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Aspects of free analysis*

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Abstract. We survey the analysis around the free difference quotient derivation, which is the natural derivation for variables with the highest degree of noncommutativity. The analogue of the Fourier transform is then bialgebra duality for the bialgebra with derivation-comultiplication to which the free difference quotient gives rise and which involves fully matricial analytic functions. Some of the motivation from free probability, especially free entropy and random matrices are also discussed.

Keywords and phrases: free difference quotient, generalized resolvents, infinitesimal bialgebras, free entropy

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