Algebraic Number Theory and Related Topics 2014
Abstracts of talks

December 1st (Mon)

Speaker: Kentaro Nakamura (Hokkaido University)
Title: Local ε-isomorphisms for rank two p-adic representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ and a functional equation of Kato's Euler system
Abstract: Local ε-isomorphisms are conjectural bases of the determinants of the Galois cohomologies of families of p-adic representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$, which p-adically interpolate the de Rham ε-isomorphisms which are explicitly defined by using local constants and Bloch-Kato's exponential maps for de Rham representations. Up to now, such bases have been constructed for the rank one case by Kazuya Kato, (the cyclotomic deformation of) the crystalline case by Benois-Berger and Loeffler-Venjakob-Zerbes, and the trianguline case by the speaker. In this talk, using (a multivariable version of) Colmez's convolution pairing, we propose a conjectural definition of the local ε-isomorphisms for any families of p-adic representations. Moreover, using Colmez's theory of p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, we prove our conjecture for (almost) all rank two families of p-adic representations. As an application, we prove a functional equation of Kato's Euler systems associated to modular forms.

Speaker: Kenji Sakugawa (Osaka University)
Title: Polylogarithmic analogue of the Coleman-Ihara formula
(joint work with H. Nakamura and Z. Wojtkowiak)
Abstract: The Coleman-Ihara formula states that restrictions of Soulé characters to the absolute Galois groups of local fields are written as Coates-Wiles homomorphisms multiplied by special values of Kubota-Leopoldt p-adic L-functions at positive integers. In this talk, we give a polylogarithmic analogue of the Coleman-Ihara formula. We describe restrictions of l-adic polylogarithmic characters introduced by Nakamura-Wojtkowiak as the trace of Coates-Wiles homomorphisms multiplied by special values of Coleman's p-adic polylogarithms. This is joint work with Hiroaki Nakamura and Zdzisław Wojtkowiak.

Speaker: Chikara Nakayama (Hitotsubashi University)
Title: Log abelian varieties (Survey)
Abstract: Degenerating abelian varieties cannot preserve group structures, properness, and smoothness at the same time. However, in a world of log geometry, they can become group objects, so-called log abelian varieties, which behave well like proper smooth objects. In this talk, we discuss the idea and the status of the theory, which is in progress.

Speaker: Kazuaki Tajima (Tohoku University)
Title: Stratification of the null cone in the non-split case (joint work with A. Yukie)
Abstract: In early 80's, the notion of stratification of the null cone of reductive group actions was studied by Kempf, Ness and Kirwan. We are interested in stratifications of finite dimensional representations of reductive groups. If the group is split over a perfect field k, their works tell us that these stratifications are rationally defined over the good ground field k. In this talk, we extend these stratifications to all (not necessarily split) reductive algebraic groups over k. This is a joint work with Akihiko Yukie.

Speaker: Yasuhiro Wakabayashi (Kyoto University · Osaka City University)
Title: A theory of dormant opers on pointed stable curves
Abstract: A(n) (dormant) oper, being our central object of this talk, is a certain principal homogeneous space on a pointed stable curve (in positive characteristic) equipped with an integrable logarithmic connection. The study of dormant opers and their moduli may be linked to various fields of mathematics, e.g., the p-adic Teichmuller theory developed by Shinichi Mochizuki, representation theory in the context of the geometric Langlands program, Gromov-Witten theory, combinatorics of rational polytopes (and spin networks), etc. In this talk, we would like to give an overview of a theory of opers and to present some related results, including an explicit formula for the generic number of dormant opers, which was conjectured by Kirti Joshi. This talk is intended for a general audience.

December 2nd (Tue)

Speaker: Yuri Yatagawa (University of Tokyo)
Title: Characteristic cycle of a rank 1 sheaf on a surface
Abstract: Recently, T. Saito gave a definition of the characteristic cycle of a smooth sheaf on a surface using vanishing cycles, which is difficult to calculate explicitly. Earlier, K. Kato had given another definition in the rank 1 case using ramification theory. We will compare the two definitions.

