「代数的整数論とその周辺２０１４」講演アブストラクト

１２月１日（月）

講演者：中村健太郎（北海道大学）
題名：Local ε-isomorphisms for rank two p-adic representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ and a functional equation of Kato's Euler system
概要：Local ε-isomorphisms are conjectural bases of the determinants of the Galois cohomologies of families of p-adic representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$, which p-adically interpolate the de Rham ε-isomorphisms which are explicitly defined by using local constants and Bloch-Kato's exponential maps for de Rham representations. Up to now, such bases have been constructed for the rank one case by Kazuya Kato, (the cyclotomic deformation of) the crystalline case by Benois-Berger and Loeffler-Venjakob-Zerbes, and the trianguline case by the speaker. In this talk, using (a multivariable version of) Colmez's convolution pairing, we propose a conjectural definition of the local ε-isomorphisms for any families of p-adic representations. Moreover, using Colmez's theory of p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, we prove our conjecture for (almost) all rank two families of p-adic representations. As an application, we prove a functional equation of Kato's Euler systems associated to modular forms.

講演者：佐久川憲児（大阪大学）
題名：Coleman-伊原の等式のポリログ類似（Z. Wojtkowiak 氏，中村博昭氏との共同研究）
概要：Coleman-伊原の等式は，Soulé 指標の局所体の絶対ガロワ群上への制限が，Coates-Wiles 準同型と久保田-Leopoldt p 進 L 関数の正の特殊値の積として書ける，ということを主張している．本講演では，この公式のポリログ類似を与える．即ち，中村-Wojtkowiak により導入された l 進ポリログ指標の制限を，Coates-Wiles 準同型と Coleman の p 進ポリログ関数の特殊値の積のトレースとして表す公式を与える．この研究は，中村博昭氏と Zdzisław Wojtkowiak 氏との共同研究である．

講演者：中山能力（一橋大学）
題名：Log abelian varieties（概説講演）
概要：Degenerating abelian varieties cannot preserve group structures, properness, and smoothness at the same time. However, in a world of log geometry, they can become group objects, so-called log abelian varieties, which behave well like proper smooth objects. In this talk, we discuss the idea and the status of the theory, which is in progress.

講演者：田嶋和明（東北大学）
題名：Stratification of the null cone in the non-split case（雪江明彦氏との共同研究）
概要：In early 80's, the notion of stratification of the null cone of reductive group actions was studied by Kempf, Ness and Kirwan. We are interested in stratifications of finite dimensional representations of reductive groups. If the group is split over a perfect field k, their works tell us that these stratifications are rationally defined over the good ground field k. In this talk, we extend these stratifications to all (not necessarily split) reductive algebraic groups over k. This is a joint work with Akihiko Yukie.

講演者：若林泰央（京都大学・大阪市立大学）
題名：A theory of dormant opers on pointed stable curves
概要：A(n) (dormant) oper, being our central object of this talk, is a certain principal homogeneous space on a pointed stable curve (in positive characteristic) equipped with an integrable logarithmic connection. The study of dormant opers and their moduli may be linked to various fields of mathematics, e.g., the p-adic Teichmuller theory developed by Shinichi Mochizuki, representation theory in the context of the geometric Langlands program, Gromov-Witten theory, combinatorics of rational polytopes (and spin networks), etc. In this talk, we would like to give an overview of a theory of opers and to present some related results, including an explicit formula for the generic number of dormant opers, which was conjectured by Kirti Joshi. This talk is intended for a general audience.

１２月２日（火）

講演者：谷田川友里（東京大学）
題名：曲面上の階数 1 の層の特性サイクル
概要：Recently, T. Saito gave a definition of the characteristic cycle of a smooth sheaf on a surface using vanishing cycles, which is difficult to calculate explicitly. Earlier, K. Kato had given another definition in the rank 1 case using ramification theory. We will compare the two definitions.

講演者：陽恩林（清華大学）
題名：Logarithmic version of the Milnor formula and the characteristic cycle of a tamely ramified sheaf
概要：In SGA 7, Deligne proved a formula for the total dimension of the space of vanishing cycles at an isolated singularity of a morphism from a smooth variety to a smooth curve over an algebraically closed field of characteristic $p > 0$. As a logarithmic variant of this formula, we prove an analogous formula for vanishing cycles with a coefficient sheaf tamely ramified along a divisor with normal crossings. This implies that the characteristic cycle of a tamely ramified sheaf satisfies a Milnor formula.

