List of fundamental spectral types with index of the rigidity ≥ -12

Toshio Oshima (2024.12.28, 2026.9.30)

Numbers of fundamental tuples of all, E_8-type, 3, 4, 5 partitions with a given index

idx	0	−2	−4	−6	−8	−10	−12	−14	−16	−18
all	4	13	37	69	113	198	291	415	647	884
E_8	1	2	5	7	10	15	18	21	33	36
3	3	9	25	46	73	127	182	249	395	522
4	1	3	9	17	29	50	76	115	172	243
5	0	1	2	4	8	15	24	36	55	83

idx	−20	−22	−24	−26	−28	−30	−32	−34	−36	−38
all	1186	1682	2183	2889	3841	4831	6203	8012	9789	12290
E_8	40	55	62	70	92	96	114	142	148	168
3	680	963	1230	1577	2105	2570	3257	4187	5009	6144
4	345	478	625	861	1126	1430	1890	2411	2982	3832
5	111	164	229	310	416	570	716	940	1214	1546

idx	−40	−42	−44	−46	−48	−50	−52	−54	−56	−58
all	15379	18683	22910	28096	33387	40617	48703	57481	68622	81728
E_8	212	220	246	298	314	351	412	429	486	570
3	7731	9153	11072	13493	15768	18878	22594	26040	30819	36495
4	4721	5785	7227	8763	10484	12953	15392	18259	22056	26041
5	1939	2491	3046	3805	4675	5700	6916	8521	10114	12203

$\mathbf{m} = \big(m_{j,\nu}\big)_{\substack{0\le j<p\\ 1\le\nu\le n_j}}$: a tuple (set) of p partitions of a positive integer n (spectral type)
$= m_{0,1}\cdots m_{0,n_1}, m_{1,1}\cdots m_{1,n_1}, \cdots\cdots, m_{p-1,1}\cdots m_{p-1,n_{p-1}}$

$n = m_{j,1}+\cdots+m_{j,n_j}$, ordered $\overset{\text{def.}}{\Longleftrightarrow} m_{j,1}\ge m_{j,2}\ge\cdots\ge m_{j,n_j}>0 \quad (n_j>1,\ 0\le j<p)$

$\mathbf{m}$: fundamental $\overset{\text{def.}}{\Longleftrightarrow} 2n \le \sum_{j=0}^{p-1}\Big(n-\sum_{\nu=1}^{n_j} m_{j,\nu}\Big)$ (and $\mathbf{m}$ is indivisible if the equality holds)

$\mathbf{m}$: basic $\overset{\text{def.}}{\Longleftrightarrow}$ $\mathbf{m}$: fundamental and indivisible

index of rigidity of $\mathbf{m}$: $\operatorname{idx}\mathbf{m} := 2n^2 - \sum_{j=0}^{p-1}\Big(n^2-\sum_{\nu=1}^{n_j} m_{j,\nu}^2\Big)$, $\operatorname{ord}\mathbf{m} := n$

$\operatorname{idx} k\mathbf{m} = k^2\cdot\operatorname{idx}\mathbf{m}$

E_8-type : a_1a_2, $b_1b_2b_3$, $c_1c_2\cdots c_r$ $(p=2,\ n_0=2,\ n_1=3 \overset{\text{fund.}}{\Longrightarrow} r=n_2\ge 6)$

$\operatorname{mc}_\sigma : \mathbf{m} = \big(m_{j,\nu}\big)_{\substack{0\le j<p\\ 1\le\nu\le n_j}} \mapsto \big(m_{j,\nu}-\delta_{\nu,\sigma_j}d_\sigma(\mathbf{m})\big)_{\substack{0\le j<p\\ 1\le\nu\le n_j}}$ $\qquad (d_\sigma(\mathbf{m}) := \sum_{j=0}^{p-1} m_{j,\sigma_j}-(p-1)n)$

Getting fundamental tuples by using computer algebra `Risa/Asir`

rigidity index	0	−2	−4	−6	−8	−10	−12	−14	−16	−18	−20	−22	−60
runtime (sec)				0.02	0.05	0.09	0.19	0.34	0.58	0.97	1.61	2.34	885.1
total number	4	13	37	69	113	198	291	415	647	884	1186	1682	95216

The time for the calculation above is given under Windows 11, i7-10700 by the command `spbasic(`*idx*`,0)` in the library `os_muldif.rr` [O4] of the computer algebra `Risa/Asir` [No].

