Workshop
Johnson homomorphisms and related topics
2019
May 13–17, 2019

Lecture Hall in the Mathematical Science Building,
the University of Tokyo
3-8-1 Komaba Meguro-ku, Tokyo 153-8914, Japan

(ver.190516)

Time Table

May 13	**May 14**	**May 15**	**May 16**	**May 17**
10:00–11:00	10:00–11:00	10:00–11:00	10:00–11:00	10:00–11:00
Satoh	Darné	Konno	Vera	Song
11:20–12:20	11:20–12:20	11:20–12:20	11:20–12:20	11:20–12:20
Sakasai	Tsuji	Randal-Williams	Furusho	Kuno
14:20–15:20	14:00–15:00		14:20–15:20	13:00–
Habiro	Cha		Tadokoro	Discussion
15:40–16:40	15:20–16:20		15:40–16:40	
Nozaki	Paris		Kassabov	
	17:00–18:30			
	(TST)			
	19:00–			
	banquet			

Organizers: Shinobu Hikami (OIST), Nariya Kawazumi (Tokyo),
Gwénaël Massuyeau (Dijon), Hiroaki Nakamura (Osaka),
Takuya Sakasai (Tokyo) and Christine Vespa (Strasbourg).

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Schedule with Titles

May 13 (Monday)

10:00–11:00,
Takao Satoh (Tokyo University of Science)
On the Andreadakis conjecture of the automorphism groups of free groups

11:20–12:20,
Takuya Sakasai (The University of Tokyo)
Johnson homomorphisms up to degree 7

14:20–15:20,
Kazuo Habiro (RIMS, Kyoto University)
Cyclic nerves of stratified categories

15:40–16:40,
Yuta Nozaki (Meiji University)
Finiteness of the image of the Reidemeister torsion of a splice

May 14 (Tuesday)

10:00–11:00,
Jacques Darné (Université Grenoble Alpes)
The Andreadakis problem and Milnor invariants of (welded) braids up to homotopy

11:20–12:20,
Shunsuke Tsuji (RIMS, Kyoto University)
The 4th Johnson homomorphism and the 2nd term of the Ohtsuki series

14:00–15:00,
Jae Choon Cha (POSTECH)
Transfinite lower central series and Milnor invariants of 3-manifolds

15:20–16:20,
Luis Paris (Université de Bourgogne)
Commensurator and commensurability of braid groups

17:00–18:30, Tuesday Seminar on Topology

19:00–,
Banquet at the Seminar Room in the Komaba Faculty House

May 15 (Wednesday)

10:00–11:00,
Hokuto Konno (RIKEN iTHEMS)
Characteristic classes defined using 4-dimensional gauge theory

11:20–12:20,
Oscar Randal-Williams (University of Cambridge)
Cohomology of Torelli groups

May 16 (Thursday)

10:00–11:00,
Anderson Vera (IRMA, Université de Strasbourg)
Alternative versions of the Johnson homomorphisms and the LMO functor

11:20–12:20,
Hidekazu Furusho (Nagoya University)
Kontsevich's eye, Lie graphs and associators

14:20–15:20,
Yuuki Tadokoro (National Institute of Technology, Kisarazu College)
Pointed harmonic volume and its relation to the extended Johnson homomorphism

15:40–16:40,
Martin Kassabov (Cornell University)
Co-cycles in the Lie algebra of special derivations

May 17 (Friday)

10:00–11:00,
Minkyoung Song (IBS Center for Geometry and Physics)
Invariants of the homology cobordism group of homology cylinders

11:20–12:20,
Yusuke Kuno (Tsuda University)
Generalized Dehn twists on surfaces and homology cylinders

13:00–, Discussion

Abstracts

Jae Choon Cha (POSTECH)

Transfinite lower central series and Milnor invariants of 3-manifolds

In his Ph.D. thesis, John Milnor defined link invariants using lower central series of the fundamental group, and asked a question to find a method to extract invariants from the transfinite lower central series. We develop a general theory of transfinite Milnor invariants for 3-manifolds, and realize nontrivial values of the invariant for the transfinite case. This particularly gives an affirmative answer to Milnor's question for the case of 3-manifolds. This talk is based on joint work with Kent Orr.

Jacques Darné (Université Grenoble Alpes)

The Andreadakis problem and Milnor invariants of (welded) braids up to homotopy

The Andreadakis problem consists in understanding the difference between two filtrations on the group $Aut(F_n)$ of automorphisms of the free group, one of which being the Andreadakis filtration, which is analogous to the Johnson filtration of the Mapping Class Group. The pure braid group embeds into $Aut(F_n)$ via the Artin action, and so does the group of pure welded braids. The latter identifies with the group of basis-conjugating automorphisms, also known as the McCool group. When restricted to these subgroups, the Andreadakis filtration corresponds to the filtration of braids with respect to the minimal degree of the Milnor invariants detecting them. Thus this restriction of the Andreadakis problem translates into a problem about Milnor invariants of (welded) braids. In this talk, I will present a version of this problem up to (link-)homotopy, which turns out to be simpler, in the case of welded braids, than the original problem.

