

# RIMS Workshop: Developments in Representation Theory and Related Topics

Date: 9–12, July, 2019

Location: Room 420, RIMS, Kyoto University

Organizer: Yoshiki Oshima (Osaka University)

## Schedule

### 9(Tue)

13:20 – 14:20 Toshiyuki Kobayashi (University of Tokyo)  
Regular Representations on Homogeneous Spaces

14:35 – 15:35 Taito Tauchi (University of Tokyo)  
A generalized uniformly bounded multiplicity theorem

15:50 – 16:50 Yosuke Morita (Kyoto University)  
Cartan projections of abelian horospherical subgroups and proper actions on homogeneous spaces

### 10(Wed)

9:50 – 10:50 Ryotaro Kawago (Okayama University of Science)  
Multiplicities of points on Schubert varieties in the Symplectic flag variety

11:05 – 12:05 Yuichiro Tanaka (University of Tokyo)  
Visible actions of compact Lie groups on complex spherical varieties

13:20 – 14:20 Takahiro Nagaoka (Kyoto University)  
Strong Lefschetz property and prehomogeneous vector space

14:35 – 15:35 Kazuki Ishimoto (Kobe University)  
Fourier transform for prehomogeneous vector spaces over finite field

15:50 – 16:50 Hideto Nakashima (Nagoya University)  
An algebraic proof for explicit formulas of basic relative invariants of homogeneous cones

## 11(Thu)

9:50 – 10:50 Masaki Hidaka, Minoru Itoh (Kagoshima University)

The Schur polynomials in all  $n$ th primitive roots of unity

11:05 – 12:05 Genki Shibukawa (Kobe University)

Pieri type formulas and difference relations for Jack interpolation polynomials

13:20 – 14:20 Perez Victor (University of Tokyo)

Normality and Duality on Topological Groups

14:35 – 15:35 Masaru Sugawara (Tohoku University)

Convex bases for affine quantum groups and its application for quantum dilogarithm identities

15:50 – 16:50 Noriyuki Abe (University of Tokyo)

On Soergel bimodules

## 12(Fri)

9:10 – 10:10 Takashi Satomi (University of Tokyo)

A refinement of Young's inequality for the convolution product on groups

10:20 – 11:20 Ryosuke Nakahama (University of Tokyo)

Weighted Bergman inner products on subspaces of bounded symmetric domains

11:30 – 12:30 Takuma Hayashi (University of Tokyo)

Half-integrality of the closed  $SO(3)$ -orbit on the flag variety of  $SL_3$

## Abstracts

### **Toshiyuki Kobayashi (University of Tokyo)**

Title: Regular Representations on Homogeneous Spaces

I plan to discuss some basic questions about regular representations on  $X$  acted algebraically by real reductive groups  $G$ .

1. (function spaces) Does the group  $G$  have a “good control” on the space  $C(X)$  of functions on  $X$ ?

2. ( $L^2$  theory) What can we say about “spectrum” for  $L^2(X)$ ?

We highlight “multiplicities” and “temperdness” for these questions, and give their geometric criteria.

If time permits, I will mention some applications to branching problems for restriction of infinite-dimensional representations.

### **Masaki Hidaka, Minoru Itoh (Kagoshima University)**

Title: The Schur polynomials in all  $n$ th primitive roots of unity

We show that the values of the Schur polynomials in all  $n$ th primitive roots of unity are in  $\{1, 0, -1\}$ , if  $n$  has at most two distinct odd prime factors. This result can be regarded as a generalization of properties of the cyclotomic polynomial. Indeed, when the partition is  $(1^k)$ , our result is equivalent to the well-known fact that the coefficients of the  $n$ th cyclotomic polynomial are in  $\{1, 0, -1\}$ , if  $n$  has at most two distinct odd prime factors. Moreover, when the partition is  $(k)$ , our result is equivalent to a similar property of the coefficients of the multiplicative inverse of the  $n$ th cyclotomic polynomial which was proved by P. Moree about 10 years ago.

### **Genki Shibukawa (Kobe University)**

Title: Pieri type formulas and difference relations for Jack interpolation polynomials.

We obtain new Pieri type formulas for Jack polynomials. From these new Pieri formulas, we give Pieri type formulas and difference relations for Jack interpolation polynomials, and their applications.

**Perez Victor (University of Tokyo)**

Title: Normality and Duality on Topological Groups

This talk can be divided essentially in two parts. In the first one, we introduce basic concepts and classical results of the class of topological groups. Concretely, we deal with the Pontryagin Duality Theorem for locally compact abelian groups. In the second one, we present a result of normality using techniques of duality theory of topological groups. A group topology  $\nu$  in a topological group  $(G, \tau)$  is said to be a compatible topology with the duality  $(G, G^\wedge)$  if the dual groups of  $G$  induced by  $\tau$  and  $\nu$  coincide, i.e.  $(G, \nu)^\wedge = (G, \tau)^\wedge$ .

The result we show states that for a concrete group (namely, the product of  $\mathbb{R}$ -copies of  $\mathbb{Z}$ ), there do not exist normal topologies compatible with the duality  $(G, G^\wedge)$  that belongs to a particular family; namely, the family of the locally quasi-convex topologies.

**Ryosuke Nakahama (University of Tokyo)**

Title: Weighted Bergman inner products on subspaces of bounded symmetric domains

Let  $H_r(\mathbb{F})$  ( $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ ) be the space of Hermitian matrices, and we consider the bounded symmetric domain  $D \subset H_r(\mathbb{F})^{\mathbb{C}}$ . In this talk we present a result on the computation of the weighted Bergman inner product on  $D$  of a polynomial on the subspace  $H_{r'}(\mathbb{F}) \oplus H_{r''}(\mathbb{F})$  and an exponential function on  $H_r(\mathbb{F})$ . Also, as an application, we present a result on explicit construction of intertwining operators from representations of  $Sp(r, \mathbb{R})$  to those of the subgroup  $U(r', r'')$ .

**Takuma Hayashi (University of Tokyo)**

Title: Half-integrality of the closed  $\mathrm{SO}(3)$ -orbit on the flag variety of  $\mathrm{SL}_3$

In representation theory over  $\mathbb{C}$  of a real reductive group  $G_{\mathbb{R}}$ , the geometry of  $K$ -orbits on the flag variety of  $G$  over  $\mathbb{C}$  plays an important role. For example, we can attach fundamental series representations of  $G_{\mathbb{R}}$  from closed  $K$ -orbits.

Recently, several people have started to work on rational and integral structures of Harish-Chandra modules. In this talk, I will show that the closed  $\mathrm{SO}(3)$ -orbit on the flag variety of  $\mathrm{SL}_3$  is defined over  $\mathbb{Z}[1/2]$ , motivated by Michael Harris' work on a construction of rational structures of Harish-Chandra modules and Fabian Januszewski's descent theorem for certain Harish-Chandra modules. This is a joint work with Fabian Januszewski.