Classified by orbits of Harnad dual and confluence of Poincaré rank 1 with unfolding[*1]

$$\text{index}: 2-2m \ (m \geq 1)$$

$$[11]_{m+3}: \overbrace{11,11,\ldots,11}^{m+3} = [11]_{m+3} \qquad [11]^T_{m+3}: (m+2)^2, (m+1)^2 2, 1^{2m+4}$$

$$FS_m: m1, m1, 1^{m+1}, 1^{m+1} \qquad Sa_m: m^2, m^2, (2m-1)1, 1^{2m}$$

$$D_4^{(m)}: m^2, m^2, m^2, m(m-1)1 \qquad E_6^{(m)}: m^3, m^3, m^2(m-1)1$$

$$E_7^{(m)}: (2m)^2, m^4, m^3(m-1)1 \qquad E_8^{(m)}: (3m)^2, (2m)^3, m^5(m-1)1$$

Harnad dual

$$\left(m^{(0)}_{1,1}\cdots m^{(0)}_{1,R_1}\right)\cdots\left(m^{(0)}_{n_1,1}\cdots m^{(0)}_{n_1,R_{n_1}}\right), m_{2,1}\cdots m_{2,n_2}, \ldots, m_{q,1}\cdots m_{q,n_q} \quad \left(m_{1,\nu} := \sum_{k=1}^{R_\nu} m^{(0)}_{\nu,k}\right)$$

$$= m^{(0)}_{1,1}\cdots m^{(0)}_{1,R_1}\cdots m^{(0)}_{n_1,1}\cdots m^{(0)}_{n_1,R_{n_1}} \,|\, m_{1,1}\cdots m_{1,n_1}, m_{2,1}\cdots m_{2,n_2}, m_{q,1}\cdots m_{q,n_q}$$

$$\xrightarrow{\text{Hd}} \left(m_{2,2}\cdots m_{2,n_2}\right)\cdots\left(m_{q,2}\cdots m_{q,n_q}\right), \underline{m^{(0)}_{1,0}} m^{(0)}_{1,1}\cdots m^{(0)}_{1,R_1}, \ldots, \underline{m^{(0)}_{n_1,0}} m^{(0)}_{n_1,1}\cdots m^{(0)}_{n_1,R_{n_1}}$$

$$E_8^{(m)}: m^3\cdot m^2(m-1)1 | (3m)^2, (2m)^3 \xrightarrow{\text{Hd}} E_7^{(m)}: (2m)^2, \underline{m}m^3, \underline{m}m^2(m-1)1$$

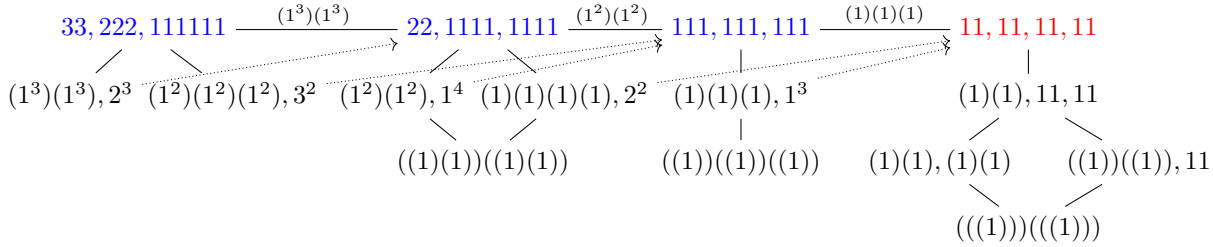
$$E_7^{(m)}: m^2\cdot m^2 | (2m)^2, m^3(m-1)1 \xrightarrow{\text{Hd}} E_6^{(m)}: m^2(m-1)1, \underline{m}m^2, \underline{m}m^2$$

$$E_6^{(m)}: m^3 | m^3, m^2(m-1)1 \xrightarrow{\text{Hd}} D_4^{(m)}: m(m-1)1, \underline{m}m, \underline{m}m, \underline{m}m$$

$$1^{m+1}\cdot 1^{m+1}\cdot 1^2 | (m+1)^2 2, (m+2)^2 \xrightarrow{\text{Hd}} m+2, \underline{1}1^{m+1}, \underline{1}1^{m+1}, m1^2 \to 1^{m+2} | 1^{m+2}, m1^2 \xrightarrow{\text{Hd}} 1^2, [\underline{1}]_{m+2}$$

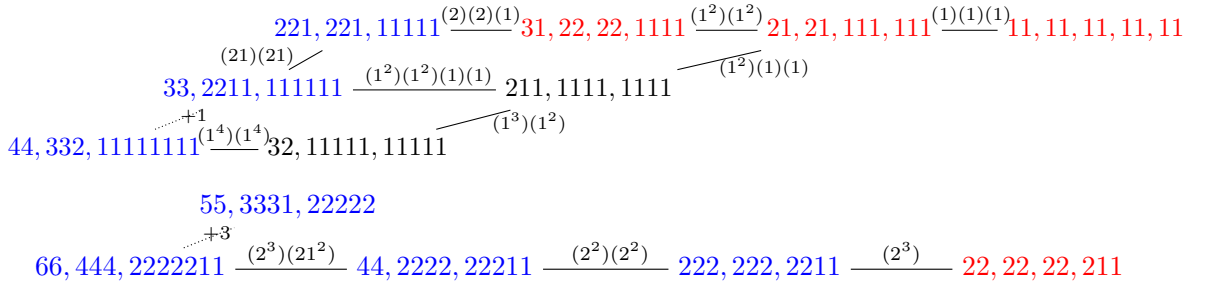
idx=0 (Pidx=1), 4 basic (fundamental) tuples