Speaker: Enlin Yang (Tsinghua University)
Title: Logarithmic version of the Milnor formula and the characteristic cycle of a tamely ramified sheaf
Abstract: In SGA 7, Deligne proved a formula for the total dimension of the space of vanishing cycles at an isolated singularity of a morphism from a smooth variety to a smooth curve over an algebraically closed field of characteristic $p > 0$. As a logarithmic variant of this formula, we prove an analogous formula for vanishing cycles with a coefficient sheaf tamely ramified along a divisor with normal crossings. This implies that the characteristic cycle of a tamely ramified sheaf satisfies a Milnor formula.

Speaker: Yûsuke Okuyama (Kyoto Institute of Technology)
Title: Potential geometry and (non-)archimedean dynamics on the Berkovich projective line
Abstract: We will give a talk on a local proximity estimate between the iteration f^n of a rational function f of degree > 1 and a rational function a of degree > 0 on the projective line over a product formula field (e.g., a number field or a function field) of characteristic 0, using the formalism of Berkovich spaces and potential theory on the Berkovich projective line. This estimate is a dynamical analog of Diophantine approximation, and is based on a more general quantitative equidistribution result for a sequence of algebraic zeros divisors on $\mathbb{P}^1$ over a product formula field (e.g., Galois conjugacy classes of algebraic numbers) having small diagonals and small weighted heights.

Speaker: Hirofumi Niibo (Kyushu University)
Title: Idelic class field theory for 3-manifolds (joint work with J. Ueki)
Abstract: This is a joint work with Jun Ueki. Following the analogies between 3-dimensional topology and number theory, we will study a topological analogue of idèlic class field theory for 3-manifolds. We firstly introduce a notion of a *very admissible link* $\mathcal{K}$ in a 3-manifold M, which plays a role similar to the set of primes of a number field, and define an *idèle class group* for $(M, \mathcal{K})$. Then we present analogues of Artin's global reciprocity law and the existence theorem of idèlic class field theory.

Speaker: Masanori Morishita (Kyushu University)
Title: Arithmetic Milnor invariants and multiple power residue symbols in number fields
(joint work with F. Amano)
Abstract: We introduce arithmetic Milnor invariants and multiple power residue symbols for primes in number fields, following the analogies between primes and knots. Our symbols generalize the Legendre, power residue symbols and the Redei triple symbol, and describe the decomposition law of a prime in certain nilpotent extensions of number fields. As a new example, we deal with triple cubic residue symbols by constructing concretely Heisenberg extensions of degree 27 over the cubic cyclotomic field with prescribed ramification. We also give a cohomological interpretation of our multiple power residue symbols by Massey products. This is a joint work with Fumiya Amano.

December 3rd (Wed)

Speaker: Jerome Dimabayao (Kyushu University)
Title: On the cohomological coprimality of Galois representations of a p-adic field
Abstract: Let K be a local or a global field and G_K its absolute Galois group. Given two continuous representations V and V' of G_K, we are interested in determining when and how they are "independent". Motivated by our efforts to generalize some results of Coates, Sujatha and Wintenberger, we introduce the notion of "cohomological coprimality" of such representations. We say that the two representations V and V' of G_K are "cohomologically coprime" if all the Galois cohomology groups corresponding to the field cut out by the representation V' (resp. V) having coefficients in V (resp. V') vanish. We consider the situation where K is a p-adic field and V and V' come from proper smooth varieties X and X' over K with potential good reduction, respectively. Then it can be shown that in many cases where X and X' have "quite different" nature, V and V' are cohomologically coprime. We will also discuss cohomological coprimality among elements of a system of ℓ-adic representations of G_K associated with a fixed X as above.

