

教授 (Professor)

小林 俊行 (KOBAYASHI Toshiyuki)

A. 研究概要

1. 非対称空間の大域解析

非対称空間上の大域解析は殆ど未知の世界である。その研究の基盤として、以下の基礎理論の構築を手がけた。

1.A. (実旗多様体の解析的理論) 誘導表現の既約分解(プランシェル型定理) および表現の制限の既約分解(分岐則)における重複度の有限性および一様有界性に関する判定条件を、それぞれ実旗多様体および複素旗多様体の幾何的な言葉で与え、超局所解析の手法を用いて証明した ([4]; 大島利雄と共同, Adv. Math. 2013) .

1.B. (実旗多様体の分類理論) 無限次元表現の分岐則において重複度が常に有限となるような対称対を **1.A** の判定条件を用い、リー理論の手法を用いて分類した (松木敏彦と共同 [7]).

1.C. 対称空間の調和解析において有効である偏微分方程式系の理論の代替物として、幾何学的な測度の評価式を用い、非対称空間の正則表現が L^p 緩増加となるための必要十分条件を与えた (Y.Benoist と共同 [1]).

2. 極小表現の大域解析

単純リーモード群の極小表現は、分解・誘導という観点において最も根源的なユニタリ表現の1つであり、多くの代数的研究がなされている。筆者は極小表現をモチーフとする大域解析に焦点を当て、異種の新しい幾何的モデルを通して、極小表現の大きな対称性が数学の異なる分野と結びつくような理論の将来性を提唱し ([Publ. RIMS 2011])、共形幾何を用いた極小表現の構成 ([Adv. Math. 2003]) 以来、約 1000 頁の論文を著し、この分野を主導してきた。

2.A. (シュレーディンガーモデル) ジョルダン代数の共形変換群の枠組で、極小表現の L^2 -モデルを構築した [9]. さらに、二次錐上にフーリエ変換に相当するユニタリ反転変換を決定した (著書 [アメリカ数学会メモワール 1000 号, 2011]).

2.B. (フーリエ変換の変形理論) C 型単純群の極小表現である Weil 表現と D 型単純群の極小表現を連続的に結ぶ複素解析的半群を構成し、長編の論文 [Compositio Math. 2012] で、古典的な Fourier 変換, Hankel 変換, Dunkl 変換, Hermite 半群等が特殊値として現れる作用素の変形理論を与えた (Ben Saïd, Ørsted と共同)。

2.C. (特殊関数) 極小表現から生じる 4 階の微分

方程式を満たす、新しい“特殊関数”の基礎的性質 ([Ramanujan J. 2011], [6] 他).

2.D. (Kostant–閑口対応の量子化) Jordan 代数の共形変換群の極小幕零軌道の量子化として、Fock モデルを一般化して構成した [JFA2012].

2.E. (極小表現の分岐則) ‘極小表現’の半單純対称対に関する分岐則を解析的手法で A 型の場合に完全に決定した [JFA2011].

3. 可視的作用と無重複表現

複素多様体における可視的な作用という概念と無重複性の伝播という視点を導入し (口頭発表 [4]), 無重複表現の統一的な理論をめざしている.

4. 不連続群

私の長年のモチーフである非リーマン対称空間における不連続群に関して、スペクトル理論の新たな研究を行い、その第一歩として安定スペクトラムの理論を発表した ([C. R. Acad. Paris 2011], [arXiv:1209.0475], [14], 口頭発表 [10]).

5. 分岐則の理論

5.A. カテゴリー \mathcal{O} における分岐則の離散性に関する幾何的判定条件を発見し [Transf. Groups 2012], さらに “特異ベクトル” を微分方程式によって求める手法 (‘ F -method’) を提唱した ([3,10,13], [arXiv:1301.2111]).

5.B. 対称対に制限した時に離散的に分解するのはいつかという問に関して、Zuckerman 加群 [Adv. Math. 2012] および極小表現等 [2] の場合に完全に分類した (with Y.Oshima).