6: 33,222,111111 $E_8^{(1)}$	4: 22,1111,1111 $E_7^{(1)}$	3: 111,111,111 $E_6^{(1)}$	2: 11,11,11,11 $D_4^{(1)}$



idx=−2 (Pidx=2), 13 basic (fundamental) tuples

8: 44,332,11111111 $[11]^T_5$	6: 33,2211,111111	5: 32,11111,11111	5: 221,221,11111
4: 211,1111,1111	4: 31,22,22,1111 Sa	3: 21,21,111,111 FS	2: 11,11,11,11,11 $[11]_5$
12: 66,444,2222211 $E_8^{(2)}$	10: 55,3331,22222	8: 44,2222,22211 $E_7^{(2)}$	6: 222,222,2211 $E_6^{(2)}$
4: 22,22,22,211 $D_4^{(2)}$			



$$44, 332, 11111111 \xrightarrow[+1]{\text{mc}} 11111\cdot 211 | 54, 333 \xrightarrow{\text{Hd}} 33, \underline{1}111\underline{1}2211$$

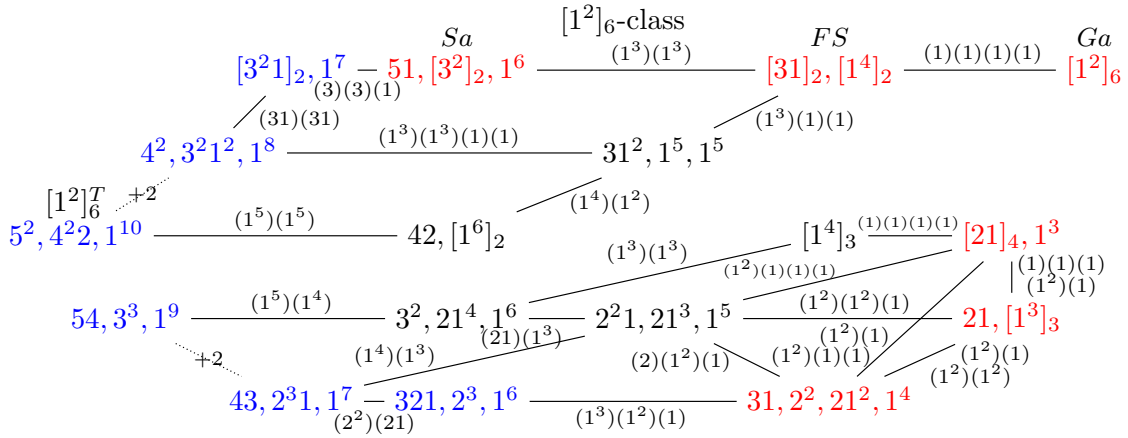
$$66, 444, 2222211 \xrightarrow[+3]{\text{mc}} 2^4\cdot 3^2 1 | 87, 5^3 \xrightarrow{\text{Hd}} 5^2, \underline{2}2^4, \underline{3}3^2 1 = 55, 3331, 22222$$

$$66, 444, 2^5 1^2 \xleftarrow{\text{mc}} 76, 544, 2^6 1 \xleftarrow{\text{mc}} 77, 554, 32^5 1 \xleftarrow{\text{mc}} 87, 555, 3^2 2^4 1$$

[*1] The classification in this note is based on the idea due to H. Kawakami (cf. [Ka]) and the result [O3] with programs (cf. [O4]) under a compuer algebra `Risa/Asir`

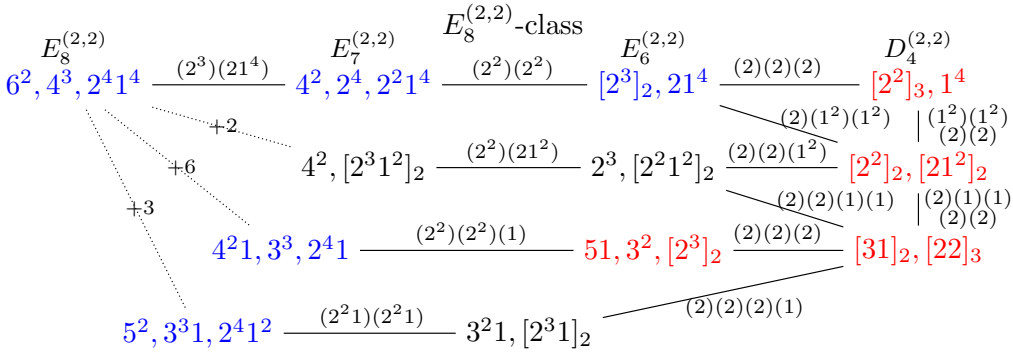
idx=−4 (Pidx=3), 37 basic (fundamental) tuples