講演者：奥山裕介（京都工芸繊維大学）
題名：Berkovich 射影直線上のポテンシャル幾何と（非）アルキメデス的力学系
概要：We will give a talk on a local proximity estimate between the iteration f^n of a rational function f of degree > 1 and a rational function a of degree > 0 on the projective line over a product formula field (e.g., a number field or a function field) of characteristic 0, using the formalism of Berkovich spaces and potential theory on the Berkovich projective line. This estimate is a dynamical analog of Diophantine approximation, and is based on a more general quantitative equidistribution result for a sequence of algebraic zeros divisors on $\mathbb{P}^1$ over a product formula field (e.g., Galois conjugacy classes of algebraic numbers) having small diagonals and small weighted heights.

講演者：新甫洋史（九州大学）
題名：Idelic class field theory for 3-manifolds（植木潤氏との共同研究）
概要：This is a joint work with Jun Ueki. Following the analogies between 3-dimensional topology and number theory, we will study a topological analogue of idèlic class field theory for 3-manifolds. We firstly introduce a notion of a *very admissible link* $\mathcal{K}$ in a 3-manifold M, which plays a role similar to the set of primes of a number field, and define an *idèle class group* for $(M, \mathcal{K})$. Then we present analogues of Artin's global reciprocity law and the existence theorem of idèlic class field theory.

講演者：森下昌紀（九州大学）
題名：Arithmetic Milnor invariants and multiple power residue symbols in number fields
（天野郁弥氏との共同研究）
概要：素イデアルと結び目の類似に基づき，代数体の素イデアルに対し，数論的ミルナー不変量と多重べき剰余記号を導入する．我々の記号は，古典的なルジャンドル記号，べき剰余記号およびレダイのトリプル記号を一般化するものであり，代数体のあるべき零拡大における素イデアルの分解法則を記述する．新しい例として，円の 3 分体上にある 27 次のハイゼンベルグ拡大を具体的に構成することにより，トリプルキュービック記号を論じる．また，我々の多重べき剰余記号のマッセイ積によるコホモロジー的な解釈を与える．これは，天野郁弥氏との共同研究である．

１２月３日（水）

講演者：Jerome Dimabayao（九州大学）
題名：On the cohomological coprimality of Galois representations of a p-adic field
概要：Let K be a local or a global field and G_K its absolute Galois group. Given two continuous representations V and V' of G_K, we are interested in determining when and how they are “independent”. Motivated by our efforts to generalize some results of Coates, Sujatha and Wintenberger, we introduce the notion of “cohomological coprimality” of such representations. We say that the two representations V and V' of G_K are “cohomologically coprime” if all the Galois cohomology groups corresponding to the field cut out by the representation V' (resp. V) having coefficients in V (resp. V') vanish. We consider the situation where K is a p-adic field and V and V' come from proper smooth varieties X and X' over K with potential good reduction, respectively. Then it can be shown that in many cases where X and X' have “quite different” nature, V and V' are cohomologically coprime. We will also discuss cohomological coprimality among elements of a system of ℓ-adic representations of G_K associated with a fixed X as above.

講演者：三原朋樹（東京大学）
題名：保型形式に伴う Galois 表現の族の新たな幾何的構成について
概要：We define a new étale sheaf on a modular curve. Every p-adic Galois representation associated to a cusp p-adic Hecke eigenform is obtained as a quotient of the étale cohomology of the sheaf. In particular, we give an alternative construction of a Λ-adic Galois representation associated to an ordinary cusp Λ-adic Hecke eigenform.

講演者：津嶋貴弘（東京大学）
題名：Perfectoid 空間 I　— 基礎理論について —（概説講演）
概要：本講演では Fontaine-Wintenberger のノルム体の理論の特殊な場合を復習し，それの自然な一般化としての perfectoid 体と perfectoid 空間の概念の定義を正確に述べ，基本技術である tilting 同値とその応用について解説することを目標とする．perfectoid 空間は Huber の adic 空間の特別な場合として定義されるので上記の目標のために adic 空間についての最小限の必要事項を述べる．tilting 同値の応用として perfectoid 空間論の基礎をなす「構造前層が層になる」という事実とその証明方針を簡単に紹介する．時間に余裕があれば perfectoid 空間のエタールサイトとその tilt のエタールサイトが同型になるという定理も紹介したい．

１２月４日（木）

講演者：甲斐亘（東京大学）
題名：On the Albanese cokernel of varieties over p-adic fields
概要：S. Lichtenbaum は 1969 年の論文で，p 進体 K 上の曲線の双対性理論の帰結として，K 上の非特異射影曲線 X に対する標準的単射 $\mathrm{Pic}^0(X) \to J_X(K)$（$J_X$ は X のヤコビ多様体）の余核を X のピカールスキームの連結成分で記述する式を得た．この結果を X が K 上の高次元非特異射影多様体である場合へ一般化するには曲線のヤコビ多様体をピカール多様体と看做すかアルバネーゼ多様体と看做すかにより二通りの方向が考えられる．この違いは曲線の Pic^0 を多様体のピカール群と看做すか 0 次元サイクルのチャウ群と看做すかの違いと対応する．前者の方向での一般化は 2004 年に van Hamel によって為されている．

