SW 類

X/k proper smooth $\dim = n$ $\ell \neq \mathrm{ch}$ $\mathrm{ch}\,k \neq 2$

$H^n(X_{\bar{k}}, \mathbb{Q}_\ell)$ orthogonal or symplectic acc. to parity of n

n odd.

n even $\det H^n = \underset{e_n}{\underset{\shortparallel}{\text{位数}2}} \cdot \left(-\frac{n \cdot b_n}{2}\right)$

$e_n \in$ ~~…~~ $H^1(\mathbb{G}k, \mathbb{Z}/2\mathbb{Z}) = k^\times/k^{\times 2}$

$$e_n = \{d\} + \begin{cases} r\{-1\} & n \equiv 0 \ (4) \\ (r + b_{dR,n})\{-1\} & \equiv 2 \ (4) \end{cases}$$

X proj
JAG
Lefschetz pencil.

$r = \sum_{g<n} (-1)^g b_{dR,g}$ d. disc. of deRham coh $H^n_{dR}(X/k)$. $\mathrm{ch}\,k \neq 2$

H^n の問題ではなく. $R\Gamma(X)$.

H^1 det vs disc. SW_1 of $\overset{\text{ortho}}{\text{Gal rep}}$, vs sym. form

- 2次の不変量の定義
- 予想の定式化
- 定理 X n次元 smooth hypersurface $\ell > n+1$

証明. - degeneration
- p-adic Hodge
- Hodge str
- transcendental
- moduli.

cf. 第2 S.W 類 $\xrightarrow{\text{Deligne}}$ local ε-factor $\xrightarrow{\text{類似}}$ 関数等式の符号 $= +$.

- dep'd on ℓ.

1. 第2 SW 類

π. profinite C. 有限 Abel. $1 \to C \to E \to \pi \to 1$ central ext

$[E] \in H^2(\pi, C) \leftarrow H^1(\pi, \pi)$
$\longmapsfrom$ id

$\rho: \pi \to O(V)$ orthogonal ℓ-adic rep'n

$$1 \to \mu_2 \to \tilde{O}(V) \to O(V) \to 1$$

$\tilde{O}(V)$ Spinor Clifford alg $Cl(V) = T^\bullet(V)/(v \otimes v - q(v))$
$\dim Cl(V) = 2^{\dim V}$

$\widetilde{O}(V) = \ker(C(V)^\times \to \mathbb{G}_m)$

$x \mapsto g(x)$

$\downarrow \quad \overset{x}{\mathrm{I}} \quad$ gen. by $x \in V$. st. $g(x) \neq 0$

$O(V)$

reflection.

$y \mapsto \cancel{\ldots}\, y - 2b(x.y)x \qquad b(x.y) = 0 \Rightarrow y$

$x \mapsto -x.$

$b(x.x) = g(x) = 1$

$\ker =$ $x_1, \ldots, x_n$ 正規直交基底

$x_1 \cdots x_n.$

$sw_2(\rho) = [$ pull back of $\widetilde{O}$ by $\rho: \pi \to O(V)]$

$1 + \det\rho + sw_2(\rho).$ $\qquad \pi = G_K \quad V = H^{n}(X_{\bar{K}}, \mathbb{Q}_\ell(\tfrac{n}{2}))$

$D.$ $x_1, \ldots, x_n$ 直交基底 $\quad a_i = \langle x_i, x_i \rangle$

$\operatorname{disc} D = a_1 \cdots a_n. \qquad hw_2(D) = \sum_{1 \le i < j \le n} \{a_i, a_j\} \in H^2(K, \mathbb{Z}/2)$

$K^\times / K^{\times 2}$

$1 + \operatorname{disc} D + hw_2(D).$

2、予想

X/K proper smooth $\quad q. \qquad \det H^q = e_q(-\frac{b_q \cdot q}{2})$ order 2.

$e = \sum_{q<n} e_q \qquad q \cdot b_q$ odd $\Rightarrow K \supset \mathbb{F}_{p^2}$

n . even

$\beta = \frac{1}{2} \sum_{q<n} (n-q) \cdot b_q \qquad$ char 0

c_ℓ gen of $H^2(\pi_1(S_p \mathbb{Z}[\frac{1}{\ell}])^{ab}, \mathbb{Z}/2\mathbb{Z}) \simeq \mathbb{Z}/2\mathbb{Z}.$ の生成元

char $\neq 0$ なら 0

$sw_2(H^n_\ell) + \{e, -1\} + \beta \cdot c_\ell$

$= hw_2(H^n_{dR})$

$$+ \begin{cases} r\{d_X, -1\} + \binom{n}{2}\{-1,-1\} & n \equiv 0 \ (4) \\ (r + b_{dR,n} - 1)\{d_X, -1\} + \binom{r + b_{dR,n}}{2}\{-1,-1\} & n \equiv 2 \ (4) \end{cases}$$