5.C. ローレンツ群の球主系表現の対称性の破れの作用素を完全に分類し、明示式および関数等式を決定した (著書 [15], B. Speh と共同)

1. Analysis on non-symmetric spaces

This is a challenge to the global analysis on homogeneous spaces beyond symmetric spaces.

1.A I introduced a notion of *real spherical manifolds* and established a geometric criterion for finite multiplicities in the induced/restricted representations [Adv. Math. 2013] with T.Oshima,

1.B classified all symmetric pairs that yield finite-multiplicity branching laws in [7].

1.C Jointly with Y. Benoist [1], we proved a criterion for L^p -temperedness of the regular representation on G/H in the generality that $G \supset H$ are pair of reductive groups.

2. Analysis on minimal representations

Minimal representations such as the Weil representation are one of building blocks of unitary representations. I proposed a *geometric approach* to minimal representations, by which we could expect a fruitful theory on global analysis by *maximal symmetries*. It includes a conformal construction of minimal representations with B. Ørsted [Adv.Math.2003]), a theory of *unitary inversion operator* on the L^2 -model that generalizes the Euclidean Fourier transform with G. Mano ([Memoirs of AMS, **1000**, (2011)]), a deformation theory of the Fourier transform in [Compositio Math. 2012], new “special functions” satisfying a certain ODE of *order four* with G. Mano, Hilgert, and Möllers in [Ramanujan J. 2011], and a generalization of the Schrödinger/Fock model [9] among others.

3. Multiplicity-free representations

The paper gives a full proof of the propagation theorem of multiplicity-freeness, which produces various multiplicity-free results as synthetic applications of the original theory of *visible actions* on complex manifolds.

4. Discontinuous groups

Developing my continuing motif on discontinuous groups for non-Riemannian homogeneous spaces, I initiated the study on discrete spectrum on locally non-Riemannian symmetric spaces with F. Kassel [arXiv:1209.0475, 14].

5. Restriction of representations

I accomplished the classification of the triple $(\mathfrak{q}, \mathfrak{g}, \mathfrak{h})$ such that Zuckerman’s derived functor modules $A_{\mathfrak{q}}(\lambda)$ decompose discretely with respect to a reductive symmetric pair $(\mathfrak{g}, \mathfrak{h})$ in [Adv. Math. 2012] and also some other small representations in [2] with Y.Oshima. In the BGG category \mathcal{O} , I developed a theory of discretely decomposable restrictions [Transf. Groups 2012], proposed an effective method to find singular vectors (‘F-method’ [10,13]), and joint with B.Ørsted, V.Souček, P.Somberg, M.Pevzner, and T.Kubo determined explicit formulae of covariant differential operators in various geometric settings ([arXiv:1301.2111], [arXiv:1303.3541], [3]). With B.Speh, I classified symmetry breaking operators of spherical principal series of Lorentz groups [16].

B. 発表論文

1. Y. Benoist and T. Kobayashi: “Temperedness of reductive homogeneous spaces”, 28 pp. To appear in Journal of the European Mathematical Society (JEMS). arXiv: 1211.1203.
2. T. Kobayashi and Y. Oshima: “Classification of symmetric pairs with discretely decomposable restrictions of (\mathfrak{g}, K) -modules”. Crelles Journal (to appear), 19 pp. DOI:10.1515/crelle-2013-0045.
3. T. Kobayashi, T. Kubo, and M. Pevzner. Vector-valued covariant differential operators for the Möbius transformation. Springer Proceedings in Mathematics & Statistics, **111**, (2015) pp. 67–86.
4. T. Kobayashi. Shintani functions, real spherical manifolds, and symmetry breaking operators. Developments in Mathematics, **37**, (2014) pp. 127–159.
5. T. Kobayashi. Symmetric pairs with finite-multiplicity property for branching laws of admissible representations. Proc. Japan Acad., Ser. A, Mathematical Sciences, **90**, pp. 79–83, 2014.
6. T. Kobayashi: “Special functions in minimal representations”, Perspectives in Representation Theory in honor of Igor Frenkel on his 60th birthday, (eds. P. Etingof, M. Khovanov, and A. Savage), Contemporary Mathematics **610**, (2014), 253–266. Amer. Math. Soc.
7. T. Kobayashi and T. Matsuki: “Classification of finite-multiplicity symmetric pairs”, Transformation Groups **19** (2014), 457–493, Special Issue in honour of Professor Dynkin for his 90th birthday.
8. T. Kobayashi and B. Speh: “Intertwining operators and the restriction of representations of rank one orthogonal groups”. C. R. Acad. Sci. Paris, Ser. I, **352**, (2014), 89–94.