本講演では後者の方向に沿い，0 次元サイクルのアルバネーゼ写像の余核

$$\mathrm{coker}\left(\mathrm{CH}_0(X)^{\deg=0} \to \mathrm{Alb}_X(K)\right)$$

(ここで Alb_X は X のアルバネーゼ多様体) を Néron-Severi 群を用いて記述する式を予想として提示し，X の整数環上のモデルに良い条件を仮定した上でその式を証明する．証明の重要な要素は斎藤-佐藤による Lichtenbaum 双対の或る高次元化と，Gabber や de Jong によるコホモロジー的 Brauer 群と東屋代数 Brauer 群の比較定理である．

時間が許せばアルバネーゼ余核の局所大域問題を紹介する.「局所」側の群は，本講演の主定理により有限群であることが分かる．

講演者：佐野昂迪（慶應義塾大学）
題名：一般の代数体上の岩澤主予想について（D. Burns 氏，栗原将人氏との共同研究）
概要：Burns-Greither は 2003 年に，$\mathbb{Q}$ 上の円分岩澤主予想を用いて，$\mathbb{Q}$ 上アーベルな拡大体に対する同変玉河数予想の大部分を解いた．彼らの “降下議論” において重要な役割を果たすのが，Ferrero-Greenberg の公式と，Solomon の “cyclotomic p-units” に関する定理である．本講演では，一般の代数体上の岩澤主予想を定式化し，Burns-Greither の議論を一般の代数体に対して一般化する．我々の降下議論においては，近年 Mazur-Rubin と講演者により独立に定式化された予想が重要な役割を果たす．本講演の内容は David Burns 氏と栗原将人氏との共同研究である．

講演者：加塩朋和（東京理科大学）
題名：有理数体上の Stark 予想とフェルマー曲線の CM 周期
概要：We will define a “period ring-valued beta function” and give a reciprocity law on its special values, by using some results on Fermat curves due to Rohrlich and Coleman. There is the following application: One can show that (a version of) Stark’s conjecture holds true when the base field is the rational number field by using Euler’s formulas and cyclotomic units. We will provide an alternative (and partial) proof by our reciprocity law. In other words, the reciprocity law given in this talk is a refinement of the reciprocity law on cyclotomic units.

講演者：伊藤哲史（京都大学）
題名：Perfectoid 空間 II　— 数論への応用について —（概説講演）
概要：This is a survey talk sequel to Tsushima’s talk on foundations of the theory of perfectoid spaces. It is now well-understood that perfectoid spaces have several striking applications to arithmetic and geometric problems such as the weight-monodromy conjecture for complete intersections, comparison theorems in p-adic Hodge theory, duality isomorphisms between the Rapoport-Zink towers at infinite level, construction of Galois representations associated with torsion elements in the cohomology of Shimura

varieties as well as regular algebraic cuspidal automorphic representations of $GL(n)$ over totally real or imaginary CM fields. We plan to explain some ideas behind these applications briefly. The exposition will be very brief. Almost no proofs will be given.

講演者：呼子笛太郎（東北大学）
題名：Mass formula for abelian varieties
概要：The Eichler-Deuring mass formula says that the weighted number of isomorphism classes of supersingular elliptic curves over an algebraically closed field of characteristic p is expressed as a simple polynomial in p. In 2009, C.-F. Yu and J.-D. Yu generalized this formula for supersingular abelian surfaces. In this talk, we show a mass formula for supersingular abelian three-folds.

講演者：丸山拓也（東京大学）
題名：主偏極化されたアーベルスキームの個数の評価について
概要：Arakelov and Parshin showed that there are only finitely many isomorphism classes of nonisotrivial families of curves of given genus parameterized by a fixed base curve over $\mathbb{C}$. Gordon Heier gave an effective uniform bound for the number of such families. In this talk, I will explain how a similar bound for the number of families of principally polarized Abelian varieties is obtained when the base curve is proper.

１２月５日（金）

講演者：岸康弘（愛知教育大学）
題名：偶数周期の連分数展開と末尾急増型主要対称部分
（河本史紀氏，鈴木浩志氏，冨田耕史氏との共同研究）
概要：For a non-square positive integer d with $4 \nmid d$, put $\omega(d) := (1+\sqrt{d})/2$ if d is congruent to 1 modulo 4 and otherwise $\omega(d) := \sqrt{d}$. Moreover, for a positive integer ℓ, let A_ℓ denote the set of non-square positive integers d with $4 \nmid d$ such that the minimal periods of the simple continued fraction expansions of $\omega(d)$ are equal to ℓ. According to numerical experiments, for each ℓ with $1 \leq \ell \leq 63948$, the class number of real quadratic field $\mathbb{Q}(\sqrt{d_\ell})$ is equal to 1 except for $\ell = 7, 11, 49, 225, 299$, where d_ℓ is the minimal element of A_ℓ. Thus, in order to find many real quadratic fields of class number 1 we will have to know how to get the minimal element of A_ℓ. In this talk, we introduce a notion of “extremely large end (ELE)” for a finite string of positive integers to look for the minimal element and study their properties in even periods ℓ.

