Higher dimensional algebraic geometry
March 12–16, 2018, University of Tokyo

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Organizers
Yoshinori Gongyo (University of Tokyo)
Keiji Oguiso (University of Tokyo)
Shunsuke Takagi (University of Tokyo)

Schedule

	Mar. 12	Mar. 13	Mar. 14	Mar. 15	Mar. 16
10:00–10:50	Yukinobu Toda	Jungkai Chen	Yongnam Lee	Nobuyoshi Takahashi	Taro Sano
11:10–12:00	Takehiko Yasuda	Chen Jiang	Fabrizio Catanese	Yasunari Nagai	Kenji Matsuki
14:00–14:50	Paolo Cascini	Yusuke Nakamura	Yoshinori Namikawa	Shinnosuke Okawa	free afternoon
15:10–16:00	Masayuki Kawakita	Atsushi Ito	Akira Ishii	Hokuto Uehara	free afternoon
16:20–17:10	Caucher Birkar	Jun-Muk Hwang	Yukari Ito	Hiromichi Takagi	free afternoon
18:00–20:00			Reception		

Titles and Abstracts

Caucher Birkar (University of Cambridge)
Title: *Some problems about singularities*

Abstract: In this talk, I will discuss some problems about singularities. The aim would be to understand the relation between positivity properties of divisors on the one hand and their singularities on the other.

Paolo Cascini (Imperial College London)
Title: *Minimal Model Program for foliations*

Abstract: I will survey some recent results on the study of the birational geometry of foliations over complex projective varieties. Work in progress with C. Spicer.

Fabrizio Catanese (Universität Bayreuth)
Title: *Canonical surfaces of high degree and low genus*

Abstract: Let Y be the image of a surface S under the canonical map, and assume that Y is a surface. The degree d of Y is called canonical degree, and d is bounded by the canonical volume K^2. By the BMY inequality we have $d \leq K^2 \leq 9\chi = 9(1 - q + p_g)$.

The first main Question is: what is the maximum value of d for $p_g = 4, 5, 6$? Can we find surfaces realizing large canonical degree?

I will recall several older and some new results. For instance about ball quotients in the above range.

The second main Question is : what is the maximum value of d for a canonically embedded surface S? (this means: Y is isomorphic to the canonical model Z of S).

This question is interesting for $p_g = 6$, since for $p_g = 4$, Z must be a 5-ic, and, for $p_g = 5$, the canonical model Z must be a complete intersection of type $(2, 4)$ or $(3, 3)$ (hence $d = 8$ or 9). I shall explain that ($p_g = 5$) this is a consequence of Severi's double point formula, and of its extension done in joint work with Keiji Oguiso for surfaces with isolated singularities. For $p_g = 6$, if S is canonically embedded, there are interesting ties with methods and questions of homological algebra (Walter's bundle Pfaffians), which led to the question whether 18 would be the upper bound for d (the range $[11, 17]$ can be easily filled by bundle methods). Degree $d = 24$ was achieved by myself with some regular surfaces ($q = 0$), and by Cesarano with a family of surfaces having $q = 3$, polarizations of type $(1, 2, 2)$ in an Abelian 3-fold. I shall then give results concerning question I, surfaces S of general type with $p_g = 5, 6$, whose canonical map has image Σ of very high degree, $d = 48$ for $p_g = 5$, $d = 56$ for $p_g = 6$. These surfaces are surfaces isogenous to a product of curves, $S = (C_1 \times C_2)/G$, with G abelian. Ball quotients S with $p_g = 6$, $K^2 = 63$, are constructed as unramified $\mathbb{Z}/7$ covers of some fake projective planes X, and in work in progress with JongHae Keum we are studying their canonical map. As a preliminary result, we showed that the bicanonical map of these fake projective planes is an embedding.

Jungkai Alfred Chen (National Taiwan University)
Title: *On Noether type inequality, I – general theory*

Abstract: For varieties of general type, it is natural to study the distribution of birational invariants and relations between invariants. We are interested in the relation between two fundamental birational invariants: the geometric genus and the canonical volume. For a minimal projective surface S, M. Noether proved that $K_S^2 \geq 2p_g(S) - 4$, which is known as the Noether inequality. It is thus natural and important to study the higher dimensional analogue.

In cooperated with Chen Jiang, we are going to present our recent advances in three dimensional Noether Inequality. This is our joint work with Meng

Chen. In these two talks , we will talk about our recent work on the Noether inequality for projective 3-folds. We will show that the inequality $\mathrm{vol}(X) \geq \frac{4}{3}p_g(X) - \frac{10}{3}$ holds for all projective 3-folds X of general type with either $p_g(X) \leq 4$ or $p_g(X) \geq 27$, where $p_g(X)$ is the geometric genus and $\mathrm{vol}(X)$ is the canonical volume. This inequality is optimal due to known examples found by M. Kobayashi in 1992. This proves that the optimal Noether inequality holds for all but finitely many families of projective 3-folds (up to deformation and birational equivalence).

