GCOE Spring School on Representation Theory

DATE March 12 (Thu)–17 (Tue), 2009
PLACE Graduate School of Mathematical Sciences, the University of Tokyo

Thu, 12 March	Fri, 13 March	Sat, 14 March	Mon, 16 March	Tue, 17 March
09:30–10:30 Zierau	09:30–10:30 Zierau	09:00–10:00 Zierau	10:00–11:00 Krötz	11:00–12:00 Zierau
11:00–12:00 Krötz	11:00–12:00 Trapa	10:15–11:15 Mehdi	11:15–12:15 Trapa	13:30–14:30 Mehdi
13:30–14:30 Trapa	13:30–14:30 Krötz	11:45–12:45 Krötz	13:30–14:30 Zierau	15:00–16:00 Krötz
	15:00–16:00 Mehdi	13:00–14:00 Trapa	15:20–16:20 Mehdi	16:30–17:30 Trapa

SPEAKER **Bernhard Krötz** (Germany)

TITLE Harish-Chandra modules

ABSTRACT We plan to give a course on the various types of topological globalizations of Harish-Chandra modules. It is intended to cover the following topics:

1. Topological representation theory on various types of locally convex vector spaces.
2. Basic algebraic theory of Harish-Chandra modules
3. Unique globalization versus lower bounds for matrix coefficients
4. Dirac type sequences for representations
5. Deformation theory of Harish-Chandra modules

The new material presented was obtained in collaboration with Joseph Bernstein and Henrik Schlichtkrull. A first reference is the recent preprint “Smooth Frechet Globalizations of Harish-Chandra Modules” by J. Bernstein and myself, downloadable at arXiv:0812.1684v1.

SPEAKER **Salah Mehdi** (France)

TITLE Enright-Varadarajan modules and harmonic spinors

ABSTRACT The aim of these lectures is twofold. First we would like to describe the construction of the Enright-Varadarajan modules which provide a nice algebraic characterization of discrete series representations. This construction uses several important tools of representations theory. Then we shall use the Enright-Varadarajan modules to define a product for harmonic spinors on homogeneous spaces.

Speaker **Peter Trapa** (USA)

Title Special unipotent representations of real reductive groups

Abstract These lectures are aimed at beginning graduate students interested in the representation theory of real Lie groups. A familiarity with the theory of compact Lie groups and the basics of Harish-Chandra modules will be assumed. The goal of the lecture series is to give an exposition (with many examples) of the algebraic and geometric theory of special unipotent representations. These representations are of considerable interest; in particular, they are predicted to be the building blocks of all representation which can contribute to spaces of automorphic forms. They admit many beautiful characterizations, but their construction and unitarizability still remain mysterious.
The following topics are planned:

1. Algebraic definition of special unipotent representations and examples.
2. Localization and duality for Harish-Chandra modules.
3. Geometric definition of special unipotent representations.

Speaker **Roger Zierau** (USA)

Title Dirac Cohomology

Abstract Dirac operators have played an important role in representation theory. An early example is the construction of discrete series representations as spaces of L^2 harmonic spinors on symmetric spaces G/K. More recently a very natural Dirac operator has been discovered by Kostant; it is referred to as the cubic Dirac operator. There are algebraic and geometric versions. Suppose G/H is a reductive homogeneous space and $\mathfrak{g} = \mathfrak{h} + \mathfrak{q}$. Let $S_\mathfrak{q}$ be the restriction of the spin representation of $SO(\mathfrak{q})$ to $H \subset SO(\mathfrak{q})$. The algebraic cubic Dirac operator is an H-homomorphism $\mathcal{D} : V \otimes S_\mathfrak{q} \to V \otimes S_\mathfrak{q}$, where V is an $\mathfrak{g}$-module. The geometric geometric version is a differential operator acting on smooth sections of vector bundles of spinors on G/H. The algebraic cubic Dirac operator leads to a notion of Dirac cohomology, generalizing $\mathfrak{n}$-cohomology.
The lectures will roughly contain the following.

1. Construction of the spin representations of $\widetilde{SO}(n)$.
2. The algebraic cubic Dirac operator $\mathcal{D} : V \otimes S_\mathfrak{q} \to V \otimes S_\mathfrak{q}$ will be defined and some properties, including a formula for the square, will be given.
3. Of special interest is the case when $H = K$, a maximal compact subgroup of G and V is a unitarizable $(\mathfrak{g}, K)$-module. This case will be discussed.
4. The Dirac cohomology of a finite dimensional representation will be computed. We will see how this is related to $\mathfrak{n}$-cohomology of V.
5. The relationship between the algebraic and geometric cubic Dirac operators will be described. A couple of open questions will then be discussed.

The lectures will be fairly elementary.

Organizer Toshiyuki Kobayashi