$CC'(\mathcal{F})$ $\quad\quad$ $\mathrm{Art}(\mathcal{F}) = D(CC'(\mathcal{F}))$

(定理1.2は使わない)

命題3.2 $X \subset \mathbb{P}$ proj smooth surface, H very ample.
Deligne Notes sur E-P 参.

$$\chi(X,\mathcal{F}) - (CC\mathcal{F}, T^*_X X) = -2(\mathrm{Art}\,\mathcal{F} - D(CC\mathcal{F}), H)$$

系1. $\mathrm{Art}\,\mathcal{F}$ は $D(CC\mathcal{F})$ と数値的同値

系2 $\quad \chi(X,\mathcal{F}) = (CC\mathcal{F}, T^*_X X)$

定理3.1.2 の$\overline{\text{証明}}$ $\dim X = 2$ の場合の

$[L_x]$ の係数が等しいことをいえばよい。

étale local. additive 1つだけとしてよい。

$\mathrm{Art}\,\mathcal{F} = D(CC\mathcal{F})$ をいえばよい。

$$\begin{tikzcd} X & X' \arrow[l] \\ & Y \arrow[u] \arrow[ul, "\pi"] \end{tikzcd}$$

$X \leftarrow X'$ 点ではétale それ以外では分岐を持つ

Y resolution.

~~株~~ ~~株~~ $(\mathrm{Art}\,\mathcal{F}' - D(CC\mathcal{F}'), \pi^*H) = 0.$

4 Vanishing topos と Swan 導手の有理性

Oriented product

$X \overleftarrow{\times}_S Y \qquad T \to X \overleftarrow{\times}_S Y$

4.1 nearby cycle & van. cycle

$$\begin{array}{ccc} T & \to & Y \\ \downarrow & \swarrow f & \\ X & \to & S \end{array}$$

くわしくは Illusie さんの講義.

vanishing topos $X \overleftarrow{\times}_S S$ $\quad x \to X$ geom pt

$x \overleftarrow{\times}_S S = S_{(s)}$ strict localization.

$X \overleftarrow{\times}_S S$ の点 $x \leftarrow t$

$$\Psi_f : X \to X \overleftarrow{\times}_S S \xrightarrow{p_2} S, \qquad X \overleftarrow{\times}_S S \xrightarrow{p_1} X$$

$R\Psi_{f}, p_1^* : D^+(X) \to D^+(X \overleftarrow{\times}_S S)$

$R\Phi_f$ $\qquad p_1^* \to R\Psi_f \to R\Phi_f \to$

S henselian d.v.r. $\qquad D^+(X \overleftarrow{\times}_S \eta) \to D^+(X \overleftarrow{\times}_S \bar{\eta}) \simeq D^+(X_s)$

$i_s^* \to R\Psi_f \to R\Phi_f$

$R\Psi_f \mathcal{F}_{x \leftarrow t} = R\Gamma(X_{(x)} \times_{S_{(s)}} S_{(t)}, \mathcal{F})$ Milnor tube

補題 1.1 次は同値.

(1) $f: X \to S$ は $\mathcal{F}$ に関して loc. acyclic

(2) $p_1^* \mathcal{F} \to R\Psi_f \mathcal{F}$ は同形 ($\Leftrightarrow R\Phi_f \mathcal{F} = 0$) かつ $R\Psi_f \mathcal{F}$ の構成は任意の有限射と可換

$$\begin{array}{ccc} S \leftarrow T & & \\ X & \leftarrow & X_T \\ \downarrow & & \downarrow \\ X \overleftarrow{\times}_S S & \leftarrow & X_T \overleftarrow{\times}_T T \end{array} \qquad \begin{array}{ccc} \mathcal{F} & \to & \mathcal{F}_T \\ \downarrow & & \downarrow \\ R\Psi_f \mathcal{F} & \to & R\Psi_{f_T} \mathcal{F}_T \end{array}$$

(2)⇒(1)

T を t の像の閉包とすれば $S \leftarrow T$ は closed im

$$\begin{array}{ccc} R\Psi_f \mathcal{F}_{x \leftarrow t} & \to & R\Psi_{f_T} \mathcal{F}_{T\, x \leftarrow t} \\ \| & & \| \\ R\Gamma(X_{(x)} \times_{S_{(s)}} S_{(t)}, \mathcal{F}) & \to & R\Gamma(X_{(x)} \times_{S_{(s)}} t, \mathcal{F}) \end{array}$$

(1)⇒(2) App to Th finitude Cor. 2.6

of noetherian sdim

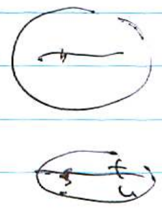
命題1.2 (Ongogozo) $X \to S$ finite type $Z \subset X$ q-F/S
$\mathcal{F}$ const complex on X $X-Z \to S$ univ. loc. acyclic

⇒ 1. $R\Psi_f\mathcal{F}$, $R\Phi_f\mathcal{F}$ const. on $X \overleftarrow{\times}_S S$. formation cpt with b.c
$R\Phi_f\mathcal{F}$ supp on $Z \overleftarrow{\times}_S S$

2 $\to R\Psi_f\mathcal{F}_{x\leftarrow t} \to R\Psi_f\mathcal{F}_{x\leftarrow u} \to \bigoplus_{z \in Z_{(x)} \times_{S_{(s)}} t} R\Phi_f\mathcal{F}_{z\leftarrow u}$
$S_{(s)} \leftarrow t \leftarrow u$

proper base change.

