BRANCHING PROBLEMS IN REPRESENTATION THEORY OF REDUCTIVE LIE GROUPS

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In the series of lectures, I plan to discuss recent progress on branching problems for representations of real reductive Lie groups.

Branching problems ask how irreducible representations π of groups G "decompose" when restricted to subgroups G'. Branching problems include various important special cases such as decomposition of tesor products, theta correspondences, Blattner formulæ, etc. however, it is notorious that wild features such as "infinite multiplicities" and "continuous spectra" may well happen for real reductive groups even if G' is a maximal subgroup in G.

In the first lecture, I plan to give a necessary and sufficient condition on the pair of reductive groups (G, G') for the multiplicities of the restriction $G \downarrow G'$ to be always finite (and also to be of uniformly bounded) by using analysis on real spherical varieties [4, 6, 8]. Further, we discuss "discretely decomposable restrictions" which allow us to apply algebraic tools in branching problems [2, 9].

In the second lecture, I plan to focus on a construction of *symmetry breaking operators* (SBO in short), which are intertwining operators from irreducible representations of G to irreducible representations of the subgroup G'. Some of SBOs are given by integral operators, and some others are by differential operators such as Rankin–Cohen brackets [10, 12] and Juhl's conformal differential operators [1]. We present a new method called F-*method* [4, 7, 10] to construct differential SBOs. If time permits, some classification results [11] will be also presented.

Survey article and textbooks for the lectures

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