

The 17th Takagi Lectures

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ABSTRACT

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*Categorification of Invariants in Gauge Theory
and Symplectic Geometry*

Categorification of invariants is a topic studied in various branches of geometry. The first proposal of this kind seems to be G. Segal's axiom of conformal field theory, which was in the first half of 1990's. Around the same time, Instanton Floer homology of 3 manifolds appeared and its relation to Donaldson invariant of 4 manifold was discovered. To extend this Donaldson–Floer story to one including 2 dimensional case was studied also in the first half of 1990's. Atiyah–Floer conjecture can be regarded as a part of such study. The motivation I defined a category out of Lagrangian Floer theory of symplectic manifold was actually to apply it to this gauge theory problem. After Seiberg–Witten invariant appeared and Ozsváth–Szabó re-construct it with minimum use of PDE, the mainstream of the research goes away from instanton Floer homology and its categorification. However actually the basic idea in the categorification of instanton Floer homology are used in the very basic part of the construction of Ozsváth–Szabó's construction. It seems that gauge theory in 2-3-4 dimension is now being built in that formulation. On the other hand, the categorification of Lagrangian Floer theory making progress now and it turns out the final result is a construction of (higher) functor from the 'category of all symplectic manifold' to the 'category of all A infinity categories'. The categorification of Lagrangian Floer theory can serve as a foundation of categorification of Instanton Floer homology also. I would like to explain some of such study.