

Hasse-Arf theorem in high dim. (j.w/k-kato) |

- Plan.
1. Classical H.A. thm
  2. Gen'l to high dim.
  3. Rank 1 case
  4. Ramification of char.

1.  $k$  local fld. (= c.d.v. f w/ perfect res. fld.)

$L/k$  finite Galois ext.  $G = \text{Gal}(L/k)$

$M$ . rep'n of  $G$

$$\text{Sw}_F M = \frac{1}{|G|} \sum_{\sigma \in G, \neq 1} S_G(\sigma) (\text{Tr}(\sigma: M) - \dim M)$$

$$S_G(\sigma) = \begin{cases} -\text{ord}_L(\sigma(\pi_k)/\pi_k - 1) & \sigma \in I, \neq 1 \\ 0 & \sigma \notin I \end{cases}$$

H.A. thm

$$\text{Sw}_F M \in \mathbb{N}.$$

2.  $S = \text{Sp}_n \mathbb{O}_K$ .  $U/S$ . any flat sep. sds of f. z  
 $f: U \rightarrow U$  finite étale Galois cov.  $G = \text{Gal}$ .

$S \times V_k \rightarrow U_k$  tamely ramified.

$M$ . rep'n of  $G$

$$\text{Sw}_U M = \frac{1}{|G|} \sum_{\sigma \in G, \neq 1} S_{G|K}(\sigma) (\text{Tr}(\sigma: M) - \dim M)$$

$$\in F_0 G(X_F) \mathbb{Q}$$

$$V \hookrightarrow Y$$

$$\downarrow \quad \downarrow f$$

$$U \hookrightarrow X \text{ proper/S}$$

$$S_G(\sigma) = \bar{f}_* (-((T_\sigma, \Delta_U)))$$

~~Assume~~

$Y$  regular

$V = Y \setminus D$  cpt of SNCD.

$(Y \times Y)$  log product.

$$G \curvearrowright Y$$

$$\langle \cdot, \cdot \rangle = (-1)^d ([\text{Tor}_d^{\mathcal{O}_{(Y \times Y)}}(\mathcal{O}_F, \mathcal{O}_{\Delta_Y})] - [\text{Tor}_{d+1}^{\mathcal{O}_{(Y \times Y)}}])$$

In general. alteration.

classical case  $Y \approx \Theta_L = \Theta_F[T]/f(T)$   $f(T)$  Eisenstein

$$(Y \approx Y) \approx : \Theta_L \Theta_F \Theta_L [U^{1/\tau}] / (1 - \tau - \tau \theta \cdot U)$$

$$= \Theta_L [U^{1/\tau}] / f(\tau U) = A$$

$$f(\tau U) = (U - \sigma(\tau)/\tau) \cdot g_\sigma$$

$$A \xrightarrow{g_\sigma} A \xrightarrow{U - \sigma(\tau)/\tau} A \rightarrow \Theta_F \rightarrow 0$$

$\Theta_A \Theta_L$

$$\Theta_C \rightarrow \Theta_L$$

0

$$\Theta_F \xrightarrow{(-\sigma(\tau)/\tau)} \Theta_L$$

Theorem (Kato-S.) If  $\dim U_F = 1$ ,

$Sw_0 M$  is in ~~the~~ Image  $(CH_0(X_F) \rightarrow F_0 G(X_F) \otimes \mathbb{Q})$ .

Application to Artin char.

A: ~~regular~~ regular local.  $G$  finite gp of  $A$ .  
 s.t.  $A^G$  noetherian,  $A/I_\sigma$  finite length for  $\sigma \neq 1$   
 $I_\sigma = (a - \sigma(a); a \in A)$

Define  $a_G(\sigma) = \begin{cases} - \text{length } A/I_\sigma & \sigma \neq 1 \\ - \sum_{\tau \neq 1} a_G(\tau) & \sigma = 1. \end{cases}$

Conj (Serre)  $a_G$  is a char of  $G$ .  
 Thm (Kato) Conj is true if  $\dim A \leq 2$ .

3. Pf of Thm.

Redn to v.l. case: Brauer + Induction formula.  
 Rank 1 case  $\Gamma \leftarrow X \in H^1(U, \mathbb{Q}(\mathbb{Z}))$ .

Define  $C_X \in CH_0(X_F)$  and show  $C_X \mapsto Sw_0 M$ .  
 $\dim X = 2$   $X$  regular,  $U = X \setminus D$   $D$  snc d.

- Swan divisor  $R_X = \sum v_i D_i$ .
- $X$  "clean".