$+ \{2, d_X\} + \eta(c_\ell - c_2)$

$\eta = \sum_{q < \frac{n}{2}} (-1)^q (\frac{n}{2} - q)\chi(X, \Omega^q) , \qquad r = \sum_{q<n} (-1)^q b_{dR, q}$

l := dep'd. $K = \mathbb{Q}_p$ X/K. good red. $p \neq 2$ $hw_2 = 0$

$l \neq p \Rightarrow$ $sw_2 = 0$,

d, e. unramified

$X = A$ abelian surface $\eta = 0$ $\beta = \frac{1}{2}(2\cdot 1 - 1\cdot 4) = -1$

A a-s $/\mathbb{R}$

$X = \mathrm{Spec}\, L$. L/K fin. sep. ext

$$sw_2(H^0) = hw_2 \mathrm{Tr} + \{2, d\}$$ Serre.

略証..

cx. version

$$sw_2(H^n_l) = hw_2(H^n_{dR}) + \underset{\substack{\uparrow \\ \text{geom reason}}}{\{2, d_X\}} + \eta(c_l - c_2)$$

Theorem 予想は次の場合に成立する

1. $K/\mathbb{Q}_p$ finite $p \neq 2, l$ $\exists X_{\mathcal{O}_K}$ proj reg. flat model.
   s.t X_F has at most isolated ord. double pts.
2. $K/\mathbb{Q}_p$ unramified $p = l > n+1$. good red.
3. $K = \mathbb{R}$ X projective
4. $k \supset \overline{\mathbb{Q}}$.
5. X smooth hyper surface of dim n. $l > n+1$

方針. 2 p進Hodge Fontaine-Laffaille

good red $hw_2 = 0$

3 Hodge str. Lefschetz decomposition polarization $\sim$ positivity.

4 $k \supset \mathbb{C}$ finitely gen $X \to S/\mathbb{C}$ $\underset{\substack{\uparrow \\ \text{locally free}}}{\mathcal{H}^n_{dR}} = \underset{\substack{\uparrow \\ \text{local system}}}{H_B} \otimes_{\mathbb{C}} \mathbb{Q}$.

1. blup

$$\begin{array}{ccc} & X' & \leftarrow & (X' \otimes_{O_K} O_L)^{\text{[illegible]}} \text{ の正規化.} \\ \swarrow & & & \downarrow \\ X & \text{blup} & & \\ \downarrow & & & \\ O_K & \longleftarrow & & O_L \end{array}$$

[illegible]

$D \leftarrow D'$ double covering

irred. cpt

∂sw_2. ∂hw_2 $\quad H^2(K, \mathbb{Z}/2) \xrightarrow{\partial} H^1(F, \mathbb{Z}/2)$

$\det H^{n}_{\ell}(D') / \det H^{n}_{\ell}(D)$, $\quad$ ~~det~~ $\operatorname{disc} H^{n}_{dR}(D') / \operatorname{disc} H^{n}_{dR}(D)$

ここでずれがでる

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hypersurface $\qquad$ universal family について示す.

$$\mathbb{P}(\Gamma(\mathbb{P}^{n+1}_{\mathbb{Z}}, \mathcal{O}(m))^*) = P \supset U = P - D \quad \text{discriminant.}$$

X_D smoothなところ

$$\begin{array}{ccc} X & \hookrightarrow & P \times \mathbb{P}^{n+1} \\ & \searrow & \downarrow \\ & & P \end{array}$$

univ. hypersurface

$H^2(\mathbb{Z}[\tfrac{1}{2}], \mathbb{Z}/2)$

$$\begin{array}{ccccc} & & H^2(\mathbb{Z}[\tfrac{1}{2}], \mathbb{Z}/2) & & \\ & & \downarrow & & \\ H^2(P_{\mathbb{Z}[\frac{1}{2}]}, \mathbb{Z}/2) & \longrightarrow & H^2(U_{\mathbb{Z}[\frac{1}{2\ell}]}, \mathbb{Z}/2) & \xrightarrow[1,2]{\partial} & H^1(D_{\mathbb{Z}[\frac{1}{2\ell}]}, \mathbb{Z}/2) \oplus H^1(U_{\mathbb{F}_\ell}, \mathbb{Z}/2) \\ \downarrow & & \downarrow \; 4 & \searrow^{3} & \\ H^2(P_{\bar{\mathbb{Q}}}, \mathbb{Z}/2) & \to & H^2(U_{\bar{\mathbb{Q}}}, \mathbb{Z}/2) & & H^2(\mathbb{R}, \mathbb{Z}/2\mathbb{Z}) \end{array}$$

$\mathbb{Z}/2\mathbb{Z} \to H^2(P_{\bar{\mathbb{Q}}}, \mathbb{Z}/2)$

$1 \longmapsto [D] = 0$