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The AD$^+$ duality program, the HOD conjecture, and the Ultimate-L conjecture*

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Abstract. These notes cover and expand on the material presented in the 25th Takagi Lectures, on October 18 and October 19, in 2025, after a lengthy pandemic pause.

The entire subject of Set Theory has reached a critical stage where there is the possibility of essentially eliminating independence obtained through Cohen's method forcing. The technique of forcing has been developed and refined over the last 60 years, and it is the major tool for showing statements are unsolvable on the basis of the ZFC axioms.

This would be achieved by simply adding one additional axiom which is formally is the axiom, $V =$ Ultimate-L, to the standard ZFC axioms. The main issue is whether this new axiom is compatible with large cardinal axioms.

For this a key conjecture has emerged which is the Ultimate-L Conjecture, and that conjecture in turn is closely related to the AD$^+$ Duality Program.

This, and the broader context, was the central focus of the lectures.

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