

# Algebraic Geometry in Positive Characteristic and Related Topics

Dates : December 17 (Mon.) – 20 (Thur.), 2018  
Venue : Auditorium, Graduate School of Mathematical Sciences,  
University of Tokyo

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## Program Committee

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## Schedule

	Dec. 17	Dec. 18	Dec. 19	Dec. 20
10:00– 10:50	Igor <b>Dolgachev</b>	Luc <b>Illusie</b>	Piotr <b>Pragacz</b>	Shigeru <b>Mukai</b>
11:10– 12:00	Keiji <b>Oguiso</b>	Gerard <b>van der Geer</b>	Toshiaki <b>Maeno</b>	Ichiro <b>Shimada</b>
14:00– 14:50	Tomohide <b>Terasoma</b>	Yongnam <b>Lee</b>	Stefan <b>Schröer</b>	Jun-Muk <b>Hwang</b>
15:10– 16:00	Hiromu <b>Tanaka</b>	JongHae <b>Keum</b>	Masayuki <b>Hirokado</b>	Yoichi <b>Miyaoka</b>
16:20– 17:10	Sho <b>Ejiri</b>	Toshiyuki <b>Katsura</b>	Hiroyuki <b>Nakaoka</b>	
18:00– 20:00		Reception		

# Titles and Abstracts

**Igor Dolgachev** (University of Michigan)

Title: *Linear systems of quadrics in characteristic 2*

Abstract: The usual assumption in the study of quadrics is that the characteristic of the ground field is different from 2. I will discuss some peculiar properties of linear systems of quadrics in characteristic 2 and concentrate on the particular example of construction of Enriques surfaces in characteristic 2 from webs of quadrics in a 3-dimensional projective space.

**Sho Ejiri** (Osaka University)

Title: *On direct images of pluricanonical bundles in positive characteristic*

Abstract: In this talk, we will consider surjective morphisms between projective varieties in positive characteristic that are not necessarily separable. We will discuss the positivity of the direct images of pluricanonical bundles, and will then prove a positivity theorem of Popa-Schnell type.

**Gerard van der Geer** (Universiteit van Amsterdam)

Title: *Algebraic curves and modular forms of degree two and three*

Abstract: Siegel modular forms of degree 2 (resp. 3) are intimately connected with moduli of curves of genus 2 (resp. 3). We show that classical invariant theory yields an effective way to construct modular forms and then give attention to modular forms of degree 2 and low weight. This is joint work with Clery and Faber.

**Masayuki Hirokado** (Hiroshima City University)

Title: *On non-taut rational triple points in positive characteristic*

Abstract: M. Artin and G. Tjurina studied rational triple points in 1960's and it is now well-known that, over the field of complex numbers, 1) these surface singularities are classified into nine classes, 2) each rational triple point is taut, i.e., its local structure is determined uniquely by the resolution dual graph. I will talk about the question whether their results and classification can be extended to positive characteristic, as well as on canonical/index 1 coverings.

**Jun-Muk Hwang** (KIAS)

Title: *Euler-symmetric projective varieties*

Abstract: A projective variety is Euler-symmetric if for each general point there is a multiplicative group action which fixes the given point and acts on

the tangent space as a scalar multiplication. We explain how Euler-symmetric varieties are classified by their symbol-systems, a class of algebraic objects and how they are related to the compactifications of vector groups. This is a joint-work with Baohua Fu.

**Luc Illusie** (Université Paris-Sud)

Title: *A new approach to the de Rham-Witt theory, after Bhatt, Lurie and Mathew*

Abstract: Bhatt, Lurie, and Mathew have recently constructed de Rham-Witt-like complexes for schemes over a perfect field of positive characteristic, which coincide with the classical one in the smooth case and are of interest in certain singular cases. I will explain their approach, which exploits the (fashionable) Deligne-Ogus décalage  $\eta_p$  functor, and uses only elementary homological algebra (in particular, avoids the laborious calculations involved in the so-called canonical bases for Deligne complexes of integral forms).

**Toshiyuki Katsura** (Hosei University)

Title: *Algebraic geometry in positive characteristic*

Abstract: In this talk, I will give a survey of results on algebraic varieties in positive characteristic which are related to my works. The subjects will be the moduli of principally polarized abelian varieties, the moduli of K3 surfaces, and the classification of Enriques surfaces with finite automorphism group, etc.

**JongHae Keum** (KIAS)

Title: *Explicit equations of a fake projective plane*

Abstract: A compact complex surface with the same Betti numbers as the projective plane is called a fake projective plane (FPP) if it is not biholomorphic to the projective plane. A FPP has ample canonical divisor, so it is a smooth proper (geometrically connected) surface of general type with  $p_g = 0$  and  $K^2 = 9$  (this definition extends to arbitrary characteristic.) The existence of a FPP was first proved by Mumford in 1979 based on the theory of 2-adic uniformization, and later two more examples by Ishida-Kato (1998) in this abstract method.

It has long been of great interest since Mumford to find equations of a FPP. With Lev Borisov we find explicit equations of a conjugate pair of fake projective planes by using the geometry of the quotients of such FPP.

