

The Kervaire invariant problem^{*}

Michael J. Hopkins

Received: 7 March 2015 / Revised: 14 January 2016 / Accepted: 21 January 2016

Published online: 25 March 2016

© The Mathematical Society of Japan and Springer Japan 2016

Communicated by: Kaoru Ono

Abstract. The history and solution of the Kervaire invariant problem is discussed, along with some of the future prospects raised by its solution.

Keywords and phrases: Kervaire invariant, framed manifold, homotopy groups of spheres

Mathematics Subject Classification (2010): 55Q45, 55Q91, 55N22, 57R05, 57R55, 57R60, 57R85

Contents

| | |
|---|----|
| 1. Introduction | 1 |
| 2. Pontryagin's work of the 1930s..... | 2 |
| 3. Topology around 1960: smooth structures | 5 |
| 4. Browder's work and the connection to homotopy theory | 8 |
| 5. The homotopy groups of spheres | 8 |
| 6. Brief sketch of the proof | 10 |
| 7. Open questions..... | 11 |

^{*} This article is based on the 7th Takagi Lectures that the author delivered at the University of Tokyo on November 21–23, 2009.

M.J. HOPKINS

Department of Mathematics, Harvard University, Cambridge, MA 02138, USA

(e-mail: mjh@math.harvard.edu)