講演者：John C. Miller（Rutgers University）
題名：On class numbers of cyclotomic fields and $\mathbb{Z}_p$-extensions
概要：The class number of cyclotomic fields has only been calculated for fields of rather small conductor, due to the difficulty of finding the “plus part” of the class number. By counting principal prime ideals, we establish class number upper bounds, allowing us to calculate the class number for real cyclotomic fields of larger conductor than has been previously possible. We also will survey some recent results and conjectures regarding the class numbers of fields in cyclotomic $\mathbb{Z}_p$-extensions over the rationals.

講演者：尾崎学（早稲田大学）
題名：$\mathbb{Z}_p$-拡大の非アーベル岩澤理論（概説講演）
概要：In this talk, I will give a survey on non-abelian Iwasawa theory of $\mathbb{Z}_p$-extensions, namely, theory of non-abelian restricted ramified (especially unramified and p-ramified) extensions over $\mathbb{Z}_p$-extensions

of number fields.

講演者：都地崇恵（東海大学）
題名：$\mathbb{Q}(\sqrt{p})$ の円分 $\mathbb{Z}_2$ 拡大における岩澤 λ 不変量について
（福田隆氏，小松啓一氏，尾崎学氏との共同研究）
概要：In the preceding works, Fukuda and Komatsu developed criteria for Greenberg conjecture of the cyclotomic $\mathbb{Z}_2$-extension of $k = \mathbb{Q}(\sqrt{p})$ with prime number p and showed $\lambda_2(k) = 0$ for all p less than 10^5 except $p = 13841, 67073$. All the known criteria at present can not handle $p = 13841, 67073$. We develop the structure theorem of cyclotomic units in the cyclotomic $\mathbb{Z}_2$-extension of the quadratic field k. Therefore, we obtain another criterion for $\lambda_2(k) = 0$, which is considered a slight modification of the method of the Ichimura and Sumida. Our new criterion fits the numerical examination and quickly shows that $\lambda_2(\mathbb{Q}(\sqrt{p})) = 0$ for $p = 13841, 67073$.

講演者：平田典子（日本大学）
題名：New Diophantine criterion of polylogarithms over an algebraic number field
（S. David 氏，伊藤勝氏，鷲尾勇介氏との共同研究）
概要：In the talk, we show the fisrt linear independence criterion concerning with the $s + 1$ numbers: 1 and s polylogarithms over an algebraic number field of degree ≥ 2, in the p-adic case as well as in the complex case.

Around the value of usual logarithmic function at non-zero point $\in \overline{\mathbb{Q}} \subset \mathbb{C}$, although the transcendence is only known, no algebraic independence result is known in the complex neither in the p-adic case.
For the polylogarithms, NO transcendence neither algebraic independence result exists in the cases. The only proven results are in positive characteristic ones which are much easier to be dealt with.

In 2003, T. Rivoal showed a lower bound for the dimension of the linear space spanned by polylogarithms by means of the method of Yu. V. Nesterenko. The result shows the existence of infinitely many irrational polylogarithms, however, his result does not imply any irrationality of a chosen polylogarithm.

Here we prove the first linear independence criterion of polylogarithms in the p-adic and the complex cases, over a number field of arbitrary finite degree over $\mathbb{Q}$. We also construct infinitely many explicit examples of irrational, or linearly independent polylogarithms over a number field of given degree over $\mathbb{Q}$.

Let $Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}$, for $z \in \mathbb{C}, |z| \leq 1 (z \neq 1$ if $s = 1)$ and consider $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha| < 1$. We obtain:

if the absolute value of α is relatively small, then the $s + 1$ numbers:
$Li_1(\alpha), Li_2(\alpha), \cdots, Li_s(\alpha)$ and 1 are linearly independent over $\mathbb{Q}(\alpha)$. In the p-adic case, for $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha|_p < 1$, we also give a criterion of similar nature relying on Diophantine approximations so-called Padé approximation.
The p-adic case is proven in collaboration with Sinnou David (University of Paris VI). The complex case together with construction of examples is a joint work with Masaru Ito (Tokyo Institute of Technology) and Yusuke Washio (Nihon University).

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