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On the classification of unitary highest weight modules

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Abstract. In the 1980s, Enright, Howe and Wallach [EHW] and independently Jakobsen [J3] gave a complete classification of the unitary highest weight modules. In the survey part of this paper we describe their results, and also several other related papers. In the research part of the article, we give a more direct and elementary proof of the same result for the (universal covers of the) Lie groups $Sp(2n, \mathbb{R})$, $SO^*(2n)$ and $SU(p, q)$. (The rest of the cases are treated in our paper [PPSST].) We also show how to describe the set of unitary highest weight modules with a given infinitesimal character.

Keywords and phrases: Lie group, representation, Harish-Chandra module, highest weight module, unitary highest weight module, Dirac operator, Dirac inequality

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