10: 55,442,1111111111 $[11]_6^T$	8: 44,3311,11111111	7: 331,331,1111111	6: 42,111111,111111
6: 51,33,33,111111 Sa	5: 311,11111,11111	4: 31,31,1111,1111 FS	2: 11,11,11,11,11,11 $[11]_6$
18: 99,666,3333321 $E_8^{(3)}$	12: 66,3333,33321 $E_7^{(3)}$	9: 333,333,3321 $E_6^{(3)}$	6: 33,33,33,321 $D_4^{(3)}$
9: 54,333,111111111	7: 43,2221,11111111	6: 321,222,111111	6: 33,21111,11111
5: 221,2111,11111	4: 1111,1111,1111	4: 31,22,211,1111	3: 21,111,111,111
3: 21,21,21,21,111			
12: 66,444,22221111 $E_8^{(2,2)}$	10: 55,3331,222211	9: 441,333,22221	8: 44,22211,22211
8: 44,2222,221111 $E_7^{(2,2)}$	7: 331,2221,2221	6: 222,2211,2211	6: 222,222,21111 $E_6^{(2,2)}$
6: 51,33,222,222	4: 22,22,211,211	4: 31,31,22,22,22	4: 22,22,22,1111 $D_4^{(2,2)}$
14: 77,554,2222222	10: 55,3322,22222	8: 332,332,2222	6: 42,33,33,222



$$55,442,1111111111 \xrightarrow[+2]{\text{mc}} 1111111\cdot 311|75,444 \xrightarrow{\text{Hd}} 44,\underline{1}1111111,\underline{3}311$$

$$54,333,111111111 \xrightarrow[+2]{\text{mc}} 1^6\cdot 221|65,443 \xrightarrow{\text{Hd}} 43,\underline{1}111111,\underline{2}221$$



$$66,444,22221111 \xrightarrow[+2]{\text{mc}} 2221\cdot 2221|77,644 \xrightarrow{\text{Hd}} 44,\underline{1}2221,\underline{1}2221$$

$$66,444,22221111 \xrightarrow[+6]{\text{mc}} 2221\cdot 33\cdot 41|765,99 \xrightarrow{\text{Hd}} \underline{2}2221,\underline{3}33,\underline{4}41$$

$$99,765,43^22^31^2 \xrightarrow{\text{mc}} 97,655,3^22^41^2 \xrightarrow{\text{mc}} 77,554,32^41^3 \xrightarrow{\text{mc}} 76,544,2^51^3 \xrightarrow{\text{mc}} 66,444,2^41^4$$

$$66,444,22221111 \xrightarrow[+3]{\text{mc}} 22211\cdot 331|87,555 \xrightarrow{\text{Hd}} 55,\underline{2}22211,\underline{3}331$$

$$77,554,2222222 \xrightarrow[+1]{\text{mc}} 2^4\cdot 32^2|87,5^3 \xrightarrow{\text{Hd}} 5^2,\underline{2}2^4,\underline{3}32^2 = 55,3322,22222$$

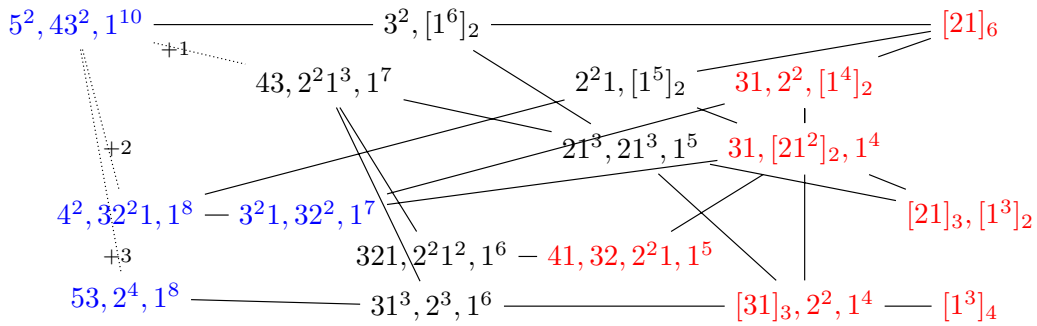
$$55,3322,22222 \to 32\cdot 32|55,22222 \xrightarrow{\text{Hd}} 2222,\underline{3}32,\underline{3}32$$

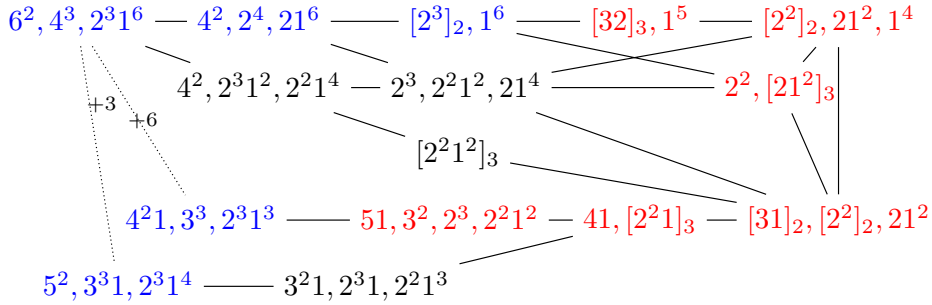
$$332,332,2222 \to 3\cdot 3\cdot 2|332,2222 \xrightarrow{\text{Hd}} 222,\underline{3}3,\underline{3}3,\underline{4}2$$

