The 10th Takagi Lectures

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Lecture Hall (Room No. 420)
Research Institute for Mathematical Sciences
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ABSTRACT

Y. Benoist: *Random Walks on Homogeneous Spaces*

Let a_0 and a_1 be two matrices in $\mathrm{SL}(2, \mathbb{Z})$ which span a non-solvable group. Let x_0 be an irrational point on the torus $\mathbb{T}^2$. We toss a_0 or a_1, apply it to x_0, get another irrational point x_1, do it again to x_1, get a point x_2, and again. This random trajectory is equidistributed on the torus. This phenomenon is quite general on any finite volume homogeneous space.

A. Naor: *The Ribe Program—Ultrametric Skeletons*

Talk 1: The Ribe program

A theorem of M. Ribe from 1976 asserts that finite dimensional linear properties of normed spaces are preserved under uniformly continuous homeomorphisms. Thus, normed spaces exhibit a strong rigidity property: their structure as metric spaces determines the linear properties of their finite dimensional subspaces. This clearly says a lot about the geometry of normed spaces, but one can also use it to understand the structure of metric spaces that have nothing to do with linear spaces, such as graphs, manifolds or groups. After all, there is a deep and rich theory of finite dimensional linear invariants of Banach spaces with far reaching structural consequences. In view of Ribe's theorem we know that these invariants are preserved under homeomorphisms that are "quantitatively continuous", so in principle one can reformulate them using only the notion of distance; without referring to the linear structure in any way. Once this is achieved, one can study these properties in the context of general metric spaces using insights that originally made sense only in the context of linear spaces, and use these insights to solve problems in areas that do not have a priori connections to normed spaces. Thus, Ribe's rigidity theorem inspired a research program, known today as the Ribe program, which was formulated by Bourgain in 1986, the goal being to find explicit metric reformulations of key concepts and theorems from the theory of normed spaces. Major efforts by many mathematicians over the past 25 years led to a range of remarkable achievements within the Ribe program, with applications to areas such as group theory, harmonic analysis, and computer science. This talk will be a self-contained and elementary introduction to the Ribe program. We will explain some of the milestones of this research program, describe some recent progress, and discuss some challenging problems that remain open.

Talk 2: Ultrametric skeletons

This talk is devoted to the description of an example of a step in the Ribe program. Let (X, d) be a compact metric space, and let μ be a Borel probability measure on X. We will show that any such metric measure space (X, d, μ) admits an "ultrametric skeleton": a compact subset S of X on which the metric inherited from X is approximately an ultrametric, equipped with a probability measure ν supported on S such that the metric measure space (S, d, ν) mimics useful geometric properties of the initial space (X, d, μ). We will make this geometric picture precise, and explain a variety of applications of ultrametric skeletons in analysis, geometry, computer science, and probability theory.