$t = s$ とする. $pr_i^* \to \Psi \to \Phi$.



$Z_{(x)} \to S_{(s)}$ の像の外では loc. cont.

4.2 Swan導手の下半連続性
S noetherian
$f: X \to S$ flat curve $Z \subset X$ closed q-F/S
$X - Z \to S$ smooth.
$\mathcal{F}$ on X constructible $\mathcal{F}|_{X-Z}$ loc. cont

$s \to S$ geom pt $k(s)$ alg closed $x \in X_s$ closed pt

$\mathcal{O}_{X_s,(x)}$ の normalization $= \prod A_i$ A_i str. local d.v.r. alg closed
K_i frac.
$a_x(\mathcal{F}|X_s) = \sum \mathrm{dimtot}_{K_i} \mathcal{F}_{K_i} - \dim \mathcal{F}_x$.
Artin conductor
$\varphi_{\mathcal{F}}(x) = a_x(\mathcal{F}|X_s)$ function on Z.

命題2.1 (Deligne-Laumon)
1. $R\Psi_f\mathcal{F}$, $R\Phi_f\mathcal{F}$ constructible 構成が base change と可換

$$\dim R\Phi_f\mathcal{F}_{x\leftarrow t} = \delta(\varphi_{\mathcal{F}})(x \leftarrow t)$$

ので $\varphi_{\mathcal{F}}$ は constructible
2. $\mathcal{F} = j_!\mathcal{G}[1]$ $j: U = X - Z \to X$, Z flat/S
⇒ $\varphi_{\mathcal{F}}$ increasing, flat ⟺ univ. loc. acy

証明 1. const ⇐ 命題1.2

$= \Rightarrow \delta(\varphi_{\mathcal{F}})$ const $\Rightarrow \varphi_{\mathcal{F}}$ const

= ~~[illegible]~~ base change S = d.v.r.
devissage 2 の場合.

X proper なる $\Sigma = \Sigma$. G-O-S.
Deligne - deformation
Kato . stable reduction thm.

2 S = d.v.r. positivity R^0 以外 0
≥ 0. $= 0 \Leftarrow R^0 = 0$

4.3 Swan 導手の有理性

~~解析~~.

$Z \subset X \xrightarrow{f} Y$ $\mathcal{F}$ on X ~~p: X → S loc. acyclic~~

g.f p ↘ ↙ g smooth curve f: X → Y loc. acyclic outside Z
S

s → S gen pt k(s) alg closed

$x \in Z_s \subset X_s \xrightarrow{f_s} Y_s \ni y$ dimtot $\Phi_x(\mathcal{F}, f_s)$

$= a_y(R\Phi_f\mathcal{F}|_{X \overleftarrow{\times}_Y Y_{(y)}})$

$= \varphi_{\mathcal{F}, f}(x)$

命題3.1 p: X → S が $\mathcal{F}$ に関して loc. acyclic ⇒ $\varphi_{\mathcal{F},f}$ は S上 flat

$K = R\Phi_f\mathcal{F}$ $X \overleftarrow{\times}_Y Y$ $x \overleftarrow{\times}_Y Y = Y_{(y)}$ への制限 constructible
$Z_{(x)}$ の像の外で l.c. 命題1.2.2

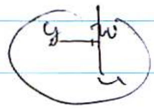
dim $R\Phi K_{y \leftarrow t} = \delta(\varphi_K)(y \leftarrow t)$

~~$\to K_{y \leftarrow t} \to R\Phi_f K_{y \leftarrow t} \to R\Phi K_{y \leftarrow t}$~~

$$
\begin{array}{ccccc}
R\Phi_{f} K_{x \leftarrow y} & \longrightarrow & R\overleftarrow{\jmath}_{*} K_{x \leftarrow t} & \longrightarrow & R\overline{\Phi}_{g} K_{y \leftarrow t} \\
\| & & \| & & \| \\
\mathcal{F}_{x} & \longrightarrow & R\overline{\Phi}_{p} \mathcal{F}_{x \leftarrow t} & \longrightarrow & R\overline{\Phi}_{p} \mathcal{F}_{x \leftarrow t}
\end{array}
$$

左辺 $= \dim R\overline{\Phi}_{p} \mathcal{F}_{x \leftarrow t} = 0$ (acyclicity)

右辺 $= \varphi_{\mathcal{F}, f}(x) - \sum_{w \in Z(x) \times_{S(x)} t} a(w) \, a_w \left(R\overline{\Phi}_{f} \mathcal{F}|_{Y(y)}, f_t \right)$



$$
\sum_{z \in Z(x) \times_{S(x)} t} \dim \operatorname{tot}^{u} \left(\Phi_{z} (\mathcal{F}|_{X_t}, f_t) \right)
$$

$$
\to R\Phi_{f} \mathcal{F}_{x \leftarrow w} \to R\Phi_{f} \mathcal{F}_{x \leftarrow u} \to \bigoplus_{z \mapsto w} R\Phi_{f} \mathcal{F}_{z \leftarrow u} \to
$$

命題1.2.2

$$
= \varphi_{\mathcal{F}, f}(x) - \sum_{z} \varphi_{\mathcal{F}, f}(z)
$$

$$
= \delta(\varphi_{\mathcal{F}, f})(x \leftarrow t)
$$