This talk is going to serve as the first part. In which we are going to work on general theory, and examples. The natural approach is to consider the canonical map. Depending on the geometry of the canonical map, one can obtain various geographical inequalities. The most technical case of $(1, 2)$ surface fibration will be taken care by Chen Jiang.

Jun-Muk Hwang (KIAS)
Title: Rigidity of Legendrian singularities

Abstract: Let (M, D) be a holomorphic contact manifold, i.e., a complex manifold M of dimension $2n+1$ equipped with a holomorphic contact structure D. An n-dimensional complex analytic subvariety V in M is called a Legendrian subvariety if the smooth locus of V is tangent to D. A Legendrian singularity means the germ of a Legendrian subvariety at a point. We discuss some rigidity results on Legendrian singularities.

Akira Ishii (Hiroshima University)
Title: *G-constellations and the maximal resolution of a quotient surface singularity*

Abstract: For a finite group acting on a variety, a G-constellation is a certain G-equivariant coherent sheaf with finite support. When G is in $\mathrm{GL}(2, \mathbb{C})$, we consider the moduli spaces of G-constellations on $\mathbb{C}^2$, which are resolutions of the quotient singularity and which depend on stability parameters. Under some assumption, we obtain every resolution dominated by the maximal one in the sense of Kollár and Shepherd-Barron.

Atsushi Ito (Nagoya University)
Title: *A remark on higher syzygies on abelian varieties*

Abstract: In this talk, I will explain a slight improvement of a result of A. Küronya and V. Lozovanu about higher syzygies on abelian surfaces.

Yukari Ito (IPMU/Nagoya University)
Title: *Higher dimensional McKay correspondence*

Abstract: Two dimensional McKay correspondence was observed by John McKay in 1979 and studied the relation with the minimal resolution of rational double points in Algebaric Geometry. It was generalized in three dimension

for crepant resolution of Gorenstein quotient singularities. Now let us consider higher dimensional McKay correspondence!

Chen Jiang (IPMU)
Title: *On Noether type inequality, II – 3-folds with* $(1,2)$ *surface fibration*

Abstract: This talk is the second part of the talks on Noether type inequality, incorporated with Jungkai Chen's talk explaining our joint work with Meng Chen. We aim to prove the Noether Inequality of the form $\mathrm{vol}(X) \geq \frac{4}{3}p_g(X) - \frac{10}{3}$. The most difficult case is that the canonical map induces a fibration to a curve with $(1,2)$ surface fibration. By a $(1,2)$ surface, we mean a surface S of general type with $\mathrm{vol}(S) = 1$ and $p_g(S) = 2$. We would like to recall that these are the only class of surfaces of general type such that $|4K_S|$ is not birational. Their canonical models are known to be weighted hypersurfaces of degree 10. With careful studies of $(1,2)$ surfaces, we are able to prove that the Noether Inequality holds as long as $p_g(X) \geq 27$.

Masayuki Kawakita (RIMS)
Title: *Minimal log discrepancies on smooth threefolds*

Abstract: The minimal log discrepancy is an important invariant of singularities in the minimal model program. We will discuss several equivalent conjectures on the minimal log discrepancies on smooth threefolds.

Yongnam Lee (KAIST)
Title: *Exceptional collections on Dolgachev surfaces associated with degenerations*

Abstract: Dolgachev surfaces are simply connected minimal elliptic surfaces with $p_g = q = 0$ and of Kodaira dimension 1. These surfaces are constructed by logarithmic transformations of rational elliptic surfaces. In this talk, we explain the construction of Dolgachev surfaces via $\mathbb{Q}$-Gorenstein smoothing of singular rational surfaces with two cyclic quotient singularities. This construction is based on the paper by Lee-Park. Also, some exceptional bundles on Dolgachev surfaces associated with $\mathbb{Q}$-Gorenstein smoothing have been constructed based on the idea of Hacking. In the case if Dolgachev surfaces were of type $(2,3)$, we describe the Picard group and present an exceptional collection of maximal length. Finally, we prove that the presented exceptional collection is not full, hence there exists a nontrivial phantom category in the derived category. This is a joint work with Yonghwa Cho.

Kenji Matsuki (Purdue University)
Title: *Toward embedded resolution of singularities of 3-folds in positive characteristic*

Abstract: I will discuss the strategy to carry out embedded resolution of singularities in positive characteristic. The strategy works well for surfaces. I will

present what difficulties await us in higher dimensions, focusing on the case of 3-folds. This is a joint work with Hiraku Kawanoue.

