

# SW 類

$X/K$  proper smooth  $\dim = n$   $l \neq \text{ch}$   $\text{ch} \neq 2$

$H^i(X_{\bar{K}}, \mathbb{Q}_\ell)$  orthogonal or symplectic acc. to parity of  $i$

$n$  odd  
 $n$  even

$$\det H^n = i^{\frac{n}{2}} (-1)^{\frac{n}{2}} \cdot \left(-\frac{n \cdot b_n}{2}\right)$$

$e_n + \dots H^i(\mathbb{Q}_\ell, \mathbb{Z}/\ell\mathbb{Z}) = K^* \otimes_{K^*} \mathbb{Z}/\ell\mathbb{Z}$

$e_n = \{d\} + \begin{cases} r \} - 14 \\ (r + b_{dR, n}) \} - 14 \end{cases} \quad n \equiv 0 \pmod{4} \\ \equiv 2 \pmod{4}$

$X$  proj  
JAG  
Lefschetz pencil

$$r = \sum_{g < n} (-1)^g b_{dR, g}$$

d. disc. of de Rham coh

$H^i$  a problem:  $\Gamma(X)$

$H^i_{\text{deR}}(X/K)$   $\text{ch} \neq 2$

$H^1$  det vs disc.

SW<sub>1</sub> of Gal. rep, ortho vs sym. form

- 2次の不变量の計算
- 予想の証明
- ~~証明~~  $X$   $n$ -次元 smooth hypersurface  $\dim > n+1$

- 証明.
- degeneration.
  - p-adic Hodge.
  - Hodge. str
  - transcendental.
  - moduli.

## 例. $\neq 2$ SW 類

Deligne local  $\varepsilon$ -factor  $\rightarrow$  関数方程式の符号 = +  
証明

dep'd on  $l$ .

## 1. $\neq 2$ SW 類

$\pi$  profinite C. 有限 Abel.  $1 \rightarrow \mathbb{C} \rightarrow E \rightarrow \pi \rightarrow 1$  central ext

$$[E] \in H^2(\pi, \mathbb{C}) \leftarrow H^1(\pi, \mathbb{C}) \leftarrow \text{id}$$

$P: \pi \rightarrow O(V)$  orthogonal  $l$ -adic rep'n

$$1 \rightarrow \mathbb{Z} \rightarrow \hat{O}(V) \rightarrow O(V) \rightarrow 1$$

$\hat{O}(V)$  Spinor Clifford alg  $Cl(V) = T^*(V)/(v \otimes v - q(v))$   
 $\dim Cl(V) = 2^{\dim V}$

$$\widehat{\mathcal{O}}(V) = \ker(C(V)^X \rightarrow \mathbb{F}_m)$$

$$\downarrow \quad \begin{array}{c} x \\ \uparrow \\ \text{gen. by } x \in V. \text{ st. } g(x) \neq 0 \end{array} \quad x \mapsto g(x)$$

$\mathcal{O}(V)$  reflection

$$g \mapsto \cancel{g} \rightarrow g - 2b(x, g)x$$

$$b(x, g) = 0 \Rightarrow g$$

$$\& x \mapsto -x.$$

$$b(x, x) = g(x) = 1$$

$$\ker = x_1, \dots, x_n \text{ 正交且两两垂直}$$

$$x_1, \dots, x_n.$$

$$SW_2(P) = [\text{pull back of } \widehat{\mathcal{O}} \text{ by } P: \mathbb{A}^n \rightarrow \mathcal{O}(V)]$$

$$1 + \det P + SW_2(P).$$

$$\pi = \text{Gr}_k \quad V = H^m(X, \mathcal{O}_2(\frac{n}{2}))$$

$$D. \quad x_1, \dots, x_n \text{ 正交且两两垂直} \quad a_i = \langle x_i, x_i \rangle$$

$$\text{disc } D = a_1 \dots a_n.$$

$$k^{\times} / k^{\times 2}$$

$$hw_2(D) = \sum_{1 \leq i < j \leq n} \{a_i, a_j\} \in H^2(k, \mathbb{Z}/2)$$

$$1 + \text{disc } D + hw_2(D).$$

2.  $\mathbb{F}_2$

$X/k$  proper smooth

$g$ .

$$\det H^2 = \sum_{g \text{ order 2.}} \left(-\frac{b_g \cdot g}{2}\right)$$

$$e = \sum_{g < n} e_g$$

$$g - b_g \text{ odd} \Rightarrow \mathbb{F}_k \supset \mathbb{F}_p$$

$n$  even

$$\beta = \frac{1}{2} \sum_{g < n} (n - g) \cdot b_g \quad \text{char } 0$$

$$c_e \text{ gen of } H^2(\pi_1(S_p, \mathbb{Z}[\frac{1}{2}]), \mathbb{Z}/2) \cong \mathbb{Z}/2. \quad \text{a } \mathbb{F}_2$$

$$\text{char} \neq 0 \Rightarrow \beta = 0$$

$$SW_2(H^2) = \{e, -1\} + \beta \cdot c_e$$

$$= hw_2(H^2_{\text{dr}})$$

$$+ \left\{ \begin{array}{l} r \{dx, -1\} + \binom{r}{2} \{-1, -1\} \end{array} \right. \quad n \geq 0 \quad (A)$$

$$\left\{ \begin{array}{l} (r + b_{\text{dr}, n} - 1) \{dx, -1\} + \binom{r + b_{\text{dr}, n}}{2} \{-1, -1\} \end{array} \right. \quad n \geq 2 \quad (B)$$

$$+ \{2 \cdot dx\} + \eta (c_e - c_2)$$

$$\eta = \sum_{g < \frac{n}{2}} (-1)^g \binom{n}{2-g} \chi(x, \Omega^g), \quad r = \sum_{g < n} (-1)^g b_{\text{dr}, g}$$