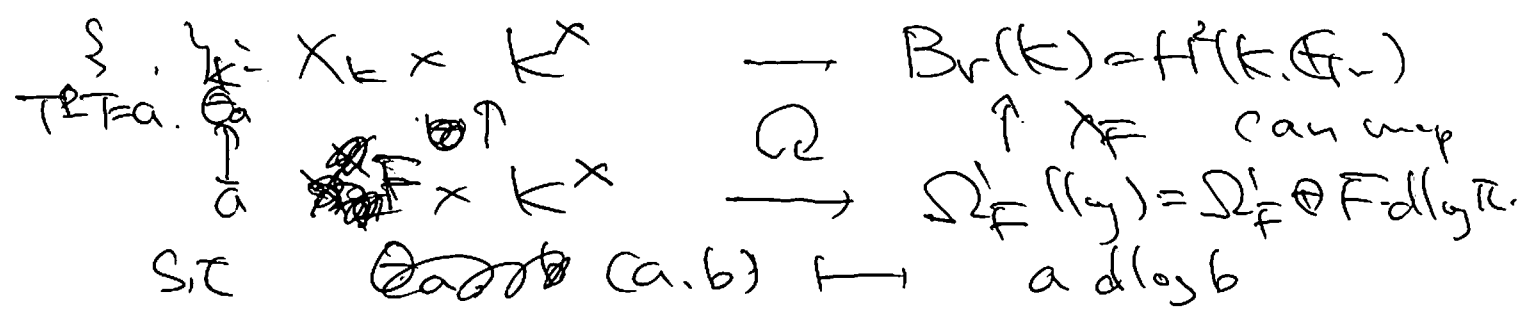
$$C_X = \sum_i c_i (\Omega_{X/\mathbb{F}}^1(\log D) / D_i) \wedge v_i D_i \in \mathbb{R}^2$$

- $C_X \mapsto Sw_0 M$ .
- ved to odd  $X = p^n$ .

$n=1$  Compute then explicitly ~~the~~  
 general induction on  $n$

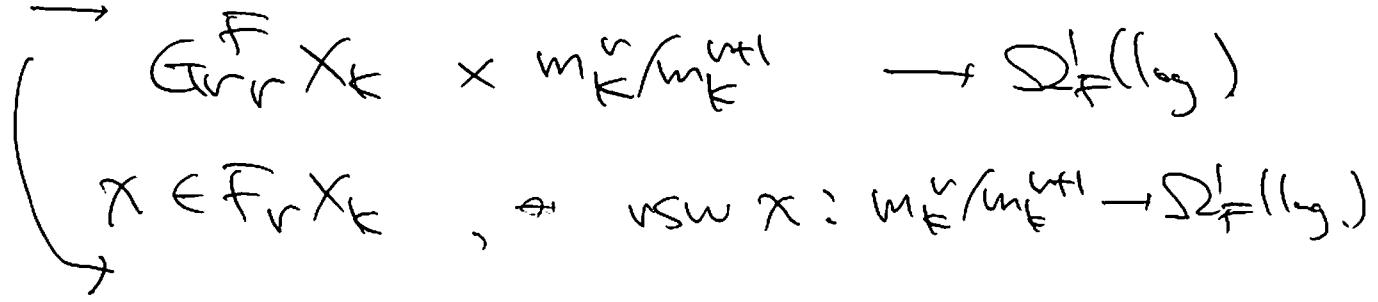
4.  $K$  cd.v.f. vcs fd may not be prof.

$$\text{Fil'n on } G_K^{ab} \leftrightarrow X_K = H^1(K, \mathbb{Q}(2)) = H^2(K, \mathbb{Z})$$



$$\text{Fr } X_K = \{ x \in X_K \mid \exists x, (1 + m_K^r \otimes_L) \subset (I - \lambda_{F_L}) \text{ for } \forall L/K. \}$$

Kummer



$X \supset U = X \setminus D$   $D_i$   $K_i$ : local fd at gen pt  
 $M \rightarrow X$  char of  $G_K^{ab}$   $v_i$   $R = \sum v_i D_i$

$$\text{vsw}_x : \mathcal{O}(-R)|_{D_i} \rightarrow \Omega_{X/S}^1(\log D)|_{D_i}$$

Clear.  $\text{vsw}_x$  locally direct sum.

$K$  mixed char  $\sum_p \in K$   $\overset{m}{\circlearrowleft} = p \cdot \text{ord}_K(\sum_p - 1) = \frac{p}{p-1} \text{ord}_K p$

$$X_K[p] = H^1(K, \mathbb{Z}/p\mathbb{Z}) \xrightarrow{\sim} K^x / (K^x)^p = H^1(K, \mathcal{O}_K/p)$$

$$\cup \quad \cup$$

$$\text{Fil}^r X_K[p] \quad \cong \quad \text{Im } I + M_K^{m-r}$$

$$\chi = \quad \quad \quad \leftarrow \quad I + b$$

vs  $\chi$   $c \mapsto \frac{1}{2^r} c db = \frac{1}{2^r} bc \cdot d \log b$   $v < u$

$c \mapsto \frac{c}{2^r} d \log a.$   $v = u,$