Hidekazu Furusho (Nagoya University)

Kontsevich's eye, Lie graphs and associators

After I will review on the notion of Kontsevich's eye and his weights associated with Lie graphs, I will discuss on the associator constructed by Alekseev and Torossian.

Kazuo Habiro (RIMS, Kyoto University)

Cyclic nerves of stratified categories

The Hochschild-Mitchell homology of a linear category C is a fundamental invariant of linear categories, which generalizes the Hochschild homology of an algebra. A stratified linear category is a linear category C equipped with a distinguished linear subcategory C^0 and a filtration on the set of objects satisfying a certain factorization property. Then the inclusion functor from C^0 to C induces an isomorphism on Hochschild-Mitchell homology. Similar constructions work also for ordinary (i.e., non-linear) categories. I plan to give some examples as well.

Martin Kassabov (Cornell University)

Co-cycles in the Lie algebra of special derivations

I will explain how to modify the construction for co-cycles in the algebra of symplectic derivations of the free Lie algebra and apply it to the case of special derivations. In the stable case, the resulting co-cycles are closely related to the cohomology groups of $\mathrm{Out}(F_n)$ with twisted coefficients, however, this is probably not the case for the small number of points (e.g the lie algebra $sder_2$).

(base on joint work with J. Conant and K. Vogtmann and ongoing project with A. Alexeev and M. Felder)

Hokuto Konno (RIKEN iTHEMS)

Characteristic classes defined using 4-dimensional gauge theory

For a given fiber bundle whose fiber is a 4-manifold, we shall define a series of characteristic classes using gauge theory, more precisely, Seiberg-Witten theory. As well as the usual gauge theoretic invariants, such as the Seiberg-Witten invariant, may detect the difference between topological 4-manifolds and smooth 4-manifolds, these characteristic classes can detect the difference between topological 4-manifold bundle and smooth 4-manifold bundle. In particular, I will explain that, using these characteristic classes, we can detect a "non-smoothable fiber bundle" whose fiber is a 4-manifold. Some parts of this talk include joint work with David Baraglia, and with Tsuyoshi Kato and Nobuhiro Nakamura.

Yusuke Kuno (Tsuda University)

Generalized Dehn twists on surfaces and homology cylinders

This is a joint work with Gwénaël Massuyeau (University of Burgundy). Lickorish's trick describes Dehn twists along simple closed curves on an oriented surface in terms of surgery of 3-manifolds. We discuss one possible generalization of this description to the situation where the curve under consideration may have self-intersections. Our result generalizes previously known computations related to the Johnson homomorphisms for the mapping class groups and for homology cylinders. In particular, we obtain an alternative and direct proof of the surjectivity of the Johnson homomorphisms for homology cylinders, which was proved by Garoufalidis-Levine and Habegger.

Yuta Nozaki (Meiji University)

Finiteness of the image of the Reidemeister torsion of a splice

We regard the $SL(2,\mathbb{C})$-Reidemeister torsion of a 3-manifold M as a $\mathbb{C}$-valued function on the character variety of M and consider the image $RT(M)$ of this function. The set $RT(M)$ is known to be infinite when M is the complement of the figure-eight knot or its double. In contrast, we prove that $RT(M)$ is a finite set if M is the splice of two certain knots in S^3. The proof is based on an observation on the character varieties and A-polynomials of knots. This is a joint work with Teruaki Kitano.

Luis Paris (Université de Bourgogne)

Commensurator and commensurability of braid groups

Joint work with Maria Cumplido.

An Artin group A is a group defined by relations of the form $aba\cdots = bab\cdots$, the words in both sides having the same length. The quotient of A by the relations $a^2 = 1$, where a ranges among the generators, is the Coxeter group associated to A. We say that A is of spherical type if its associated Coxeter group is finite. The braid groups are the seminal examples of Artin groups of spherical type. We say that two groups A and B are commensurable if there exist finite index subgroups U of A and V of B such that U and V are isomorphic. On the other hand, to a group A we can associate a group $\mathrm{Com}(A)$, called the commensurator of A, which is to the commensurability what is the automorphism group to the isomorphy. In particular, if two groups A and B are commensurable, then they commensurators are isomorphic.

Let Σ be an oriented closed surface and let $\mathcal{P}$ be a finite collection of punctures in Σ. Then, with a few exceptions, by Ivanov, McCarthy, and Kormaz, the commensurator of the mapping class group $\mathcal{M}(\Sigma, \mathcal{P})$ is equal to the extended mapping class group, $\mathcal{M}^*(\Sigma, \mathcal{P})$, which is also equal to the automorphism group of $\mathcal{M}(\Sigma, \mathcal{P})$. A consequence of these results/techniques, due to Charney and Crisp, is that the commensurator of the quotient of the braid group $\mathcal{B}_n$ by its center is equal to the extended mapping class group of the $n+1$-punctured sphere. The purpose of this talk is to tell this story and to show how it can be used to classify the Artin groups of spherical type that are commensurable to a given braid group.