**Yongnam Lee** (KAIST)

Title: *On function fields and deformations of hypersurfaces*

Abstract: In this talk, we consider dominant rational maps from very general hypersurfaces and smooth deformations of hypersurfaces under some suitable conditions. The first part is based on a joint work with Gian Pietro Pirola, and the second part is a joint work with Fabrizio Catanese.

These works are over the complex numbers. But I would like to discuss them also over the fields in positive characteristics if the time is allowed.

**Toshiaki Maeno** (Meijo University)

Title: *Lefschetz property for Artinian Gorenstein algebras*

Abstract: The Lefschetz property is a ring-theoretic abstraction of the Hard Lefschetz Theorem. It often has combinatorial implications. We discuss the Hessian criterion of the strong Lefschetz property for Artinian Gorenstein algebras. As its application, we show the Lefschetz property for algebras associated with a certain class of matroids. Our result implies the Sperner property for modular geometric lattices.

**Yoichi Miyaoka** (Chuo University)

Title: *Foliations in algebraic geometry*

Abstract: Foliations potentially play important roles in algebraic geometry. We review some recent developments in the theory of foliations.

**Shigeru Mukai** (RIMS)

Title: *Decomposition group of 6 lines and supersingular examples*

Abstract: I will discuss the group  $\text{Dec}(L)$  of Cremona transformations preserving a 6-line arrangement  $L$  on the projective plane, and determine its structure in several cases where the pair  $(\mathbb{P}^2, L)$  is covered by a supersingular K3 surface (in positive characteristic).

**Hiroyuki Nakaoka** (Kagoshima University)

Title: *Mutation via Hovey twin cotorsion pairs and model structures in extriangulated categories*

Abstract: As a simultaneous generalization of exact categories and triangulated categories, we introduce the notion of an extriangulated category. We will also give a correspondence between cotorsion pairs and model structures on extriangulated categories. This is based on a joint work with Yann Palu.

**Keiji Oguiso** (University of Tokyo)

Title: *A surface in odd characteristic with discrete and non-finitely generated automorphism group*

Abstract: It was proved by T.-C. Dinh and me that there is a smooth complex projective surface, whose automorphism group is discrete and not finitely generated. In this talk, adapting a similar idea, we will show that there is also a smooth projective surface, over any algebraically closed field of odd characteristic of positive transcendental degree over the prime field, such that the automorphism group is discrete and not finitely generated.

**Piotr Pragacz** (Polish Academy of Sciences)

Title: *Gysin maps, duality, and Schubert classes*

Abstract: We establish a Gysin formula for Kempf-Laksov flag bundles and we prove a duality theorem for Grassmann bundles. We then combine them to study Schubert bundles, their push-forwards and fundamental classes. This is a joint work with Lionel Darondeau.

**Stefan Schröer** (Heinrich-Heine-Universität Dusseldorf)

Title: *Del Pezzo surfaces over imperfect fields*

Abstract: We develop a structure theory for del Pezzo surfaces that are regular but geometrically non-normal, based on work of Reid, but now in dependence on the  $p$ -degree of the ground field. This leads to existence results, as well as non-existence results for ground fields of  $p$ -degree one. In turn, we settle questions arising from Kollár's analysis on the structure of Mori fiber spaces in dimension three. This is joint work with Andrea Fanelli.

**Ichiro Shimada** (Hiroshima University)

Title: *The elliptic modular surface of level 4 and its reduction modulo 3*

Abstract: The elliptic modular surface of level 4 is a complex K3 surface with Picard number 20. This surface has a model over a number field such that its reduction modulo 3 yields a surface isomorphic to the Fermat quartic surface in characteristic 3, which is supersingular. The specialization induces an embedding of the Néron-Severi lattices. Using this embedding, we determine the automorphism group of this K3 surface over a discrete valuation ring of mixed characteristic whose residue field is of characteristic 3.

**Hiromu Tanaka** (University of Tokyo)

Title: *On Witt analogues of the Kodaira vanishing theorem*

Abstract: It is well known that the Kodaira vanishing fails for varieties in positive characteristic. On the other hand, some analogous results are known to hold for Witt sheaves. In this talk, we first summarise some known results in this direction. We then discuss to what extent these results should be generalised.

**Tomohide Terasoma** (University of Tokyo)

Title: *Sandwich resolution and Broadhurst-Kerimer conjecture*

Abstract: Broadhurst and Kreimer conjectured the dimensions of associate graded spaces for the depth filtration of multiple zeta values. In this conjecture, the dimensions of elliptic cusp forms appears.

On the other hand, the universal enveloping algebra of the fundamental group of the punctured universal elliptic curves over the moduli  $M_{13}$  of 3-pointed genus one curves has a comodule structure over the Hopf algebra of mixed elliptic motives.

In this talk, we give a relation between the fundamental group of the degenerating elliptic curve and that of the projective line minus three points. To give a clear explanation, we introduce the sandwich resolutions of free associative algebras which yields a generating function conjectured by Broadhurst and Kerimer.