Speaker: Tomoki Mihara (University of Tokyo)
Title: On a new geometric construction of a family of Galois representations associated to modular forms
Abstract: We define a new étale sheaf on a modular curve. Every p-adic Galois representation associated to a cusp p-adic Hecke eigenform is obtained as a quotient of the étale cohomology of the sheaf. In particular, we give an alternative construction of a Λ-adic Galois representation associated to an ordinary cusp Λ-adic Hecke eigenform.

Speaker: Takahiro Tsushima (University of Tokyo)
Title: Perfectoid spaces I: Foundations (Survey)
Abstract: Our aim in this talk is to introduce the notions of perfectoid field and perfectoid space, and explain the content of tilting equivalence, which plays a key role in the foundation of the theory of perfectoid space due to P. Scholze. We briefly observe a special phenomenon which is captured in the theory of the norm field built by Fontaine-Wintenberger. We explain that this phenomenon is naturally generalized in the framework of perfectoid field or perfectoid space. Perfectoid space is defined as a special case of adic space which is due to R. Huber. Hence, to define perfectoid space, we prepare fundamental facts on adic space. As an application of the tilting equivalence, we introduce a theorem which asserts that the structure presheaf of a perfectoid space is actually a sheaf. This fact is a generalization of Tate's acyclicity theorem in this setting. If time is permitted, we also introduce a theorem which asserts that the etale site of a perfectoid space is isomorphic to the etale site of the tilt of the space.

December 4th (Thu)

Speaker: Wataru Kai (University of Tokyo)
Title: On the Albanese cokernel of varieties over p-adic fields
Abstract: Let X be a smooth projective variety over a p-adic field K. When X is a curve, S. Lichtenbaum described the cokernel of the canonical injection $\mathrm{Pic}^0(X) \to J_X(K)$ (J_X is the Jacobian variety of X) in terms of the connected components of the Picard scheme of X, as a corollary of his duality theory for p-adic curves in 1969. For higher dimensional X, there are two possibilities to generalize his result depending on whether one identifies the Jacobian variety (of a curve) with the Picard variety or with the Albanese variety. This difference amounts to whether one considers the Pic^0 of a curve as the Picard group or as the Chow group of zero-cycles of a variety. The Picard case was done by van Hamel in 2004.

In this talk, studying the Albanese case, we present a conjectural formula describing the cokernel of the Albanese map for zero-cycles

$$\mathrm{coker}\left(\mathrm{CH}_0(X)^{\deg=0} \to \mathrm{Alb}_X(K)\right)$$

(Alb_X is the Albanese variery of X) in terms of the Néron-Severi group and provide a proof of it under additional assumptions on an integral model of X. The proof depends on Saito-Sato's higher dimensional generalization of Lichtenbaum's duality and on Gabber-de Jong's comparison result of cohomological- and Azumaya-Brauer groups.

We will also briefly mention the local-global problem for the Albanese-cokernel; the abelian group on the "local side" turns out to be a finite group by our main theorem.

Speaker: Takamichi Sano (Keio University)
Title: On the Iwasawa main conjecture over a general number field
(joint work with D. Burns and M. Kurihara)
Abstract: Using the cyclotomic Iwasawa main conjecture over $\mathbb{Q}$, Burns-Greither solved most part of the equivariant Tamagawa number conjecture (ETNC) for abelian extensions over $\mathbb{Q}$ in 2003. In their "descent argument", Ferrero-Greenberg's formula and Solomon's theorem on "cyclotomic p-units" play important roles. In this talk, we formulate an Iwasawa main conjecture over a general number field, and generalize the argument of Burns-Greither for general number fields. In our descent argument, the conjecture recently formulated by Mazur-Rubin and the speaker independently plays an important role. This is a joint work with David Burns and Masato Kurihara.

