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The variational Poisson cohomology*

Alberto De Sole · **Victor G. Kac*****

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To the memory of Boris Kupershmidt (11/27/1946–12/12/2010)

Abstract. It is well known that the validity of the so called Lenard–Magri scheme of integrability of a bi-Hamiltonian PDE can be established if one has some precise information on the corresponding 1st variational Poisson cohomology for one of the two Hamiltonian operators. In the first part of the paper we explain how to introduce various cohomology complexes, including Lie superalgebra and Poisson cohomology complexes, and basic and reduced Lie conformal algebra and Poisson vertex algebra cohomology complexes, by making use of the corresponding universal Lie superalgebra or Lie conformal superalgebra. The most relevant are certain subcomplexes of the basic and reduced Poisson vertex algebra cohomology complexes, which we identify (non-canonically) with the generalized de Rham complex and the generalized variational complex. In the second part of the paper we compute the cohomology of the generalized de Rham complex, and, via a detailed study of the long exact sequence, we compute the cohomology of the generalized variational complex for any quasiconstant coefficient Hamiltonian operator with

A. De Sole
Dipartimento di Matematica, Università di Roma "La Sapienza", Piazzale Aldo Moro 5, 00185 Roma, Italy
(e-mail: desole@mat.uniroma1.it)

V.G. Kac
Department of Mathematics, MIT., Cambridge, MA 02139, USA
(e-mail: kac@math.mit.edu)

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invertible leading coefficient. For the latter we use some differential linear algebra developed in the Appendix.

Keywords and phrases: bi-Hamiltonian PDE, Lie conformal algebra, Poisson vertex algebra, universal Lie superalgebra and Lie conformal superalgebra, generalized variational complex, variational polyvector field, basic and variational Poisson cohomology, linearly closed differential field

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