Lie Group and Representation Theory Seminar

Date: December 6 (Mon) 16:30–17:30
Place: RIMS Room 005
Speaker: Jacques Faraut (Paris)
Title: Infinite dimensional harmonic analysis and Polya functions

Abstract:

Spherical pairs, which have been introduced by Olshanski, are inductive limits of Gelfand pairs. For such a pair (G, K),

$$G = \bigcup_{n=1}^{\infty} G(n), \quad K = \bigcup_{n=1}^{\infty} K(n), \quad G(n) \subset G(n+1), \quad K(n) = G(n) \cap K(n+1),$$

and, for each n, $\big((G(n), K(n)\big)$ is a Gelfand pair. For some spherical pairs, the spherical functions are characterized by a multiplicative property, and a class of one variable functions comes in the theory. A basic example is the space of infinite dimensional Hermitian matrices

$$H(\infty) = \bigcup_{n=1}^{\infty} H(n),$$

where $H(n)$ is the space of $n \times n$ Hermitian matrices, for which $K(n) = U(n)$, the unitary group, and $G(n) = U(n) \ltimes H(n)$, the corresponding motion group. A continuous function Φ on $\mathbb{R}$ is said to be a Pólya function if $\Phi(0) = 1$, and if, for every n, the function φ_n, defined on $H(n)$ by $\varphi_n(x) = \det \Phi(x)$, is of positive type. The projective system (φ_n) defines a function φ on $H(\infty)$; this function φ is spherical, and all spherical functions are obtained in that way. The Pólya functions have been determined by Olshanski and Vershik, and also by Pickrell. Surprinsingly, this class of functions has been considered a long time ago by Pólya and Schoenberg in a very different setting.

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