2.4 特性サイクルの構成

$i: X \to \mathbb{P}$ immersion $E \subset \Gamma(X, \mathcal{L})$ で定義される

条件 (E) $\forall x \neq y \in X(\bar{k})$ $E \otimes \bar{k} \to \mathcal{L}_x/\mathfrak{m}_x^2\mathcal{L}_x \oplus \mathcal{L}_y/\mathfrak{m}_y^2\mathcal{L}_y$ 全射

(C) $C = \bigcup C_a$ 既約成分 $\tilde{C}_a \subset \cancel{\ } X \times_{\mathbb{P}} T^*\mathbb{P}$ ~~...~~ が 0-section に含まれない

($i: X \to \mathbb{P}$ open im & $\mathrm{Supp}\, \mathcal{F} = X$ を除けば OK)

$$\mathbb{P}(\tilde{C}) = \bigcup \mathbb{P}(\tilde{C}_a) \subset X \times_{\mathbb{P}} Q \xrightarrow{p^\vee} \mathbb{P}^\vee$$

条件 (E) $\Rightarrow$ $\mathbb{P}(\tilde{C}) \to \mathbb{P}^\vee$ generically radicial

(C) $\Rightarrow$ $\mathbb{P}(\tilde{C}_a) \neq \emptyset$

命題 $\mathbb{P}(\tilde{C})$ は $X \times_{\mathbb{P}} Q \xrightarrow{p^\vee} \mathbb{P}^\vee$ が $p^\circ C$-trans となる最大の開集合の補集合

証明

$$(X \times_{\mathbb{P}} Q) \times_{\mathbb{P}^\vee} T^*\mathbb{P}^\vee \longrightarrow T^*(X \times_{\mathbb{P}} Q) \supset p^*C$$

$$T^*(X \times_{\mathbb{P}} Q) = \mathrm{Coker}\left(T^*_{X \times_{\mathbb{P}} Q}(X \times \mathbb{P}^\vee) \xrightarrow{\mathrm{inj}} \left((X \times_{\mathbb{P}} Q) \times T^*X \oplus (X \times_{\mathbb{P}} Q) \times_{\mathbb{P}^\vee} T^*\mathbb{P}^\vee\right)\right)$$

$$(X \times_{\mathbb{P}} Q) \times T^*X \supset p^*C$$

not transversal at $T^*_{X \times_{\mathbb{P}} Q}(X \times \mathbb{P}^\vee) \to (X \times_{\mathbb{P}} Q) \times_X T^*X \supset p^*C$

$$T^*_{X \times_{\mathbb{P}} Q}(X \times \mathbb{P}^\vee) = (X \times_{\mathbb{P}} Q) \times_Q T^*_Q(\mathbb{P} \times \mathbb{P}^\vee) \to (X \times_{\mathbb{P}} Q) \times_{\mathbb{P}} T^*\mathbb{P} \supset p^*\tilde{C}$$

$$T^*_Q(\mathbb{P} \times \mathbb{P}^\vee) \supset Q \times_{\mathbb{P}} T^*\mathbb{P}$$

$Q = \mathbb{P}(T^*\mathbb{P})$ 上の univ. sub line bdle

$\rightsquigarrow$ $\Longleftrightarrow$ $\mathbb{P}(\tilde{C})$

pencil $L \subset \mathbb{P}^\vee$ 直線

$$\begin{tikzcd} X_L \arrow[r] \arrow[d, "p_L"'] \arrow[dr, phantom, "\square"] & X \times_{\mathbb{P}} Q \arrow[d, "p^\vee"] \\ L \arrow[r] & \mathbb{P}^\vee \end{tikzcd}$$

$A_L = \bigcap_{H \in L} H \subset \mathbb{P}$ (linear subspace codim 2

$A_L \cap X$ proper intersect $\Rightarrow$ $X_L \to X$ blow up at $A_L \cap X$

$$\begin{tikzcd} X_L \arrow[r] & X \\ X_L^\circ \arrow[u, phantom, "\cup"] \arrow[r, "\simeq"] & X - X \cap A_L \arrow[u, phantom, "\cup"] \end{tikzcd}$$

$p_L^\circ : X_L^\circ \to L \quad x \mapsto x$ と A_L を含む唯一の hyperplane

open $\cap$ X

命題 $X_L^\circ \cap \mathbb{P}(\check{C})$ $\quad p_L^\circ : X_L^\circ \to L$ が C-tran となる最大の開の補

補題 1. $V \xrightarrow{g} W \xrightarrow{h} X$ $\quad h$ C-trans

g $h^\circ C$-tran $\Leftrightarrow$ $h \circ g$ C-tran

$V \times_X T^*X - V \times_W T^*W - T^*V$ $\quad$ V上のvector束

$\cup \qquad \cup$

$(h\circ g)^\circ C \xrightarrow[\text{finite}]{} g^\circ(h^\circ C)$

finite $\Leftarrow$ finite

補題 2

$$\begin{tikzcd} X \arrow[d, "f"'] & W \arrow[l, "h"'] \arrow[d, "g"] \\ Y & V \arrow[l] \end{tikzcd}$$

$\leftarrow$ reg im 同codim

(1) $f : X \to Y$ C-trans on a nbd of W

(2) $h : W \to X$ C-tran $\Leftrightarrow$ $g : W \to V$ $h^\circ C$-tran

$$\begin{tikzcd} 0 \arrow[r] & T^*_W X \arrow[r] & W \times_X T^*X \arrow[r] & T^*W \arrow[r] & 0 \\ 0 \arrow[r] & W \times_V T^*_V Y \arrow[r] \arrow[u, equal] & W \times_Y T^*Y \arrow[r] \arrow[u] & W \times_V T^*V \arrow[r] \arrow[u] & 0 \end{tikzcd}$$

(2) $\qquad \cap\, h^\circ C \qquad$ (2)

補題 1+2 $\Rightarrow$ 命題

$$\begin{tikzcd} X_L^\circ \arrow[r] \arrow[d] & X \times_{\mathbb{P}} Q \arrow[r, "(1)"] \arrow[d] & X \\ L \arrow[r] & \mathbb{P}^\vee & \end{tikzcd}$$

補題 1

補題 2