9. J. Hilgert, T. Kobayashi, and J. Möllers: “Minimal representations via Bessel operators”, *J. Math. Soc. Japan*, **66**, (2014), 349–414.
10. T. Kobayashi: “F-method for symmetry breaking operators”. *Differential Geometry and its Applications*, **33** (2014), 272–289, Special Issue “Interaction of Geometry and Representation Theory : Exploring New Frontiers” (in honor of Michael Eastwood’s 60th birthday).
11. T. Kobayashi: “Analysis on real spherical manifolds and their applications to Shintani functions and symmetry breaking operators”, *Mathematisches Forschungsinstitut Oberwolfach Report 11* (2014), pp.176–179.
12. T. Kobayashi: Symmetry breaking operators and branching problems, *Symposium on Representation Theory 2014*, (eds. J. Matsuzawa and N. Shimeno).
13. T. Kobayashi, T. Kubo, M. Pevzner, Covariant differential operators and the Rankin–Cohen bracket, *Symposium on Representation Theory 2014*, pp. 75–86, (eds. J. Matsuzawa and N. Shimeno).
14. T. Kobayashi: “局所から大域へ –リーマン幾何を超えた世界で–” *Kavli IPMU News 25* (2014), pp.30–35; (英訳)From “Local” to “Global” –Beyond the Riemannian Geometry; *ibid*, (2014), pp. 4–11.
15. T. Kobayashi, 疑問をおこして考え, そして考え方抜く, 小平邦彦(編)「新・数学の学び方」岩波書店, 2015, pp. 91–115.
- 著書.
16. T. Kobayashi and B. Speh, Symmetry breaking for representations of rank one orthogonal groups, To appear in *Memoirs of American Mathematical Society*, 118 pp. arXiv: 1310.3213.

