

Ultimate

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Goal: Define the characteristic cycle of a smooth \mathbb{Z} -adic sheaf ramified along the boundary as a cycle in the cotangent bundle.

~~Partial~~ Partial answer.

Under a certain "non degenerate" condition, $\text{codim of complement} \geq 2$.

Consequences.

1. Computability with the pull-back by "non-characteristic" morphism.

functionality of the construct

Characterization by the method of cutting-by-curves

2. Local acyclicity with respect to "non-characteristic" morphism.

reduce to the case rel dim 1. Deligne-Lusztig

3. Compute the characteristic class and the Euler number.

Missing

Compatibility with proper pushforward?

$f: X \rightarrow Y$
Spoke

Related

Further results

(a) Approach using jet bundles by Deligne.
- Tangent bundle suffices.

(b) rank 1 case by Kato.
- higher rk.

(c) logarithmic version by S. and Abh.-S.
- cutting-by-curves

~~Method~~ Method to link resolution to tangent bundle. (2)

1) Blow up at the resolution divisor embedded in the diagonal.

2) Groupoid structure of multiple self products.

Notation

k perfect field of char $p > 0$

X smooth sep scheme of f.t. k

$d = \dim X$
 k_i local field \mathbb{C}_{x_i}

$U \subset X \supset D$ div. wr. SNC D_1, \dots, D_e irred cpt

Λ ~~noetherian~~ ring finite over \mathbb{Z} $l \neq p$. condition on ass. rep in \mathbb{C}_w

\mathcal{F} smooth sheaf of Λ -modules on U . flat unique jump is \mathbb{Z} mod p ✓ satisfied

Assumptions Resolution of \mathcal{F} along D is isomorphic of slope $R = v_1 D_1 + \dots + v_e D_e$ for integers $v_i \geq 1$ (simplifying) wildly ram

Resolution is "non-degenerate" at multiplicity R (serious) (nothing particular in codim ≥ 2)

Main construction

$$\text{Char } \mathcal{F} \in \mathbb{Z}_d \left(T^d X \times_{\mathbb{Z}} \mathbb{C}_p \right)$$

linear combination of subline bundles defined over a finite covering of irreducible cpt.

+ $v_k \mathcal{F} \times \mathcal{O}$ -section, $\times (-1)^d$

Example 1. $\dim X = 1$ $D = \{x\}$

$$\text{Char } \mathcal{F} = (-1) \left(v_k \mathcal{F} \left[T^d X \right] + \dots + \text{tot}_x \mathcal{F} \cdot \left[T^d X \right] \right)$$

\uparrow \uparrow \uparrow
 \mathcal{O} -section $v_k + \text{Sw}$ fibers

2 $X = \mathbb{A}^2 \ni (x, y) \supset U = S_{\mu} \mathbb{P}^1 \ni (x, y)$ [3]

$\text{aff } \tau = 1 \quad t^D - t = \frac{1}{x^n} \quad p + n.$

$\text{Char } \tau = [T_x^k X] + (n+1) [T_D^k X]$

b) $t^D - t = \frac{y}{x^n} \quad \text{Lagrangean} \quad p \mid n \quad (n \neq 2) \text{ f.p.}$

$= [T_x^k X] + \int n = \left[\frac{dy}{D} \right] \leftarrow \text{otherwise } n=p=2$

Assume τ is trivialized by a finite non-Lagrangean Galois cover $V \rightarrow U$ of $\text{gp } G$.
 Ramification of G -torsor $F = D^{(1/p)} \rightarrow D$

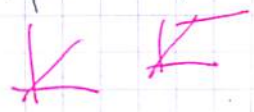
$R = \sum n_i D_i \subset X \iff X \times X$

$D^{(1/p)} = D \times_{\mathbb{F}_2} \mathbb{F}_2 \leftarrow \text{inv. of } \mathbb{F}_2$

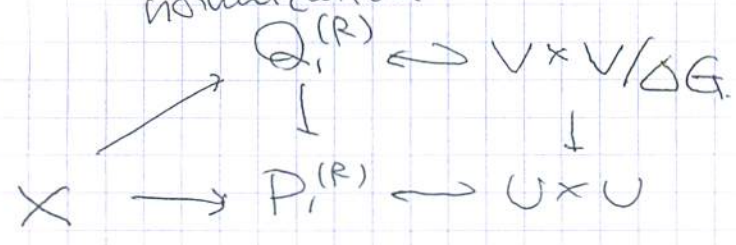
$\uparrow P_i^{(R)}$

blow up and remove prop. transf. of $D \times X \times X$

$T^{(R)} = TX(-R) \times_X D \hookrightarrow P_i^{(R)} \supset U \times U$



$V \rightarrow U$ finite étale G -torsor. G finite gp



Def'n R is bdd by R_t of

$Q_i^{(R)} \rightarrow P_i^{(R)}$ is étale on a nbd of $X \subset Q_i^{(R)}$

Groupoid $U \times U \cong U$

$(U \times U) \times (U \times U) = U \times U \times U \rightarrow U \times U$

is extended to $P_i^{(R)} \times_X P_i^{(R)} \rightarrow P_i^{(R)}$

$(V \times V / \Delta G) \times (V \times V / \Delta G) = (V \times U \times V) / \Delta G \rightarrow V \times V / \Delta G$

$W_i^{(R)} \subset Q_i^{(R)}$ max open étale over $P_i^{(R)}$

Thm. Bdd by $R \iff W_i^{(R)}$ inherits a gp d. str. existence of the unit section.

$$E_1^{(R)} = W_1^{(R)} \times_X D \rightarrow P_1^{(R)} \times_X D = TX(-R) \times_X D$$

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étale morphism of smooth gp schemes / D.

$$E_1^{(R)0} \subset E_1^{(R)}$$

max open sub gp scheme

s.t. $E_x^{(R)0} \subset E_x^{(R)}$ is the conn. cpt. containing the unit section

Def'n Non degenerate: $E_x^{(R)0} \rightarrow T^{(R)}$ is finite

$$0 \rightarrow G^{(R)} \rightarrow E^{(R)0} \rightarrow T^{(R)} \rightarrow 0$$

extension of a vector bundle by a finite étale group scheme

$G^{(R)}$. étale locally ism to \mathbb{F}_p^n for some n .

Extension is classified by

$$G^{(R)0} = \text{Hc}(G^{(R)}, \mathbb{F}_p) \rightarrow T^{(R)0} = \text{Hc}(T^{(R)}, \mathbb{F}_p)$$

$$(0 \rightarrow \mathbb{F}_p \rightarrow \mathbb{F}_p \xrightarrow{T \rightarrow TP - T} \mathbb{F}_p \rightarrow 0)$$

defined as a rad. cov. of D.

étale locally $G^{(R)} \subset G$ isodivine $\Rightarrow P_2/G^{(R)} = \oplus X$ sum of nontrivial char.

1. Computability with the pull-back.

$$\text{Char}(\mathbb{F}_p) = (-1)^d (\text{rk } T^*X + \sum_i L(x_i))$$

- functoriality of the construction

2. Local ~~ag~~ adicity

reduction to

- rel dim 1 case Deligne-Laman.

3. Characteristic class.

- vanishing of $R_{P_2}(R_{J^*} \mathcal{L}_{T^{(R)}})$