

## RIGID GEOMETRY FOLLOWING M. RAYNAUD

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Grothendieck wrote to Serre on October 19, 1961, “*Autre exposé possible : les espaces rigide-analytiques de Tate (à moins que tu les inclues dans ton cours au Collège). Peut-être cela incitera quelque Normalien (je pense à Houzel) à travailler sur le sujet ;... Je suis d’ailleurs convaincu que les espaces analytiques ordinaires, les rigides analytiques, les schémas formels et même les schémas tout court, devront tôt ou tard rentrer dans une même espèce de structure... : il y a certainement là de quoi s’amuser*”.

The main part of this program has been achieved by M. Raynaud in the beginning of the 70’s. In spite of its importance in arithmetic and algebraic geometry, Raynaud’s approach to rigid geometry remains insufficiently developed, in particular, the associated étale topology. The goal of this course is to present the foundations of this theory. The preliminary plan is the following :

- (1) Introduction. Flattening techniques of Raynaud-Gruson.
- (2) Algebraic preliminaries : Finiteness conditions ; Idyllic rings ; A flatness theorem of Gabber.
- (3) Formal geometry : Finiteness conditions ; Idyllic formal schemes ; Coherent modules on idyllic formal schemes ; The rigid closure of a module.
- (4) Admissible blow-up. Tate’s acyclicity theorem.
- (5) Rigid geometry : Raynaud’s category of coherent rigid spaces ; The admissible topology ; Quasi-separated rigid spaces ; Algebraic geometry and rigid geometry ; The admissible site and topos of a coherent rigid space ; The admissible topos as a projective limit of a fibered topos ; Coherent modules on coherent rigid spaces ; The dimension of a coherent rigid space.
- (6) Flatness : Flatness in formal geometry ; Relative devissage ; A flatness criterion ; Rig-flatness ; Flattening by admissible blow-up ; Flatness in rigid geometry ; Faithfully flat descent.
- (7) Differential invariants. Smooth morphisms.
- (8) The étale site and topos of a coherent rigid space.

The only prerequisites are the EGA, except for section (5) where we need SGA 1 Exposé VI and SGA 4 Exposés I–IV and VI.