

# Lie Group and Representation Theory Seminar, Kyoto 2006

Date: September 1 (Fri), 2006, 11:00–12:00

Place: RIMS, Kyoto University : Room 402

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Title: Invariant polynomials and invariant differential operators for multiplicity-free actions of rank 3

Abstract: Let  $V$  be a finite-dimensional vector space over  $\mathbb{C}$ , and  $K$  a compact Lie group acting on  $V$  linearly. We call  $(K, V)$  a *multiplicity-free action* if each irreducible component appears at most one in the (holomorphic) polynomial ring  $\mathcal{P}(V)$  on  $V$ . If  $(K, V)$  is multiplicity-free, then there exists a number  $r$  such that the ring  $\mathcal{P}(V_{\mathbb{R}})^K = \mathcal{P}(V) \otimes \overline{\mathcal{P}(V)}$  of  $K$ -invariant polynomials on the underlying real vector space  $V_{\mathbb{R}}$  of  $V$  is isomorphic to the polynomial ring of  $r$  variables. The number  $r$  is called the *rank* of  $(K, V)$ .

For each highest weight  $\lambda$  which appears in the irreducible decomposition of  $\mathcal{P}(V)$  there exist, up to a scalar, a unique  $K$ -invariant polynomial  $p_{\lambda}(z, \bar{z})$  and a unique  $K$ -invariant differential operator  $p_{\lambda}(z, \partial)$ . In this talk, we describe all  $K$ -invariant polynomials  $\{p_{\lambda}(z, \bar{z})\}$  and  $K$ -invariant differential operators  $\{p_{\lambda}(z, \partial)\}$  for a rank 3 multiplicity-free action  $(K, V)$  which is not derived from a Hermitian symmetric space. Moreover, we give two ‘symmetric’ slices for visibility of the action  $(K, V)$ . We show that the action of the stabilizer of one indicates the symmetry of the  $K$ -invariant polynomials, and that of the other indicates the symmetry of the eigenvalues of the  $K$ -invariant differential operators.