Lie Group and Representation Theory Seminar

Title: On the support of the Plancherel measure
Date: February 14 (Wed) 10:00–11:30
Place: 005 RIMS, Kyoto University
Speaker: Joseph Bernstein (Tel Aviv)

Abstract.

In 1970-s Harish Chandra finished his work on harmonic analysis on real reductive groups $G$. In particular, he proved the Plancherel formula for $G$ which describes the decomposition of the regular representation of $G$ as an integral of irreducible unitary representations of the group $G \times G$ (Plancherel decomposition).

The remarkable feature of this formula was the fact that only some of the unitary representations of the group $G$ contributed to this formula (so called tempered representations).

In fact this phenomenon was already known in PDE. Namely in this case it was known that one can describe the spectral decomposition of an elliptic self-adjoint differential operator $D$ in terms of eigenfunctions which have moderate growth (i.e. they almost lie in $L^2$). The general result of this sort was proven by Gelfand and Kostyuchenko in 1955.

In my paper ”On the support of Plancherel measure” (1988) I have applied the ideas of Gelfand and Kostyuchenko and gave an a priori proof of the fact that only tempered representations contribute to the Plancherel decomposition.

Moreover, I have shown that a similar statement holds for decompositions of $L^2(X)$ for a large class of homogeneous $G$-spaces $X$.

Examples are:

(i) $X = G/K$, where $K$ is the maximal compact subgroup
(ii) more generally, $X = G/H$, where $H$ is a symmetric subgroup (subgroup of fixed points of some involution of $G$);
(iii) $X = G/\Gamma$, where $\Gamma$ is a discrete lattice in $G$.
(iv) $G$ a reductive $p$-adic group, $X = G/H$, where $H$ is either an open compact subgroup or a symmetric subgroup.

I have discovered that the corresponding statement depends on some geometric structure on the space $X$ (I called it ”the structure of large scale space”) and that this structure has the same properties in all the cases listed above.

In my lecture I will discuss all these questions.