C. 口頭発表

- Hironaka, H. Ochiai; scientific advisors: Rubenthaler and F. Sato). Rikkyo University, Tokyo, Japan, 1–5 September 2014.
6. Branching, Multiplicities, and Real Spherical Varieties. **6.A.** Group Actions with applications in Geometry and Analysis: in honour of Toshiyuki Kobayashi 50th birthday. Reims, France, 3–6 June 2013. **6.B.** Representations of Reductive Groups Salt Lake City, USA, 8–12 July 2013. **6.C.** 松木敏彦教授還暦記念研究集会. Tottori, Japan, 8–9, February 2014. **6.D.** Representation Theory and Analysis of Reductive Groups: Spherical Spaces and Hecke Algebras Oberwolfach, Germany, 19–25 January 2014. **6.E.** Representation Theory and Analysis of Reductive Groups: Spherical Spaces and Hecke Algebras (organized by Bernhard Krötz, Eric M. Opdam, Henrik Schlichtkrull, and Peter Trapa). Oberwolfach, Germany, 19–25 January 2014.
7. Geometric Analysis on Minimal Representations. **7.A.** Mathematical Physics and Representation Theory (Igor Frenkel 教授 60 歳記念研究集会) (organized by P. Etingof, M. Khovanov, A. Kirillov Jr., A. Lachowska, A. Licata, A. Savage and G. Zuckerman). Yale University, USA, 12–16 May 2012. **7.B.** International summer research school of CIMPA 2013: Hypergeometric functions and representation theory. Mongolia, 5–16 August 2013.
8. Natural Differential Operators in Parabolic Geometry and Branching Laws. **8.A.** The Interaction of Geometry and Representation Theory: Exploring New Frontiers (M. Eastwood 60 歳記念研究集会) ESI, Vienna, 10–14 September 2012. **8.B.** Workshop on the Interaction of Representation Theory with Geometry and Combinatorics. Hausdorff Institute, Bonn, Germany, March 2011. **8.C.** Special day on Lie groups. Utrecht University, the Netherlands, May 2011. **8.D.** (2 lectures), Representa-
- tion Theory XII. Dubrovnik, Croatia, June 2011. **8.E.** Lie Groups: Geometry and Analysis (JSPS/DFG seminar). Paderborn, Germany, September 2011. **8.F.** Symposium on Representation Theory 2012. Kagoshima, Japan, 4–7 December 2012. **8.G.** Workshop on Geometric Analysis on Euclidean and Homogeneous Spaces (S. Helgason 教授 85 歳記念研究集会). Tufts University, USA, January 2012. **8.H.** International Workshop: Lie Theory and Its Applications in Physics (LT-10). Varna, Bulgaria, 17–23 June 2013. **8.I.** Analysis Seminar. Chalmers University of Technology and the University of Gothenburg, Sweden, 14 May 2013.
9. Finite Multiplicity Theorems and Real Spherical Varieties. **9.A.** (closing lecture), Seminar Sophus Lie. Erlangen, Germany, July 2011. **9.B.** Analysis on Lie Groups. Max Planck Institute for Mathematics, Bonn, Germany, September 2011. **9.C.** (closing lecture), Lie Groups, Lie Algebras and their Representations (organized by Joseph Wolf). University of California, Berkeley, USA, November 2011. **9.D.** Branching Laws, IMS, Singapore, March, 2012. Harmonic Analysis Seminar. Charles University in Prague, Czech, 14 December 2012. **9.E.** Harmonic Analysis, Operator Algebras and Representations. Centre International de Rencontres Mathématiques (CIRM), Luminy, France, 22–26 October 2012. **9.F.** Special Program “Branching Laws” (11–31 March 2012). Institute for Mathematical Sciences, NUS, Singapore, 19 March 2012. **9.G.** Workshop on Representations of Lie Groups and their Subgroups (organized by G. Zhang). Chalmers University of Technology, Sweden, 19–20 September 2013.
10. Global Geometry and Analysis on Locally Symmetric Spaces—Beyond the Riemannian Case. **10.A.** (S. S. Chern 生誕 100 周年記念集会). Mathematical Science Research Institute (MSRI) at

Berkeley, California, USA, October 2011.

10.B. Cohomology of Arithmetic Groups (M. S. Raghunathan 教授 70 歳記念研究集会). Tata Institute of Fundamental Research, Mumbai, India, December 2011.

10.C. JSPS-DST Asian Academic Seminar 2013: Discrete Mathematics & its Applications. the University of Tokyo, Japan, 3–10 November 2013.

10.D. Sophus Lie Days. Cornell, USA, 11 October 2013.

10.E. Japan–Netherlands Seminar. Nagoya University, Japan, 26–30 August 2013.

10.F. Hayama Symposium on Complex Analysis in Several Variables XVI. Kanagawa, Japan, 20–23 July 2013.

10.G. (2 lectures), Workshop d’analyse harmonique. Reims, France, 2 November 2012.

10.H. 談話会, Colloquium Lorrain. Université de Lorraine - Metz, France, 16 October 2012.

10.I. 談話会, University of Chicago, USA, May 2011.

10.J. 談話会, IPMU, the University of Tokyo, Japan, December 2011.

10.K. 談話会, Kyushu University, Fukuoka, Japan, 15 January 2015.

10.L. 談話会. Tohoku University, Sendai, Japan, 15 December 2014.

10.M. 談話会. The University of Tokyo, Tokyo, Japan, 11 July 2014.

10.N. Lie Groups: Structure, Actions and Representations (J. Wolf 教授 75 歳記念研究集会). Ruhr-Universität, Bochum, Germany, January 2012.