Speaker: Kashio Tomokazu (Tokyo University of Science)
Title: Stark's conjecture over the rational number field and CM-periods of Fermat curves
Abstract: We will define a "period ring-valued beta function" and give a reciprocity law on its special values, by using some results on Fermat curves due to Rohrlich and Coleman. There is the following application: One can show that (a version of) Stark's conjecture holds true when the base field is the rational number field by using Euler's formulas and cyclotomic units. We will provide an alternative (and partial) proof by our reciprocity law. In other words, the reciprocity law given in this talk is a refinement of the reciprocity law on cyclotomic units.

Speaker: Tetsushi Ito (Kyoto University)
Title: Perfectiod Spaces II: Applications to number theory (Survey)
Abstract: This is a survey talk sequel to Tsushima's talk on foundations of the theory of perfectoid

spaces. It is now well-understood that perfectoid spaces have several striking applications to arithmetic and geometric problems such as the weight-monodromy conjecture for complete intersections, comparison theorems in p-adic Hodge theory, duality isomorphisms between the Rapoport-Zink towers at infinite level, construction of Galois representations associated with torsion elements in the cohomology of Shimura varieties as well as regular algebraic cuspidal automorphic representations of $GL(n)$ over totally real or imaginary CM fields. We plan to explain some ideas behind these applications briefly. The exposition will be very brief. Almost no proofs will be given.

Speaker: Fuetaro Yobuko (Tohoku University)
Title: Mass formula for abelian varieties
Abstract: The Eichler-Deuring mass formula says that the weighted number of isomorphism classes of supersingular elliptic curves over an algebraically closed field of characteristic p is expressed as a simple polynomial in p. In 2009, C.-F. Yu and J.-D. Yu generalized this formula for supersingular abelian surfaces. In this talk, we show a mass formula for supersingular abelian three-folds.

Speaker: Takuya Maruyama (University of Tokyo)
Title: An effective upper bound for the number of principally polarized abelian schemes
Abstract: Arakelov and Parshin showed that there are only finitely many isomorphism classes of non-isotrivial families of curves of given genus parameterized by a fixed base curve over $\mathbb{C}$. Gordon Heier gave an effective uniform bound for the number of such families. In this talk, I will explain how a similar bound for the number of families of principally polarized Abelian varieties is obtained when the base curve is proper.

December 5th (Fri)

Speaker: Yasuhiro Kishi (Aichi University of Education)
Title: Continued fraction expansion with even period and a primary symmetric part with extremely large end (joint work with F. Kawamoto, H. Suzuki, K. Tomita)
Abstract: For a non-square positive integer d with $4 \nmid d$, put $\omega(d) := (1 + \sqrt{d})/2$ if d is congruent to 1 modulo 4 and otherwise $\omega(d) := \sqrt{d}$. Moreover, for a positive integer ℓ, let A_ℓ denote the set of non-square positive integers d with $4 \nmid d$ such that the minimal periods of the simple continued fraction expansions of $\omega(d)$ are equal to ℓ. According to numerical experiments, for each ℓ with $1 \leq \ell \leq 63948$, the class number of real quadratic field $\mathbb{Q}(\sqrt{d_\ell})$ is equal to 1 except for $\ell = 7, 11, 49, 225, 299$, where d_ℓ is the minimal element of A_ℓ. Thus, in order to find many real quadratic fields of class number 1 we will have to know how to get the minimal element of A_ℓ. In this talk, we introduce a notion of "extremely large end (ELE)" for a finite string of positive integers to look for the minimal element and study their properties in even periods ℓ.

Speaker: John C. Miller (Rutgers University)
Title: On class numbers of cyclotomic fields and $\mathbb{Z}_p$-extensions
Abstract: The class number of cyclotomic fields has only been calculated for fields of rather small conductor, due to the difficulty of finding the "plus part" of the class number. By counting principal prime ideals, we establish class number upper bounds, allowing us to calculate the class number for real cyclotomic fields of larger conductor than has been previously possible. We also will survey some recent results and conjectures regarding the class numbers of fields in cyclotomic $\mathbb{Z}_p$-extensions over the rationals.

