰之), 東京大学出版会.
- [学会・他大学の委員など]
18. ある国際数学者賞の授賞委員会: Prize Committee (International Prize, 数学部門) 2018, 国外.
 19. ある国際賞の授賞委員会: Prize Committee (mathematics), 2019 および 2020, 国外.
 20. ICM2022 における招待講演者および Plenary lecturers の Selection Committee の責任者 (Chair, Lie Theory and its generalizations, ICM2022), 2019–2022.
 21. ある国際賞 (数学部門) の授賞委員会: Prize Committee (mathematics), 2020–2021, 国外.
 22. ある国内の数学の賞 (複数) の授賞委員会 (anonymous, various years).
 23. 京都大学数理解析研究所運営委員 (2015–2017; 2017–2019).
 24. 京都大学数理解析研究所専門委員 (2007–2009; 2009–2011; 2015–2017; 2017–2019; 2021–).
 25. 科学研究費等の審査委員: 日本 (JSPS), 米国 (NSF-AMS), EU, ドイツ, ルクセンブルク, 中華人民共和国・香港 (various years).
 26. OIST の数学部門における国際 Advisory Board (2021–).
- [国際研究集会のオーガナイザーなど]
27. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), August 17–21, 2022 (オンライン).
 28. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), August 17–21, 2021(オンライン)

- ン).
29. オーガナイザー, Integral Geometry, Representation Theory and Complex Analysis, Kavli Institute for the Physics and Mathematics of the Universe, 27–28 January 2020.
 30. オーガナイザー, 高木レクチャー, 第 24 回 (東京大学 IPMU, 2019 年 12 月); 第 23 回 (京都大学数理研, 2019 年 6 月) (with Y. Kawahigashi, T. Kumagai, H. Nakajima, K. Ono and T. Saito).
 31. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, (virtual 玉原国際セミナーハウス), 18–22 August 2020 (オンライン).
 32. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, 玉原国際セミナーハウス, 20–24 August 2019.
 33. オーガナイザー, Summer School on Representation Theory, リー群の群作用と大域解析に関するセミナー, 玉原国際セミナーハウス, 19–23 August 2018.
 34. オーガナイザー, 高木レクチャー, 第 21 回 (京都大学数理研, 2018 年 6 月), 第 22 回 (東京大学, 2018 年 11 月) (with Y. Kawahigashi, H. Nakajima, K. Ono and T. Saito)
 35. オーガナイザー, リー群論・表現論セミナー (2007–present 東大; 2003–2007 RIMS; 1987–2001 東大)
- G. 受賞・受章
1. Doctrat Honoris Causa (University of Reims), 2022, France.
 2. 日本数学会出版賞 (2019) 『数学の現在 i, e, π 』東京大学出版会, (斎藤毅氏, 河東泰之氏との共同受賞).
 3. アメリカ数学会フェロー (2017) 「簡約リー群の構造論と表現論に対する貢献」 (Contribution to Structure Theory and Representation Theory of Reductive Lie groups).
 4. 紫綬褒章 (Medal with Purple Ribbon)(2014) 「数学研究」.
5. [学生の受賞] 東京大学学位記授与式における総代, 甘中一輝 (2020 年度 (2021 年 3 月), 総代・答辞), 田森宥好 (2019 年度 (2020 年 3 月), 総代).
- H. 海外からのビジター
1. Jonathan Ditlevsen (Aarhus University, Denmark) Mentor as a Ph. D. study on Symmetry Breaking Operators, (短期留学), 29 August –26 November 2022.
 2. F. Kassel (IHES, France), Delivered an invited lecture at the conference “Geometry, Analysis, and Representation Theory of Lie groups”. The University of Tokyo, Japan, 5–9, September 2022.
 3. M. Pevzner (University of Reims, France), Delivered an invited lecture at the conference “Geometry, Analysis, and Representation Theory of Lie groups”. The University of Tokyo, Japan, 5–9, September 2022.
 4. B. Speh (Cornel, USA), Delivered an invited lecture at the conference “Geometry, Analysis, and Representation Theory of Lie groups”. The University of Tokyo, Japan, 5–9, September 2022.
 5. J. A. Wolf (University of California, Berkeley, USA), Delivered an invited lecture at the conference “Geometry, Analysis, and Representation Theory of Lie groups”. The University of Tokyo, Japan, 5–9, September 2022.