$l=2$  dep'd.  $K = \mathbb{C}_p \ X/k$ . good red.  $p \neq 2$   $hw_2 = 0$

$l \neq p \Rightarrow Sw_2 = 0$ .

d.s.e. unramified

$X = A$  abelian surface  $\eta = 0$   $\beta = \frac{1}{2}(2 - 1 - 1 \cdot 4) = -1$

$A$  a.s.  $\mathbb{R}$

$X = S_p L$ .  $L/k$  fin. sep. ext

$$Sw_2(H^0) = hw_2 Tr + \int 2 \cdot dx$$

Serre.

§42E.

CX. version

$$Sw_2(H^0_L) = hw_2(H^0_{\text{Hodge}}) + \int 2 \cdot dx + \eta(c_1 - c_2)$$

↑  
geom reason

Theorem  $\exists$   $\mathbb{A}^1$   $\rightarrow \mathbb{A}^1$   $\mathbb{A}^1/\mathbb{C} = \mathbb{A}^1/\mathbb{C}$   $\mathbb{A}^1/\mathbb{C}$

1.  $K/\mathbb{C}_p$  finite  $p \neq 2, l$   $\exists X_{\mathbb{C}_K}$  proj reg. flat model.

s.t.  $X_p$  has at most isolated ord. double pts.

2.  $K/\mathbb{C}_p$  unramified  $p = l > n+1$  good red.

3.  $K = \mathbb{R}$   $X$  projective

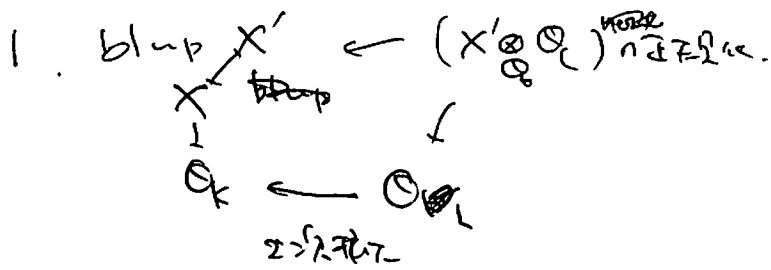
4.  $K > \mathbb{Q}$ .

5.  $X$  smooth hyper surface.  $l > n+1$   
of dim  $n$ .

Def. 2  $p \neq 2$  Hodge Fontaine-Lafaille  
good red  $hw_2 = 0$

3 Hodge ser. Lefschetz decomposition polarization or positivity.

4  $K > \mathbb{C}$  finitely gen  $X \rightarrow S/\mathbb{C}$   $H^0_{\text{ét}} = H_B \otimes_{\mathbb{C}} \mathbb{Q}$ .  
↑ ↑  
locally free local system



$D \leftarrow D'$  double covering  
 invad. cpt

$\partial Sw_2, \partial hw_2 \quad H^2(k, \mathbb{Z}/2) \xrightarrow{\partial} H^1(F, \mathbb{Z}/2)$

$\det H^2(D) / \det H^2(D), \det \text{disc } H^1(D') / \det \text{disc } H^1(D)$

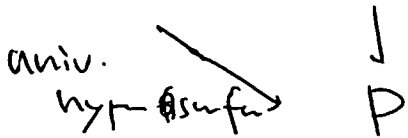
$= 2^{-2} 2^{n-2} 2^{-3}$

hypersurface ~~on~~ universal family  $1 \leq i \leq 2, \bar{F}, \bar{F}$ .

$\mathbb{P}(\Gamma(\mathbb{P}_{\mathbb{Z}}^{n+1}, \mathcal{O}(m))) = P \supset U = P - D$  discriminant.

$X \hookrightarrow P \times \mathbb{P}^{n+1}$

$X_D$  smooth  $\bar{F} \rightarrow \bar{F}$



$H^2(\mathbb{Z}[\frac{1}{2}], \mathbb{Z}/2)$

$H^2(P_{\mathbb{Z}[\frac{1}{2}]}, \mathbb{Z}/2) \rightarrow H^2(U_{\mathbb{Z}[\frac{1}{2}]}, \mathbb{Z}/2) \xrightarrow{\partial} H^1(D_{\mathbb{Z}[\frac{1}{2}]}, \mathbb{Z}/2) \oplus H^1(U_{\mathbb{Z}[\frac{1}{2}]}, \mathbb{Z}/2)$

$\mathbb{Z}/2 \rightarrow H^2(P_{\mathbb{Q}}, \mathbb{Z}/2) \rightarrow H^2(U_{\mathbb{Q}}, \mathbb{Z}/2) \rightarrow H^2(\mathbb{R}, \mathbb{Z}/2)$

$1 \mapsto [D] = 0$