

## Transgressions of the Euler class and Eisenstein cohomology of $GL_N(\mathbf{Z})$ <sup>\*</sup>

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**Abstract.** These notes were written to be distributed to the audience of the first author's Takagi Lectures delivered June 23, 2018. These are based on a work-in-progress that is part of a collaborative project that also involves Akshay Venkatesh.

In this work-in-progress we give a new construction of some Eisenstein classes for  $GL_N(\mathbf{Z})$  that were first considered by Nori [41] and Sczech [44]. The starting point of this construction is a theorem of Sullivan on the vanishing of the Euler class of  $SL_N(\mathbf{Z})$  vector bundles and the explicit transgression of this Euler class by Bismut and Cheeger. Their proof indeed produces a universal form that can be thought of as a kernel for a *regularized theta lift* for the reductive dual pair  $(GL_N, GL_1)$ . This suggests looking to reductive dual pairs  $(GL_N, GL_k)$  with  $k \geq 1$  for possible generalizations of the Eisenstein cocycle. This leads to fascinating lifts that relate the geometry/topology world of real arithmetic locally symmetric spaces to the arithmetic world of modular forms.

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In these notes we do not deal with the most general cases and put a lot of emphasis on various examples that are often classical.

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