

About the Connes embedding conjecture

Algebraic approaches

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Abstract. In his celebrated paper in 1976, A. Connes casually remarked that any finite von Neumann algebra ought to be embedded into an ultraproduct of matrix algebras, which is now known as the Connes embedding conjecture or problem. This conjecture became one of the central open problems in the field of operator algebras since E. Kirchberg's seminal work in 1993 that proves it is equivalent to a variety of other seemingly totally unrelated but important conjectures in the field. Since then, many more equivalents of the conjecture have been found, also in some other branches of mathematics such as noncommutative real algebraic geometry and quantum information theory. In this note, we present a survey of this conjecture with a focus on the algebraic aspects of it.

Keywords and phrases: Connes embedding conjecture, Kirchberg's conjecture, Tsirelson's problem, semi-pre- C^* -algebras, noncommutative real algebraic geometry

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