

## Abundance of minimal surfaces<sup>\*</sup>

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**Abstract.** This article is concerned with the existence theory of closed minimal hypersurfaces in closed Riemannian manifolds of dimension at least three. These hypersurfaces are critical points for the area functional, and hence their study can be seen as a high-dimensional generalization of the classical theory of closed geodesics (Birkhoff, Morse, Lusternik, Schnirel'mann,...). The best result until very recently, due to Almgren ([2], 1965), Pitts ([37], 1981), and Schoen–Simon ([43], 1981), was the existence of at least one closed minimal hypersurface in every closed Riemannian manifold.

I will discuss the methods I have developed with André Neves, for the past few years, to approach this problem through the variational point of view. These ideas have culminated with the discovery that minimal hypersurfaces in fact abound.

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