Local $\epsilon$-isomorphisms for rank two $p$-adic representations of Gal($\overline{\mathbb{Q}}_p/\mathbb{Q}_p$) and a functional equation of Kato’s Euler system

Local $\epsilon$-isomorphisms are conjectural bases of the determinants of the Galois cohomologies of families of $p$-adic representations of Gal($\overline{\mathbb{Q}}_p/\mathbb{Q}_p$), which $p$-adically interpolate the de Rham $\epsilon$-isomorphisms which are explicitly defined by using local constants and Bloch-Kato’s exponential maps for de Rham representations. Up to now, such bases have been constructed for the rank one case by Kazuya Kato, (the cyclotomic deformation of) the crystalline case by Benois-Berger and Loeffler-Venjakob-Zerbes, and the trianguline case by the speaker. In this talk, using (a multivariable version of) Colmez’s convolution pairing, we propose a conjectural definition of the local $\epsilon$-isomorphisms for any families of $p$-adic representations. Moreover, using Colmez’s theory of $p$-adic Langlands correspondence for $\text{GL}_2(\mathbb{Q}_p)$, we prove our conjecture for (almost) all rank two families of $p$-adic representations. As an application, we prove a functional equation of Kato’s Euler systems associated to modular forms.

Log abelian varieties

Degenerating abelian varieties cannot preserve group structures, properness, and smoothness at the same time. However, in a world of log geometry, they can become group objects, so-called log abelian varieties, which behave well like proper smooth objects. In this talk, we discuss the idea and the status of the theory, which is in progress.

Stratification of the null cone in the non-split case

In early 80’s, the notion of stratification of the null cone of reductive group actions was studied by Kempf, Ness and Kirwan. We are interested in stratifications of finite dimensional representations of reductive groups. If the group is split over a perfect field $k$, their works tell us that these stratifications are rationally defined over the good ground field $k$. In this talk, we extend these stratifications to all (not necessarily split) reductive algebraic groups over $k$. This is a joint work with Akihiko Yukie.
A theory of dormant opers on pointed stable curves

A(n) (dormant) oper, being our central object of this talk, is a certain principal homogeneous space on a pointed stable curve (in positive characteristic) equipped with an integrable logarithmic connection. The study of dormant opers and their moduli may be linked to various fields of mathematics, e.g., the $p$-adic Teichmüller theory developed by Shinichi Mochizuki, representation theory in the context of the geometric Langlands program, Gromov-Witten theory, combinatorics of rational polytopes (and spin networks), etc. In this talk, we would like to give an overview of a theory of opers and to present some related results, including an explicit formula for the generic number of dormant opers, which was conjectured by Kirti Joshi. This talk is intended for a general audience.

Recently, T. Saito gave a definition of the characteristic cycle of a smooth sheaf on a surface using vanishing cycles, which is difficult to calculate explicitly. Earlier, K. Kato had given another definition in the rank 1 case using ramification theory. We will compare the two definitions.

In SGA 7, Deligne proved a formula for the total dimension of the space of vanishing cycles at an isolated singularity of a morphism from a smooth variety to a smooth curve over an algebraically closed field of characteristic $p > 0$. As a logarithmic variant of this formula, we prove an analogous formula for vanishing cycles with a coefficient sheaf tamely ramified along a divisor with normal crossings. This implies that the characteristic cycle of a tamely ramified sheaf satisfies a Milnor formula.

We will give a talk on a local proximity estimate between the iteration $f^n$ of a rational function $f$ of degree $> 1$ and a rational function $a$ of degree $> 0$ on the projective line over a product formula field (e.g., a number field or a function field) having small diagonals and small weighted heights.

This is a joint work with Jun Ueki. Following the analogies between 3-dimensional topology and number theory, we will study a topological analogue of idèalic class field theory for 3-manifolds. We firstly introduce a notion of a very admissible link $\mathcal{K}$ in a 3-manifold $M$, which plays a role similar to the set of primes of a number field, and define an idèle class group for $(M, \mathcal{K})$. Then we present analogues of Artin’s global reciprocity law and the existence theorem of idèlic class field theory.
Arithmetic Milnor invariants and multiple power residue symbols in number fields

Let $K$ be a local or a global field and $G_K$ its absolute Galois group. Given two continuous representations $V$ and $V'$ of $G_K$, we are interested in determining when and how they are “independent”. Motivated by our efforts to generalize some results of Coates, Sujatha and Wintenberger, we introduce the notion of “cohomological coprimality” of such representations. We say that the two representations $V$ and $V'$ of $G_K$ are “cohomologically coprime” if all the Galois cohomology groups corresponding to the field cut out by the representation $V'$ (resp. $V$) having coefficients in $V$ (resp. $V'$) vanish. We consider the situation where $K$ is a $p$-adic field and $V$ and $V'$ come from proper smooth varieties $X$ and $X'$ over $K$ with potential good reduction, respectively. Then it can be shown that in many cases where $X$ and $X'$ have “quite different” nature, $V$ and $V'$ are cohomologically coprime. We will also discuss cohomological coprimality among elements of a system of $\ell$-adic representations of $G_K$ associated with a fixed $X$ as above.