10.O. Sophus Lie Days. Cornell, USA, 11 October 2013.

10.P. Journée Mathématique de la Fédération de Recherche. Logis du Roy, Amiens, France, 2 July 2013.

10.Q. Colloquium de Mathématiques de Rennes. Institut de Recherche mathématique de Rennes, France, 10 June 2013.

10.R. Colloquium. Chalmers University of Technology and the University of Gothenburg, Sweden, 20 May 2013.

10.S. Workshop: Deformation of Discrete Groups and Related Topics. Nagoya University, Nagoya, Japan, 17–18 February 2015.

D. 講義

1. 2014 夏学期, 火曜 2 限: 数物先端科学 III / Frontiers of Mathematical Sciences and Physics III: 「複素幾何と表現論」複素多様体, 再生核, 双正則変換群, ユニタリ表現の基礎事項を解説し, 可視的作用, 無重複表現の話題を紹介した。
 2. 2014 夏学期, 水曜 1 限: 数学 I, 東京大学教養学部(文系 1, 2 年生). 大きな数, 極限, 指数関数, 微分, テイラー展開, 微分方程式, 偏微分, 区分求積法, 多重積分
 3. 2014 年 5 月 7 日(水) 14:50~15:50: 数学講究 XB (数理科学概説). 「対称性と大域解析」
 4. 集中講義 2014 年 12 月 16–19 日: 「群作用とリ一群の表現論」(代数解析学特選/幾何学通論(修)/幾何学特殊講義 F), 東北大学. 複素多様体における可視的作用の基本的な性質を解説し, 多くの具体例を通して, 群の構造論および表現論の最近の話題を紹介した。
 5. 集中講義 2015 年 1 月 13–16 日: 「リ一群の無限次元表現論」(数理科学特別講義 IX), 九州大学. 誘導・制限に現れるユニタリ表現の分解に関し, 複素多様体上の可視的作用を用いた無重複定理と偏微分方程式系の境界値問題を用いた有限重複度定理を解説した。
 6. 数学講究 XA, 数学特別講究(理学部数学科 4 年生 夏・冬学期)(テキスト: 複素多様体 R.O.Wells, Jr. “Differential analysis on complex manifolds”, および、表現論 R. Howe–E.-C. Tan “Nonabelian harmonic analysis. Applications of $SL(2, \mathbb{R})$ ”)
 7. 集中講義 2014 年 6 月 6 日–11 日, Visible Actions and Multiplicity-free Representations, Summer School on Geometry, Integrability and Quantization. Bulgaria (5 回の講義).
- E. 修士・博士論文
1. (博士) 田中雄一郎 (TANAKA Yuichiro): Visible actions of reductive algebraic groups on complex algebraic varieties (簡

約代数群の複素多様体への可視的作用について)

F. 対外研究サービス

1. Kavli IPMU(数物宇宙連携機構), 上席科学研究员併任 (2009.8–); 主任研究员 (Principal Investigator) 併任 (2011.6–)

[ジャーナルのエディター]

2. Managing Editor, Japanese Journal of Mathematics (日本数学会) (2005–)
3. Editor, International Mathematics Research Notices (Oxford 大学出版) (2002–)
4. Managing Editor, Takagi Booklet, vol. 1–14 (日本数学会) (2006–)
5. Editor, Geometriae Dedicata (Springer) (2000–)
6. Editor, Advances in Pure and Applied Mathematics (de Gruyter) (2008–)
7. Editor, International Journal of Mathematics (World Scientific) (2004–)
8. Editor, Journal of Mathematical Sciences, The University of Tokyo (2007–)
9. Editor, Kyoto Journal of Mathematics (2010–)
10. Editor, Representation Theory (アメリカ数学会) (2015–)
11. Editor, Special Issue in commemoration of Professor Kuihiko Kodaira's centennial birthday (J. Math. Sciences, the University of Tokyo) (2015).
12. 共立出版, 数学叢書, 編集委員

[学会・他大学の委員など]

13. 審査委員: European Research Council (2010–)
14. 京都大学数理解析研究所専門委員 (2007–2009; 2009–2011)

