

Riemann–Hilbert correspondence for irregular holonomic \mathcal{D} -modules^{*}

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Abstract. This is a survey paper on the Riemann–Hilbert correspondence on (irregular) holonomic \mathcal{D} -modules, based on the 16th Takagi Lectures (2015/11/28). In this paper, we use subanalytic sheaves, an analogous notion to the one of indsheaves.

Keywords and phrases: irregular Riemann–Hilbert problem, irregular holonomic \mathcal{D} -modules, ind-sheaves, subanalytic sheaves, Stokes phenomenon

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