On the cohomological coprimality of Galois representations of a $p$-adic field

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Jerome Dimabayao
12月4日（木）

講演者：甲斐亘（東京大学）
題名：On the Albanese cokernel of varieties over $p$-adic fields
概要：S. Lichtenbaum は 1969 年の論文で、$p$ 進体 $K$ 上の曲線の双対性理論の帰結として、$K$ 上の非特異射影曲線 $X$ に対する標準的単射 Pic$^0(X) \to J_X(K)$ ($J_X$ は $X$ のヤコビ多様体）の余核を $X$ のピカールスキームの連結成分で記述する式を得た。この結果を $X$ が $K$ 上の高次元非特異射影多様体である場合へ一般化するには曲線のヤコビ多様体をピカール多様体と看做すかアルバネーゼ多様体と看做すかにより二通りの方向が考えられる。この違いは曲線の Pic$^0$ を多様体のピカール群と看做すか 0 次元サイクルのチャウ群と看做すかの違いと対応する。前者の方向での一般化は 2004 年に van Hamel によって為されている。

本講演では後者の方向に沿い、0 次元サイクルのアルバネーゼ写像の余核

$$\text{coker}\left(\text{CH}_0(X)_{\text{deg}=0} \to \text{Alb}_X(K)\right)$$

（ここで Albx は X のアルバネーゼ多様体）を Néron-Severi 群を用いて記述する式を予想として提示し、X の整数環上のモデルに良い条件を仮定した上でその式を証明する。証明の重要な要素は佐藤-佐藤による Lichtenbaum 双対の或る高次元化と、Gabber や de Jong によるこホモロジー的 Brauer 群と東屋代数 Brauer 群の比較定理である。

時間が許せばアルバネーゼ余核の局所大域問題を紹介する。「局所」側の群は、本講演の主定理により有限群であることが分かる。

講演者：佐野昌歩（慶應義塾大学）
題名：一般の代数体上の岩澤主予想について（D. Burns 氏、栗原将人氏との共同研究）
概要：Burns-Greither は 2003 年に、Q 上の円分岩澤主予想を用いて、Q 上アーベルな拡大体に対する同変複素数予想の大部分を解いた。彼らの “降下議論”において重要な役割を果たすのが、Ferrero-Greenberg の公式と、Solomon の “cyclotomic $p$-units”に関する定理である。本講演では、一般の代数体上の岩澤主予想を定式化し、Burns-Greither の議論を一般的の代数体に対して一般化する。我々の降下議論においては、近年 Mazur-Rubin と講演者により独立に定式化された予想が重要な役割を果たす。本講演の内容は David Burns 氏と栗原将人氏との共同研究である。

講演者：加塩朋和（東京理科大学）
題名：有理数体上の Stark 予想とフェルマー曲線の CM 周期
概要：We will define a “period ring-valued beta function” and give a reciprocity law on its special values, by using some results on Fermat curves due to Rohrlch and Coleman. There is the following application: One can show that (a version of) Stark’s conjecture holds true when the base field is the rational number field by using Euler’s formulas and cyclotomic units. We will provide an alternative (and partial) proof by our reciprocity law. In other words, the reciprocity law given in this talk is a refinement of the reciprocity law on cyclotomic units.

講演者：伊藤哲史（京都大学）
題名：Perfectoid 空間 II — 数論への応用について —（概説講演）
概要：This is a survey talk sequel to Tsushima’s talk on foundations of the theory of perfectoid spaces. It is now well-understood that perfectoid spaces have several striking applications to arithmetic and geometric problems such as the weight-monodromy conjecture for complete intersections, comparison theorems in $p$-adic Hodge theory, duality isomorphisms between the Rapoport-Zink towers at infinite level, construction of Galois representations associated with torsion elements in the cohomology of Shimura
varieties as well as regular algebraic cuspidal automorphic representations of $GL(n)$ over totally real or imaginary CM fields. We plan to explain some ideas behind these applications briefly. The exposition will be very brief. Almost no proofs will be given.

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Mass formula for abelian varieties

The Eichler-Deuring mass formula says that the weighted number of isomorphism classes of supersingular elliptic curves over an algebraically closed field of characteristic $p$ is expressed as a simple polynomial in $p$. In 2009, C.-F. Yu and J.-D. Yu generalized this formula for supersingular abelian surfaces. In this talk, we show a mass formula for supersingular abelian three-folds.

