Étale coh & char cycle. 1

Zeta f'n

X (proper smooth) alg var / $\mathbb{F}_q$ finite fd of q-elt.

$$Z(X/\mathbb{F}_q, t) = \exp\left(\sum_{m=1}^{\infty} \frac{\# X(\mathbb{F}_{q^m})}{m} t^m\right)$$

e.g. $$Z(\mathbb{P}^n_{\mathbb{F}_q}/\mathbb{F}_q, t) = \frac{1}{(1-t)(1-qt)\cdots(1-q^n t)}$$

$X = \operatorname{Spec} A$

$$Z(X/\mathbb{F}_q, q^{-s}) = \prod_{\substack{m:\text{ maxideal}\\ \text{of } A}} \frac{1}{1-\#(A/m)^{-s}}$$

$$\left(A = \mathbb{Z} \qquad = \zeta(s)\right)$$

Weil conj. good coh th with Lefschetz trace fmla.

$$Z(X/\mathbb{F}_q, t) = \prod_{i=0}^{2\dim X} \det(1-Ft : H^i(X))^{(-1)^{i+1}}$$

α eigenvalue of F on H^i

$H^i(X)$ constructed by Grothendieck $\Rightarrow$ complex abs val $|\alpha| = q^{\frac{i}{2}}$

ℓ-adic étale coh $H^i(X_{\bar{\mathbb{F}}_q}, \mathbb{Q}_\ell)$.

coefficients, rel. version.

$\mathcal{F}$ on X. ℓ-adic sheaf, complex

$f : X \longrightarrow Y$

$Rf_*, Rf_!$ $\longrightarrow$ $D(X, \mathbb{Q}_\ell) \rightleftarrows D(Y, \mathbb{Q}_\ell)$

derived cat of ℓ-adic sheaves

$f^*, Rf^!$ $\longleftarrow$ $\otimes$, $R\mathcal{H}om$. 6-operations

more fundamental than

Δ-op's $+, -, \circ, /$.

$\mathbb{Q}_\ell$ constant sheaf on X 2

k base fd ($=\mathbb{F}_q$)

$H^*(X_{\bar{k}}, \mathbb{Q}_\ell)$ with $G_k = Gal(\bar{k}/k)$-act

$a: X \to Y = Spec\ k$. $Ra_* \mathbb{Q}_\ell$

Ramanujan conj $\Delta = q \prod_{n=1}^{\infty} (1-q^n)^{24}$ mod f wt 12

$= \sum_{n=1}^{\infty} \tau(n) q^n$ $q = \exp(2\pi\sqrt{-1}z)$

$|\tau(p)| \leq 2 \cdot p^{\frac{11}{2}}$

Weil conj for sheaf coefficient

Deligne's 2nd pf. $\mathcal{F} = j_* S^{10}(R^1 f_* \mathbb{Q}_\ell)$ $H^1(X_{\bar{\mathbb{F}}_p}, \mathcal{F})$

$$\begin{array}{ll} & E \quad \text{univ. ell. curve} \\ & \downarrow f \\ X \xleftarrow{j} & Y \quad \text{modular curve} \end{array}$$

ℓ-adic sheaves $\bar{X}$ compactificat

1. Study local property of $\mathcal{F}$. particularly ramification.

~~How to understand~~

2. describe global inv. by loc inv.

Model. D-modules linear partial diff eq. on complex mfd. X

Strong analogy. 6-operations.

S-K-K Micro local analysis

T^*X cotangent bdlle.

Singular support $SS\,\mathcal{M} \subset T^*X$ closed conical $\mathbb{G}_m$-act

$\bigcup C_a$

$CC\,\mathcal{M} = \sum m_a C_a$.

K-S topological.

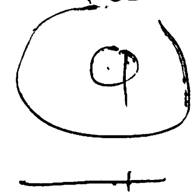
algebraic k perfect X/k smooth $\ell \neq \mathrm{char}\, k$

$\mathcal{F}$: ℓ-adic

$SS\mathcal{F} \subset T^*X$ cotangent Deligne precisely red'n mod ℓ.

related to vanishing cycles

$$\begin{array}{ccl} x & X & \\ \downarrow & \downarrow f & \text{morphism to curve} \\ y & Y & \end{array}$$



vanishing cycle.

Milnor fiber

$$\mathcal{F}_x \to R\Gamma(X_x \times_{Y_y} \bar{\eta}, \mathcal{F}) \to R\phi_x(\mathcal{F}, f) \to$$

$$SS\mathcal{F} = \overline{\{(x, df) \in T^*X \mid R\phi_x(\mathcal{F}, f) \neq 0\}}$$

Theorem (Beilinson). Every irreducible component C_a of $SS\mathcal{F} = \bigcup_a C_a$ has the same dim as X.

$\subset T^*X$

$CC\mathcal{F} := \sum m_a C_a$ $\quad m_a \in \mathbb{Z}$

~~characterized~~ satisfying

- $CC\,\mathbb{Q}_\ell = (-1)^n T^*_X X$ 0-sect
- additive $0 \to \mathcal{F}' \to \mathcal{F} \to \mathcal{F}'' \to 0 \Rightarrow CC\mathcal{F} = CC\mathcal{F}' + CC\mathcal{F}''$
- compatibility with pull-back by "C-transversal" morph
- push forward by closed imm
- Radon transform. $P \leftarrow Q \to P^\vee$

precisely speaking modulo ℓ.

Theorem. (index formula)

$$\chi(X_{\bar{k}}, \mathcal{F}) = (CC\mathcal{F}, T^*_X X)_{T^*X}$$