15. 科学研究費等の審査委員: 日本 (JSPS), 米国 (NSF-AMS), EU, ドイツ, ルクセンブルク, 中華人民共和国・香港 (various years)

16. 審査委員: Prize Committee 日本数学会春季賞・秋季賞他 (anonymous) (various years)

17. Jury, Doctor of Philosophy, Paderborn University, Germany (2010)

18. Jury, Doctor of Philosophy, Utrecht University, the Netherlands (2011)

[国際研究集会のオーガナイザーなど]

19. オーガナイザー, Winter School 2015 on Representation Theory of Real Reductive Groups, 東京大学大学院数理科学研究科, 24–26 January 2015. coorganized with Toshihisa Kubo, Hisayosi Matumoto and Hideko Sekiguchi.

20. オーガナイザー, The 14th Takagi Lectures, the University of Tokyo, 15–16 November 2014, coorganized with Yasuyuki Kawahigashi, Hiraku Nakajima, Kaoru Ono and Takeshi Saito.

21. オーガナイザー, Representation Theory and Harmonic Analysis, Oberwolfach, Germany, 14–20 November 2010 (with B. Krötz)

22. Scientific committee, Recent Developments in Harmonic Analysis and their Applications, Marrakech, Morocco, 25–29 April 2011

23. オーガナイザー, Branching Problems for Unitary Representations, Max Planck Institute for Mathematics Bonn, Germany, 25–29 July 2011 (with B. Ørsted and B. Speh)

24. Scientific committee, Harmonic Analysis, Operator Algebras and Representations, CIRM, Luminy, France, 21–26 October 2012

25. オーガナイザー, Representations of Lie Groups and Supergroups, Oberwolfach, Germany, 10–16 March 2013 (with J. Hilgert, K.-H. Neeb and T. Ratiu)

26. オーガナイザー, Session “Representation Theory” in JSPS-DST Asian Academic Seminar 2013: Discrete Mathematics & Its Applications (小谷元子他), the University of Tokyo, Japan, 7 November, 2013.
27. オーガナイザー, Winter School on Representation Theory of Real Reductive Groups, 東大, 15–18 February 2014, (with T. Kubo, H. Matumoto and H. Sekiguchi).
28. Scientific Committee, Visible Actions and Multiplicity-free Representations. XVIIth International Conference on Geometry, Integrability and Quantization. Varna, Bulgaria, 2016.
29. オーガナイザー, 高木レクチャー, 第8回 (京都大学数理研, 2010年11月), 第9回 (京都大学数理研, 2011年6月), 第10回 (京都大学数理研, 2012年5月), 第11回 (東京大学, 2012年11月), 第12回 (東京大学, 2013年5月), 第13回 (京都大学数理研, 2013年11月), 第14回 (東京大学, 2014年5月) (with Y. Kawahigashi, H. Nakajima, K. Ono and T. Saito)
30. オーガナイザー, リー群論・表現論セミナー (2007–present 東大; 2003–2007 RIMS; 1987–2001 東大)

G. 受賞

1. 2015 JMSJ 論文賞 (The JMSJ Outstanding Paper Prize) 極小表現の構成に関する論文「Minimal representations via Bessel operators」に関して (J. Hilgert, J. Möllersとの共同受賞)
2. 紫綬褒章 (Medal with Purple Ribbon)(2014) 数学研究
3. 井上学術賞 (Inoue Prize for Science) (2010) 「無限次元の対称性の解析」(Analysis on infinite dimensional symmetries)
4. [学生の受賞] 大島芳樹. 学生表彰「東京大学総長賞」(2010); 森田陽介. 学生表彰「東京大学総長賞」(2012)

H. 海外からのビジター

1. Peter Trapa (2015, January, Utah University)
2. Raul Gomez (2015, January, Cornell University)
3. Akshay Venkatesh (2014, November, Stanford University)
4. Fanny Kassel (2014, November–December, CNRS, Lille University)
5. Gordan Savin (2014, Utah University)