The 16th Takagi Lectures
November 28, 2015 (Sat) 12:40–13:40
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Riemann–Hilbert Correspondence for Holonomic D-modules

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Abstract

A classical Riemann–Hilbert problem asks if a liner ordinary differential equation with regular singularities exists for a given monodromy on a curve.

P. Deligne formulated it as a correspondence of the integrable connections with regular singularities on a complex manifold X with a pole on a hypersurface Y on X and the local systems on $X \setminus Y$.

Later the speaker formulated it as an equivalence of the triangulated category $\mathrm{D}^{\mathrm{b}}_{\mathrm{rh}}(\mathcal{D}_X)$ of D_X-modules with regular holonomic $\mathcal{D}_X$-modules as cohomologies and that of $\mathrm{D}^{\mathrm{b}}_{\mathbb{C}\text{-c}}(\mathbb{C}_X)$ of sheaves on X with $\mathbb{C}$-constructible cohomologies. The equivalence is given by the de Rham functor

$$\mathcal{DR}_X \colon \mathrm{D}^{\mathrm{b}}_{\mathrm{rh}}(\mathcal{D}_X) \to \mathrm{D}^{\mathrm{b}}_{\mathbb{C}\text{-c}}(\mathbb{C}_X).$$

Here $\mathcal{DR}_X(\mathscr{M}) = \Omega_X \otimes_{\mathcal{D}_X} \mathscr{M}$ with Ω_X the sheaf of differential forms of top degree.

However, it was a long standing problem to generalize it to the (not necessarily regular) holonomic D-module case. Recently, the speaker succeeded it by using enhanced version of indsheaves (joint work with Andrea D'Agnolo).

There are two ingredients for it. One is the notion of indsheaves. The notion of indsheaves is introduced to treat the "sheaf " of functions with tempered growth.

The other ingredient is adding an extra variable. We consider indsheaves on $M \times \mathbb{R}$, not on the base manifold M. This permits us to capture the growth of solutions at singular points.