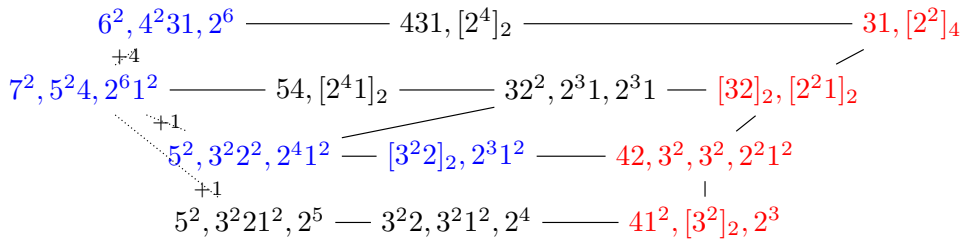
Remark. $(2222)(322),555$ is fundamental but its unfolding $87,555,3222222$ is not.

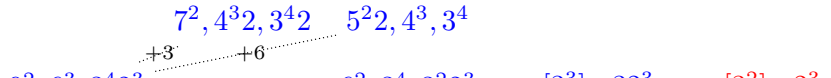
69 fundamental spectral types with idx=−6 (Pidx=4)

12: 66,552,111111111111 $[11]_7^T$	10: 55,4411,1111111111	9: 441,441,111111111	8: 71,44,44,11111111 Sa
7: 52,1111111,1111111	6: 411,111111,111111	5: 41,41,11111,11111 FS	2: 11,11,11,11,11,11,11
24: cc,888,4444431 $E_8^{(4)}$	16: 88,4444,44431 $E_7^{(4)}$	12: 444,444,4431 $E_6^{(4)}$	8: 44,44,44,431 $D_4^{(4)}$
10: 55,433,1111111111	8: 53,2222,11111111	8: 44,3221,11111111	7: 43,22111,1111111
7: 331,322,1111111	6: 33,111111,111111	6: 321,2211,111111	6: 3111,222,111111
5: 221,11111,11111	5: 2111,2111,11111	5: 41,32,221,11111	4: 31,22,1111,1111
4: 31,211,211,1111	4: 31,31,31,22,1111	3: 111,111,111,111	3: 21,21,21,111,111
3: 21,21,21,21,21,21			
12: 66,444,222111111 $E_8^{(2,3)}$	10: 55,3331,2221111	9: 441,333,222111	8: 44,2222,2111111 $E_7^{(2,3)}$
8: 44,22211,221111	7: 331,2221,22111	6: 222,222,111111 $E_6^{(2,3)}$	6: 222,2211,21111
6: 2211,2211,2211	6: 51,33,222,2211	5: 41,221,221,221	5: 32,32,32,11111
4: 22,22,211,1111 $D_4^{(2,3)}$	4: 22,211,211,211	4: 31,31,22,22,211	
14: 77,554,22222211	12: 66,4431,222222	10: 55,3322,222211	10: 55,33211,22222
9: 54,22221,22221	8: 431,2222,2222	8: 332,332,22211	8: 332,3311,2222
7: 322,2221,2221	6: 42,33,33,2211	6: 411,33,33,222	5: 32,32,221,221
4: 31,22,22,22,22			
18: 99,666,33333111 $E_8^{(3,1)}$	16: 88,5551,333331	15: 771,555,33333	12: 66,3333,333111 $E_7^{(3,1)}$
9: 333,333,33111 $E_6^{(3,1)}$	6: 33,33,33,3111 $D_4^{(3,1)}$		
18: 99,666,3333222 $E_8^{(3,2)}$	14: 77,4442,33332	12: 66,3333,33222 $E_7^{(3,2)}$	12: 552,444,3333
9: 333,333,3222 $E_6^{(3,2)}$	6: 33,33,33,222 $D_4^{(3,2)}$		

