

2.4 射影空間の浸入

$i: X \rightarrow \mathbb{P}^n$ immersion $E \subset \Gamma(X, \mathcal{O}(1))$ 定数項

条件 (E) $\forall x \neq y \in X(\mathbb{C}) \quad E \otimes \mathcal{O}_x \rightarrow \mathcal{O}_x/m_x^2 \otimes \mathcal{O}_x \oplus \mathcal{O}_y/m_y^2 \otimes \mathcal{O}_y$
(全射)

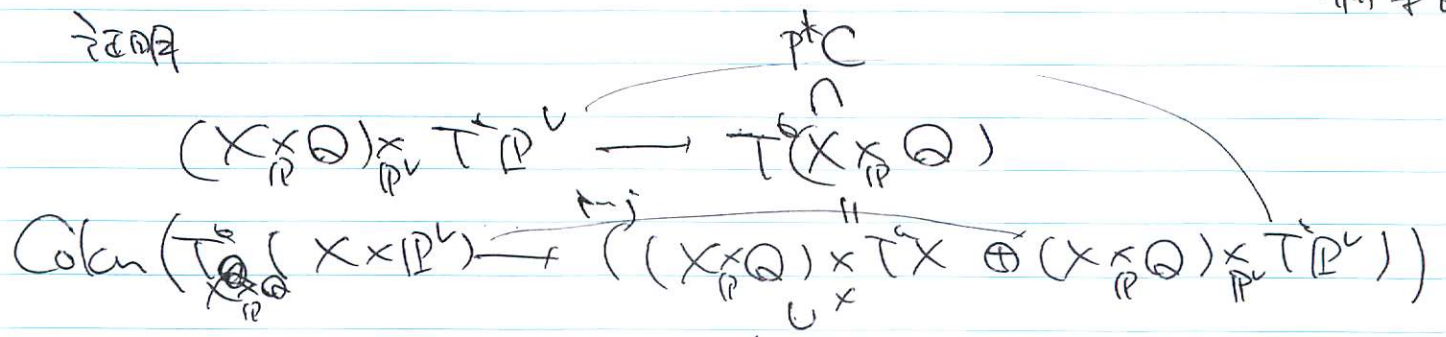
(C) $C = \cup C_a$ 既約部分 $\hat{C}_a \subset \mathbb{P}^n \times \mathbb{P}^n$
 δ -0-section (= 全射) \Rightarrow
 ($i: X \rightarrow \mathbb{P}^n$ open & $\text{Supp } \gamma = X \ni P$ 除点 OK)

$$P(\hat{C}) = \cup P(\hat{C}_a) \subset X \times_{\mathbb{P}^n} \mathbb{P}^n \xrightarrow{p^*} \mathbb{P}^n$$

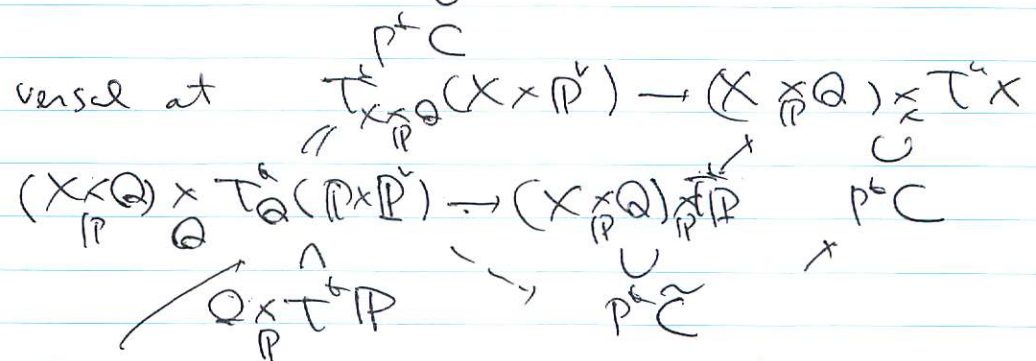
条件 (E) $\Rightarrow P(\hat{C}) \rightarrow \mathbb{P}^n$ generically radical
 (C) $\Rightarrow P(\hat{C}_a) \neq \emptyset$

命題 $P(\hat{C}) \subset X \times_{\mathbb{P}^n} \mathbb{P}^n \xrightarrow{p^*} \mathbb{P}^n$ δ -pC-trans. \Rightarrow 最大の開集合の補集合

証明



not transversal at



$\mathbb{Q} = \mathbb{P}^n / \mathbb{P} (T^* \mathbb{P}^n)$ is a univ-sub line bundle
 $\hookrightarrow P(\hat{C})$

pencil $L \subset \mathbb{P}^v$ 西線

$$\begin{array}{ccc} X_C & \longrightarrow & X_{\mathbb{P}^v} \\ p_L \downarrow & \square & \downarrow p^v \\ L & \longrightarrow & \mathbb{P}^v \end{array}$$

$A_C = \bigcap_{H \in \mathcal{L}} H \subset \mathbb{P}^v$ (line subspace codim 2)

$A_C \cap X$ propn intersect $\Rightarrow X_C \rightarrow X$ fiber up at $A_C \cap X$
 $U \neq \emptyset$
 $X_C^0 \cong X - X \cap A_C$
 $p_C^0: X_C^0 \rightarrow L \quad x \mapsto x \in \mathbb{P}^v$ 西線 is a hyp-plane
 open \square
 X

命題 $X_C^0 \cap (\mathbb{P}^v \setminus C)$ $p_C^0: X_C^0 \rightarrow L$ or C-trans $\{ \text{西線} \}$ 最大公因の補

補題 1. $V \xrightarrow{g} W \xrightarrow{e} X$ e C-trans
 g \mathbb{P}^1 -trans \Leftrightarrow $e \circ g$ C-trans

$$\begin{array}{ccc} V \times_{\mathbb{C}} T^*X & \rightarrow & U \times_{\mathbb{C}} T^*W \rightarrow T^*V \\ \downarrow \text{fibre} & & \downarrow \text{fibre} \\ (e \circ g)^* & \xrightarrow{g^*} & e^* \end{array}$$

$V \in \text{vector space}$

補題 2 $X \xleftarrow{h} W$ \leftarrow reg in \mathbb{P}^1 codim
 $f \downarrow \quad \downarrow g$
 $Y \rightarrow V$

- (1) $f: X \rightarrow Y$ C-trans on a inbd of W
- (2) $e: W \rightarrow X$ C-trans $\&$ $g: W \rightarrow V$ \mathbb{P}^1 -trans

$$\begin{array}{ccccccc} 0 & \rightarrow & T^*W \otimes X & \rightarrow & W \times_{\mathbb{C}} T^*X & \rightarrow & T^*W \rightarrow 0 \\ & & \parallel & & \uparrow & & \uparrow \\ 0 & \rightarrow & W \times_{\mathbb{C}} T^*Y & \rightarrow & W \times_{\mathbb{C}} T^*X & \rightarrow & W \times_{\mathbb{C}} T^*V \rightarrow 0 \end{array}$$

補題 1+2 \Rightarrow 命題

$$\begin{array}{ccc} X_C^0 & \rightarrow & X_{\mathbb{P}^v} \\ \downarrow & & \downarrow p^v \\ L & \rightarrow & \mathbb{P}^v \end{array}$$

(1) (2)
 補題 2 補題 1