

## **Kodaira fibrations and beyond: methods for moduli theory<sup>★</sup>**

**Fabrizio Catanese<sup>★★</sup>**

Received: 21 November 2016 / Revised: 25 March 2017 / Accepted: 11 April 2017

Published online: 31 July 2017

© The Mathematical Society of Japan and Springer Japan 2017

Communicated by: Takeshi Saito

*Contribution to the 16th Takagi Lectures  
'In Celebration of the 100th Anniversary of Kodaira's Birth'*

**Abstract.** Kodaira fibred surfaces are remarkable examples of projective classifying spaces, and there are still many intriguing open questions concerning them, especially the slope question. The topological characterization of Kodaira fibrations is emblematic of the use of topological methods in the study of moduli spaces of surfaces and higher dimensional complex algebraic varieties, and their compactifications. Our tour through algebraic surfaces and their moduli (with results valid also for higher dimensional varieties) deals with fibrations, questions on monodromy and factorizations in the mapping class group, old and new results on Variation of Hodge Structures, especially a recent answer given (in joint work with Dettweiler) to a long standing question posed by Fujita. In the landscape of our tour, Galois coverings, deformations and rigid manifolds (there are by the way rigid Kodaira fibrations), projective classifying spaces, the action of the absolute Galois group on moduli spaces, stand also in the forefront. These questions lead to interesting algebraic surfaces, for instance remarkable surfaces constructed from VHS, surfaces isogenous to a product with automorphisms acting trivially on cohomology, hypersurfaces in Bagnera-de Franchis varieties, Inoue-type surfaces.

*Keywords and phrases:* algebraic surfaces, Kähler manifolds, moduli, deformations, topological methods, fibrations, Kodaira fibrations, Chern slope, automorphisms, uniformization, projective

---

<sup>★</sup> This article is based on the 16th Takagi Lectures that the author delivered at the University of Tokyo on November 28–29, 2015.

<sup>★★</sup> The present work took place in the realm of the ERC Advanced grant n. 340258, 'TADMICAMT'.

F. CATANESE

Mathematisches Institut, Lehrstuhl Mathematik VIII, Universitätsstraße 30, D-95447 Bayreuth, Deutschland

(e-mail: [fabrizio.catanese@uni-bayreuth.de](mailto:fabrizio.catanese@uni-bayreuth.de))

classifying spaces, monodromy, fundamental groups, variation of Hodge structure, absolute Galois group, locally symmetric varieties

*Mathematics Subject Classification (2010):* 14C21, 14C30, 14D06, 14D07, 14D22, 14E20, 14G35, 14H30, 14J29, 14J50, 32Q20, 32Q30, 32J25, 32Q55, 32M15, 32N05, 32S40, 32G20, 33C60

---