Kazhdan–Lusztig polynomials were first defined by Kazhdan and Lusztig in [Invent. Math., 53 (1979), 165–184]. Since then, numerous applications have been found, especially to representation theory and to the geometry of Schubert varieties. In 1987 Deodhar introduced parabolic analogues of these polynomials. These are related to their ordinary counterparts in several ways, and also play a direct role in other areas, including geometry of partial flag manifolds and the theory of Macdonald polynomials.

In this talk I study the parabolic Kazhdan–Lusztig polynomials of the quasi-minuscule quotients of the symmetric group. More precisely, I will first show how these quotients are closely related to “rooted partitions” and then I will give explicit, closed combinatorial formulas for the polynomials. These are based on a special class of rooted partitions the “rooted-Dyck” partitions, and imply that they are always (either zero or) a power of $q$.

I will conclude with some enumerative results on Dyck and rooted-Dyck partitions, showing a connection with random walks on regular trees.

This is partly based on a joint work with Francesco Brenti and Mario Marietti.