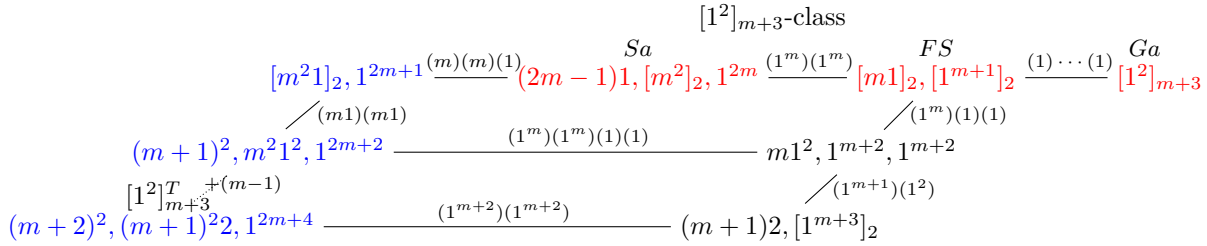






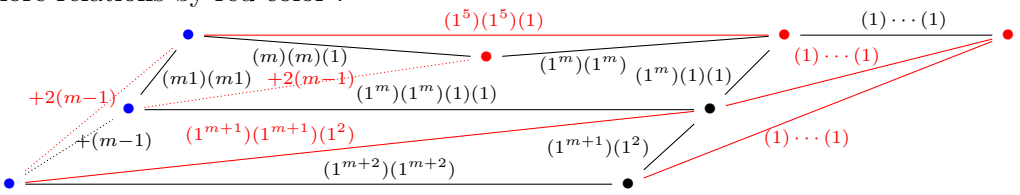


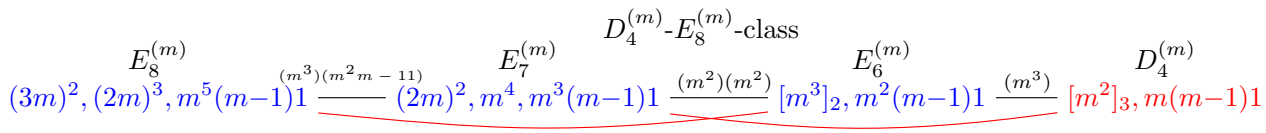
$$\mathrm{idx}\ = 2(m-1)$$



$1^{2m+4},(m{+}2)^2,(m{+}1)^22 \xleftarrow[m-1]{\text{mc}} 1^{2m+1}{\cdot}m1^2|(2m{+}1)(m{+}2),(m{+}1)^3 \xrightarrow{\text{Hd}} (m{+}1)^2,\underline{1}1^{2m+1},\underline{m}m1^2$

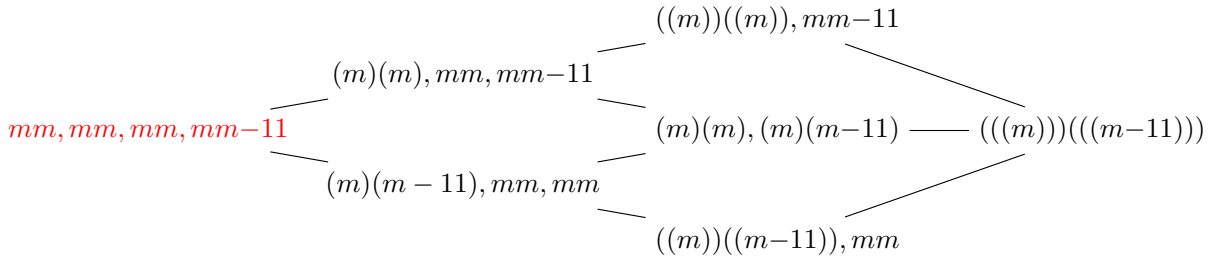
Show more relations by red color :