Arakelov and Parshin showed that there are only finitely many isomorphism classes of nonisotrivial families of curves of given genus parameterized by a fixed base curve over $\mathbb{C}$. Gordon Heier gave an effective uniform bound for the number of such families. In this talk, I will explain how a similar bound for the number of families of principally polarized Abelian varieties is obtained when the base curve is proper.

For a non-square positive integer $d$ with $4 \not| d$, put $\omega(d) := (1 + \sqrt{d})/2$ if $d$ is congruent to 1 modulo 4 and otherwise $\omega(d) := \sqrt{d}$. Moreover, for a positive integer $\ell$, let $A_\ell$ denote the set of non-square positive integers $d$ with $4 \not| d$ such that the minimal periods of the simple continued fraction expansions of $\omega(d)$ are equal to $\ell$. According to numerical experiments, for each $\ell$ with $1 \leq \ell \leq 63948$, the class number of real quadratic field $\mathbb{Q}(\sqrt{d_\ell})$ is equal to 1 except for $\ell = 7, 11, 49, 225, 299$, where $d_\ell$ is the minimal element of $A_\ell$. Thus, in order to find many real quadratic fields of class number 1 we will have to know how to get the minimal element of $A_\ell$. In this talk, we introduce a notion of "extremely large end (ELE)" for a finite string of positive integers to look for the minimal element and study their properties in even periods $\ell$.

On class numbers of cyclotomic fields and $\mathbb{Z}_p$-extensions

The class number of cyclotomic fields has only been calculated for fields of rather small conductor, due to the difficulty of finding the “plus part” of the class number. By counting principal prime ideals, we establish class number upper bounds, allowing us to calculate the class number for real cyclotomic fields of larger conductor than has been previously possible. We also will survey some recent results and conjectures regarding the class numbers of fields in cyclotomic $\mathbb{Z}_p$-extensions over the rationals.

In this talk, I will give a survey on non-abelian Iwasawa theory of $\mathbb{Z}_p$-extensions, namely, theory of non-abelian restricted ramified (especially unramified and $p$-ramified) extensions over $\mathbb{Z}_p$-extensions.
of number fields.

In the preceding works, Fukuda and Komatsu developed criteria for Greenberg conjecture of the cyclotomic $\mathbb{Z}_2$-extension of $k = \mathbb{Q}(\sqrt[p]{p})$ with prime number $p$ and showed $\lambda_2(k) = 0$ for all $p$ less than $10^5$ except $p = 13841, 67073$. All the known criteria at present can not handle $p = 13841, 67073$. We develop the structure theorem of cyclotomic units in the cyclotomic $\mathbb{Z}_2$-extension of the quadratic field $k$. Therefore, we obtain another criterion for $\lambda_2(k) = 0$, which is considered a slight modification of the method of the Ichimura and Sumida. Our new criterion fits the numerical examination and quickly shows that $\lambda_2(\mathbb{Q}(\sqrt[p]{p})) = 0$ for $p = 13841, 67073$.

Around the value of usual logarithmic function at non-zero point $\in \overline{\mathbb{Q}} \subset \mathbb{C}$, although the transcendence is only known, no algebraic independence result is known in the complex neither in the $p$-adic case. For the polylogarithms, NO transcendence neither algebraic independence result exists in the cases. The only proven results are in positive characteristic ones which are much easier to be dealt with.

In 2003, T. Rivoal showed a lower bound for the dimension of the linear space spanned by polylogarithms by means of the method of Yu. V. Nesterenko. The result shows the existence of infinitely many irrational polylogarithms, however, his result does not imply any irrationality of a chosen polylogarithm.

Here we prove the first linear independence criterion of polylogarithms in the $p$-adic and the complex cases, over a number field of arbitrary finite degree over $\mathbb{Q}$. We also construct infinitely many explicit examples of irrational, or linearly independent polylogarithms over a number field of given degree over $\mathbb{Q}$.

Let $\text{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}$ for $z \in \mathbb{C}, |z| \leq 1$ if $s = 1$ and consider $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha| < 1$. We obtain:

if the absolute value of $\alpha$ is relatively small, then the $s + 1$ numbers:
$\text{Li}_1(\alpha), \text{Li}_2(\alpha), \cdots, \text{Li}_s(\alpha)$ and 1 are linearly independent over $\mathbb{Q}(\alpha)$. In the $p$-adic case, for $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha|_p < 1$, we also give a criterion of similar nature relying on Diophantine approximations so-called Pade approximation.

The $p$-adic case is proven in collaboration with Sinnou David (University of Paris VI). The complex case together with construction of examples is a joint work with Masaru Ito (Tokyo Institute of Technology) and Yusuke Washio (Nihon University).

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