Speaker: Manabu Ozaki (Waseda University)
Title: Non-abelian Iwasawa theory of $\mathbb{Z}_p$-extensions (Survey)
Abstract: In this talk, I will give a survey on non-abelian Iwasawa theory of $\mathbb{Z}_p$-extensions, namely, theory of non-abelian restricted ramified (especially unramified and p-ramified) extensions over $\mathbb{Z}_p$-extensions of number fields.

Speaker: Takae Tsuji (Tokai University)
Title: On the Iwasawa λ-invariant of the cyclotomic $\mathbb{Z}_2$-extension of $\mathbb{Q}(\sqrt{p})$
(joint work with T. Fukuda, K. Komatsu, M. Ozaki)
Abstract: In the preceding works, Fukuda and Komatsu developed criteria for Greenberg conjecture of the cyclotomic $\mathbb{Z}_2$-extension of $k = \mathbb{Q}(\sqrt{p})$ with prime number p and showed $\lambda_2(k) = 0$ for all p less than 10^5 except $p = 13841, 67073$. All the known criteria at present can not handle $p = 13841, 67073$. We develop the structure theorem of cyclotomic units in the cyclotomic $\mathbb{Z}_2$-extension of the quadratic field k. Therefore, we obtain another criterion for $\lambda_2(k) = 0$, which is considered a slight modification of the method of the Ichimura and Sumida. Our new criterion fits the numerical examination and quickly shows that $\lambda_2(\mathbb{Q}(\sqrt{p})) = 0$ for $p = 13841, 67073$.

Speaker: Noriko Hirata-Kohno (Nihon University)
Title: New Diophantine criterion of polylogarithms over an algebraic number field
(joint work with M. Ito, Y. Washio and S. David)
Abstract: In the talk, we show the fisrt linear independence criterion concerning with the $s + 1$ numbers: 1 and s polylogarithms over an algebraic number field of degree ≥ 2, in the p-adic case as well as in the complex case.

Around the value of usual logarithmic function at non-zero point $\in \overline{\mathbb{Q}} \subset \mathbb{C}$, although the transcendence is only known, no algebraic independence result is known in the complex neither in the p-adic case.
For the polylogarithms, NO transcendence neither algebraic independence result exists in the cases. The only proven results are in positive characteristic ones which are much easier to be dealt with.

In 2003, T. Rivoal showed a lower bound for the dimension of the linear space spanned by polylogarithms by means of the method of Yu. V. Nesterenko. The result shows the existence of infinitely many irrational polylogarithms, however, his result does not imply any irrationality of a chosen polylogarithm.

Here we prove the first linear independence criterion of polylogarithms in the p-adic and the complex cases, over a number field of arbitrary finite degree over $\mathbb{Q}$. We also construct infinitely many explicit examples of irrational, or linearly independent polylogarithms over a number field of given degree over $\mathbb{Q}$.

Let $Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}$, for $z \in \mathbb{C}, |z| \leq 1 (z \neq 1$ if $s = 1)$ and consider $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha| < 1$. We obtain:

if the absolute value of α is relatively small, then the $s + 1$ numbers:
$Li_1(\alpha)$, $Li_2(\alpha), \cdots, Li_s(\alpha)$ and 1 are linearly independent over $\mathbb{Q}(\alpha)$. In the p-adic case, for $\alpha \in \overline{\mathbb{Q}}$ with $0 < |\alpha|_p < 1$, we also give a criterion of similar nature relying on Diophantine approximations so-called Padé approximation.
The p-adic case is proven in collaboration with Sinnou David (University of Paris VI). The complex case

together with construction of examples is a joint work with Masaru Ito (Tokyo Institute of Technology) and Yusuke Washio (Nihon University).

Program Committee:
Takeshi Tsuji (Univ. of Tokyo), Hiroki Takahashi (Univ. of Tokushima), Yuichiro Hoshi (RIMS, Kyoto Univ.)