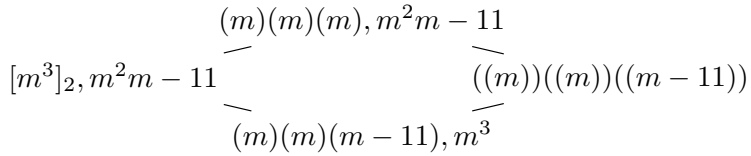




idx $= 0$: $\quad D_4^{(1)} = Ga = FS = Sa$

idx $= -2$: $D_4^{(2)}$-$E_8^{(2)}$-class contains $\ 5^3,3^31,2^5.$





113 Fundamental spectral types with idx=−8 (Pidx=5)

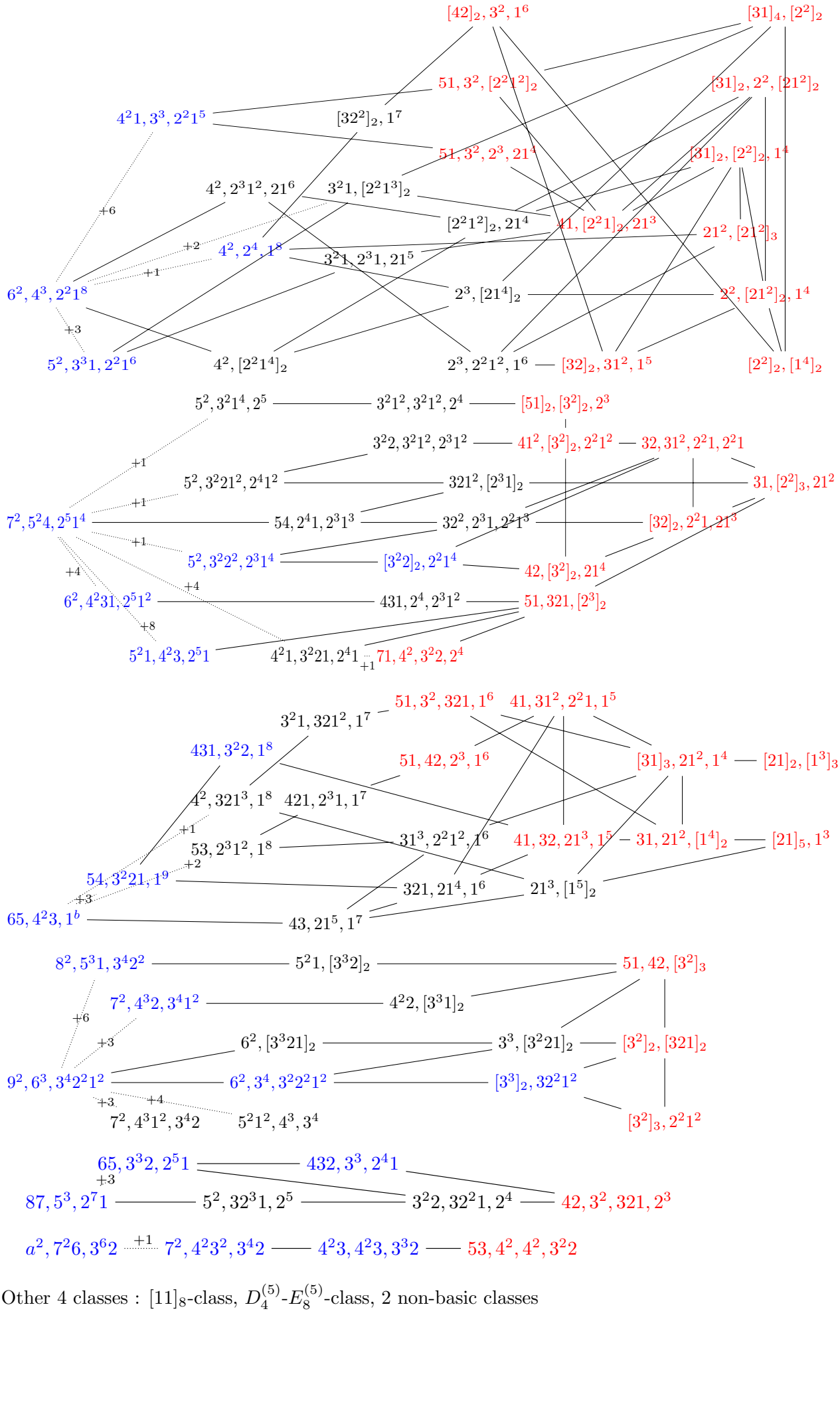
14: 77,662,11111111111111	12: 66,5511,111111111111	11: 551,551,11111111111	10: 91,55,55,1111111111
8: 62,11111111,11111111	7: 511,1111111,1111111	6: 51,51,111111,111111	2: 11,11,11,11,11,11,11,11
30: ff,aaa,5555541	20: aa,5555,55541	15: 555,555,5541	10: 55,55,55,541
12: 66,444,2211111111	10: 55,3331,22111111	9: 441,333,2211111	8: 44,2222,11111111
8: 44,22211,2111111	8: 44,221111,221111	7: 331,2221,211111	7: 331,22111,22111
7: 322,322,1111111	6: 222,2211,111111	6: 222,21111,21111	6: 2211,2211,21111
6: 51,33,222,21111	6: 51,33,2211,2211	6: 42,42,33,111111	5: 41,221,221,2111
5: 32,32,311,11111	4: 22,22,1111,1111	4: 22,211,211,1111	4: 211,211,211,211
4: 31,31,22,22,1111	4: 31,31,22,211,211	4: 31,31,31,31,22,22	
14: 77,554,222221111	12: 66,4431,2222211	11: 551,443,222221	10: 55,3322,2221111
10: 55,33211,222211	10: 55,331111,22222	9: 54,22221,222111	9: 441,3321,22221
8: 431,2222,22211	8: 332,332,221111	8: 332,3311,22211	8: 3311,3311,2222
8: 71,44,332,2222	7: 322,2221,22111	7: 3211,2221,2221	6: 51,321,222,222
6: 42,33,33,21111	6: 411,33,33,2211	6: 51,51,33,33,222	5: 32,32,221,2111
5: 32,311,221,221	4: 31,22,22,22,211		
11: 65,443,11111111111	9: 54,3321,111111111	8: 53,22211,11111111	8: 44,32111,11111111
8: 431,332,11111111	7: 43,211111,1111111	7: 421,2221,1111111	7: 331,3211,1111111
6: 321,21111,111111	6: 3111,2211,111111	6: 51,42,222,111111	6: 51,33,321,111111
5: 2111,11111,11111	5: 41,32,2111,11111	5: 41,311,221,11111	4: 31,211,1111,1111
4: 31,31,31,211,1111	3: 21,21,111,111,111	3: 21,21,21,21,21,111	
18: 99,666,33332211	16: 88,5551,333322	14: 77,4442,333311	14: 77,44411,33332
12: 66,3333,332211	12: 66,33321,33321	12: 5511,444,3333	11: 551,3332,3332
10: 442,3331,3331	9: 333,333,32211	9: 333,3321,3321	6: 33,33,33,2211
6: 33,33,321,321	6: 51,42,33,33,33		
15: 87,555,22222221	11: 65,3332,222221	10: 55,32221,22222	9: 432,333,22221
8: 332,3221,2222	6: 42,33,321,222		
20: aa,776,3333332	14: 77,4433,33332	11: 443,443,3332	8: 53,44,44,332
24: cc,888,4444422	20: aa,6662,44444	16: 88,4444,44422	12: 444,444,4422
8: 44,44,44,422			
16: 88,664,22222222	12: 66,4422,222222	10: 64,22222,22222	10: 442,442,22222
8: 422,2222,2222	8: 62,44,44,2222	6: 42,42,222,222	4: 22,22,22,22,22

The fundamental spectral types in the last two classes in the above table are not basic (their multiplicities are divisible by 2).

