Lie Group and Representation Theory Seminar

Date: April 16 (Fri), 2004, 10:30–11:30
Place: RIMS 402
Speaker: Adam Koranyi (CUNY, USA)
Title: A SIMPLE DESCRIPTION OF THE SYMMETRIC SPACES OF RANK ONE

Abstract:
These are the hyperbolic spaces over $R, C, H, O$, the corresponding four projective spaces, and the sphere. It is usually difficult to make computations in them because $O$ is hard to handle; the alternative way, using the structure theory of semisimple Lie groups, is also relatively complicated. Here a direct description of these spaces will be given, in which everything is fairly easily computable.

A Euclidean space $Z$ determines a Clifford algebra Clif($Z$). We write $C = R1 + Z$ and define a $C$-module as a Euclidean space $X$ with an orthogonal action of Clif($Z$), such that for every non-zero $x$ in $X$, $Cx$ is an invariant subspace. Then the unit ball in $X + C$ can be made in a natural way into a hyperbolic space; a certain compactification of $X + C$ gives the projective spaces and the sphere (which appears as a degenerate projective space). One can work with these without using any classification. One can also study “$C$-lines” and the collineation groups of the projective spaces.