Yasunari Nagai (Waseda University)
Title: *Degeneration of Hilbert scheme of points on surfaces*

Abstract: A degenerating family of surfaces naturally induces a degeneration of Hilbert schemes on surfaces that are general fiber of the family. We discuss constructions of a good birational model of the degeneration of Hilbert schemes.

Yusuke Nakamura (University of Tokyo)
Title: *Vanishing theorems of Witt-vector cohomology for Fano threefolds*

Abstract: In this talk, I will explain a vanishing theorem of Witt-vector cohomology. This is an analogy of the Kawamata-Viehweg vanishing theorem in positive characteristic. I will also explain its generalization to a vanishing theorem of Nadel type. Further, I will also discuss its application to the connectedness lemma and the rational point formula. This is joint work with Yoshinori Gongyo and Hiromu Tanaka.

Yoshinori Namikawa (Kyoto University)
Title: *Towards the classification of symplectic singularities*

Abstract: After introducing the finiteness theorem for symplectic singularities, I will give a characterisation of nilpotent orbit closures of a complex semisimple Lie algebra.

Shinnosuke Okawa (Osaka University)
Title: *On the definition of noncommutative del Pezzo surfaces*

Abstract: Noncommutative projective planes and noncommutative quadrics have clear definitions as the category qgr of 3-dimensional Artin-Schelter regular quadratic (resp. cubic) $\mathbb{Z}$-algebras. Other noncommutative del Pezzo surfaces lack a definition of this sort, but are constructed by blowing up noncommutative projective planes. In this talk I will talk about an attempt toward a definition of noncommutative del Pezzo surfaces without using blowups. This is a joint work in progress with Tarig Abdelgadir and Kazushi Ueda.

Taro Sano (Kobe University)
Title: *Effective non-vanishing for weighted complete intersections*

Abstract: Weighted complete intersections provide a testing ground for conjectures. In this talk, I'll confirm the effective non-vanishing conjecture for weighted complete intersections which are Fano or Calabi-Yau.

Hiromichi Takagi (University of Tokyo)
Title: *On rationality of certain moduli spaces of even spin curves via Fano 3-folds*

Abstract: I will explain how to use explicit birational geometry of Fano 3-folds to show rationality of certain moduli spaces of even spin curves. This is a joint work with Francesco Zucconi.

Nobuyoshi Takahashi (Hiroshima University)
Title: *Modules on quandle varieties*

Abstract: A quandle is an algebraic system defined by a binary operation, with ample applications in low dimensional topology. An algebraic variety endowed with a quandle operation, which we call a quandle variety, can also be regarded as a kind of generalized symmetric space.

I will talk about a definition of modules on quandle varieties, analogous to that on topological quandles by Elhamdadi-Moutou, give examples and see how such modules are related to representations of certain algebras.

Yukinobu Toda (IPMU)
Title: *Birational geometry for d-critical loci and wall-crossing in Calabi-Yau 3-folds*

Abstract: The notion of d-critical loci was introduced by Joyce in order to give classical shadows of (-1)-shifted symplectic derived schemes. In this talk, I will discuss birational geometry for d-critical loci, by introducing notions such as 'd-critical flips', 'd-critical flops', etc. They are not birational maps of the underlying spaces, but rather should be understood as virtual birational maps. I will show that several wall-crossing phenomena of moduli spaces of stable objects on Calabi-Yau 3-folds are described in terms of d-critical birational geometry. Among them, wall-crossing diagrams of Pandharipande-Thomas (PT) stable pair moduli spaces, which are relevant in showing the rationality of PT invariants, form a d-critical minimal model program. The above idea of d-critical birational geometry leads to a d-critical version of Bondal-Orlov, Kawamata's D/K equivalence conjecture, which is interpreted as a categorification of wall-crossing formula of Donaldson-Thomas invariants.

Hokuto Uehara (Tokyo Metropolitan University)
Title: *Spherical sheaves on D_n singularities*

Abstract: Spherical objects in the derived category of coherent sheaves induce autoequivalences, so called twist functors. We often need knowledge of spherical objects to study the autoequivalence groups. In my talk, I report my research on spherical sheaves on the minimal resolutions of D_n singularities on surfaces.

Takehiko Yasuda (Osaka University)
Title: *The motivic version of Serre's mass formula*

Abstract: Serre found a beautiful formula for counting totally ramified extensions of a given local field weighted by their discriminants. I will talk about the motivic version of this formula in the case of a local field of positive

characteristic (a Laurent power series field). This is a joint work with Fabio Tonini.