Oscar Randal-Williams (University of Cambridge)

Cohomology of Torelli groups

It is a basic problem in the cohomology of moduli spaces of Riemann surfaces to describe the cohomology of the Torelli group—the subgroup of the mapping class group of those diffeomorphisms which act trivially on the first cohomology of the surface—as a representation of $Sp(2g, \mathbb{Z})$, at least in a stable range depending on the genus of the surface. This question can be generalised to higher dimensions by replacing the genus g surface with its analogue $\#^g S^n \times S^n$. I will present joint work with Alexander Kupers in which we answer this question in dimensions at least 6. Our description is also valid in the classical case $2n = 2$ assuming a finiteness conjecture about the cohomology of this Torelli group.

Takuya Sakasai (The University of Tokyo)

Johnson homomorphisms up to degree 7

Following Satoh's introduction on Johnson homomorphisms, we first recall those for subgroups of the mapping class group. Their fundamental properties and tools for explicit computations, which have been developed by many researchers, will be also reviewed. Then, we will discuss how to determine the rational image of Johnson homomorphisms up to degree 7. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

Takao Satoh (Tokyo University of Science)

On the Andreadakis conjecture of the automorphism groups of free groups

In his doctrial thesis, Andreadakis began to study a certain filtration of the automorphism group of a free group, consisting of normal subgroups defined by the actions on the nilpotent quotients of the free group. We call it the Andreadakis-Johnson filtration. Andreadakis studied the difference between the filtration and the lower central series of the IA-automorphism group, and showed that they are equal for the rank of the free group is two.

Due to Dehn and Nielsen, the mapping class group of a surface with one boundary component can be considered as a subgroup of the automorphism group of a free group. By beautiful works of Johnson and Morita, at least until early 1990s, it is known that the Johnson filtration of the mapping class group differs from the lower central series of the Torelli group, by some topological reasons.

For the automorphism groups of free groups, Andreadakis conjectured that they are equal. Although this problem had been open for long years, Bartholdi gave the answer negatively in the unstable range by using a computer calculation. However the conjecture is still open in the stable range.

Minkyoung Song (IBS Center for Geometry and Physics)

Invariants of the homology cobordism group of homology cylinders

The homology cobordism group of 3-dimensional homology cylinders is an enlargement of both the mapping class group of a surface and the concordance group of string links. In this talk, I introduce extended Milnor invariants and Hirzebruch-type invariants. Extended Milnor invariants for homology cylinders are a combination of Johnson homomorphisms of the mapping class group and Milnor invariants for (string) links. Hirzebruch-type invariants from iterated p-covers are defined for homology cylinders with vanishing extended Milnor invariants. We observe structures of the group of homology cylinders via those invariants, mainly with the extended Milnor filtration which is analogous to the Johnson filtration.

Yuuki Tadokoro (National Institute of Technology, Kisarazu College)

Pointed harmonic volume and its relation to the extended Johnson homomorphism

The period for a compact Riemann surface, defined by the integral of differential 1-forms, is a classical complex analytic invariant, strongly related to the complex structure of the surface. In this talk, we treat another complex analytic invariant called the pointed harmonic volume. As a natural extension of the period defined using Chen's iterated integrals, it captures more detailed information of the complex structure. It is also one of a few explicitly computable examples of complex analytic invariants. We obtain its new value for a certain pointed hyperelliptic curve. An application of the pointed harmonic volume is presented. We explain the relationship between the harmonic volume and first extended Johnson homomorphism on the mapping class group of a pointed oriented closed surface.

Shunsuke Tsuji (RIMS, Kyoto University)

The 4th Johnson homomorphism and the 2nd term of the Ohtsuki series

Using the 4th Johnson homomorphism, we can give an explicit formula for the 2nd term of the Ohtsuki series for some integral homology 3-spheres. The proof is based on the Kauffman bracket skein algebra.

Anderson Vera (IRMA, Université de Strasbourg)

Alternative versions of the Johnson homomorphisms and the LMO functor

One of the main objects associated to a surface S is its mapping class group MCG(S). By considering the action of MCG(S) on the fundamental group of S, it is possible to define different filtrations of MCG(S) together with some homomorphisms on each term of the filtration. In this talk we present our results concerning a filtration of MCG(S), "the alternative Johnson filtration", recently introduced by Habiro-Massuyeau and whose definition involves a handlebody bounded by S. We show the relationship between the "alternative Johnson homomorphisms" and the functorial extension of the Le-Murakami-Ohtsuki (LMO) invariant of 3-manifolds.