The tables of the classification are calculated for idx ≥ -100 by `spfundclass( )` in [O4], 2026.3.19. They are available as binary files used by `Risa/Asir`, which can be loaded as `bload("<`*dir*`>/fc50.bs")` under `Risa/Asir` in the case when idx $= -50$ and the files `fc`ℓ`.bs` for $0 \leq \ell \leq 100$ are in the `zip` files : fc0-50.zip (idx : $0 \sim -50$) , fc52-70.zip, fc72-80.zip, fc82-90.zip, fc92-100.zip.

The tables for idx ≥ -12 are presented in this note.

rigidity index	0	−2	−4	−6	−8	−10	−12	−14	−16	−18	−20
run time(sec)	0.02	0.03	0.03	0.06	0.11	0.20	0.30	0.52	0.86	1.3	2.0
# fund	4	13	37	69	113	198	291	415	647	884	1186
# class	1	2	5	7	10	15	18	21	33	36	40



Other 4 classes : $[11]_8$-class, $D_4^{(5)}$-$E_8^{(5)}$-class, 2 non-basic classes

198 Fundamental spectral types with idx=−10 (Pidx=6)

16: 88,772,1111111111111111	14: 77,6611,11111111111111	13: 661,661,1111111111111	12: b1,66,66,111111111111
9: 72,111111111,111111111	8: 611,11111111,11111111	7: 61,61,1111111,1111111	2: 11,11,11,11,11,11,11,11,11,11
36: ii,ccc,6666651	24: cc,6666,66651	18: 666,666,6651	12: 66,66,66,651
14: 77,554,2222111111	12: 66,4431,22221111	11: 551,443,2222111	10: 55,3322,22111111
10: 55,33211,2221111	10: 55,331111,222211	9: 54,22221,2211111	9: 54,222111,222111
9: 441,3321,222111	9: 441,33111,22221	8: 431,2222,221111	8: 431,22211,22211
8: 332,332,2111111	8: 332,3311,221111	8: 3311,3311,22211	8: 71,44,332,22211
8: 71,44,3311,2222	7: 322,2221,211111	7: 322,22111,22111	7: 3211,2221,22111
7: 31111,2221,2221	7: 61,43,2221,2221	7: 61,331,331,2221	6: 51,321,222,2211
6: 51,3111,222,222	6: 42,33,33,111111	6: 411,33,33,21111	6: 51,51,33,33,2211
5: 32,32,221,11111	5: 32,32,2111,2111	5: 32,311,221,2111	5: 311,311,221,221
5: 41,41,32,221,221	4: 31,22,22,22,1111	4: 31,22,22,211,211	4: 31,31,31,22,22,22
12: 66,444,21111111111	10: 55,3331,211111111	9: 54,3222,111111111	9: 441,333,21111111
8: 44,22211,11111111	8: 44,221111,2111111	8: 422,332,11111111	7: 331,2221,1111111
7: 331,22111,211111	7: 322,3211,1111111	6: 222,21111,111111	6: 2211,2211,111111
6: 2211,21111,21111	6: 51,33,222,111111	6: 51,33,2211,21111	6: 42,411,33,111111
5: 41,221,221,11111	5: 41,221,2111,2111	5: 32,311,311,11111	5: 41,41,32,32,11111
4: 22,211,1111,1111	4: 211,211,211,1111	4: 31,31,22,211,1111	4: 31,31,211,211,211
4: 31,31,31,31,22,211			
12: 66,543,111111111111	10: 55,4321,1111111111	9: 63,22221,111111111	9: 54,33111,111111111
9: 441,432,111111111	8: 53,221111,11111111	8: 521,2222,11111111	8: 431,3311,11111111
7: 43,1111111,1111111	7: 421,22111,1111111	7: 4111,2221,1111111	7: 61,43,331,1111111
6: 321,111111,111111	6: 3111,21111,111111	6: 51,42,2211,111111	6: 51,411,222,111111
5: 41,32,11111,11111	5: 41,311,2111,11111	5: 41,41,41,221,11111	4: 31,31,31,1111,1111
3: 21,111,111,111,111	3: 21,21,21,21,111,111	3: 21,21,21,21,21,21,21	
15: 87,555,222222111	13: 76,4441,2222221	12: 651,444,222222	11: 65,3332,2222111
11: 65,33311,222221	10: 55,32221,222211	10: 55,322111,22222	10: 541,3331,22222
9: 441,3222,22221	9: 432,333,222111	9: 4311,333,22221	8: 332,3221,22211
8: 332,32111,2222	8: 3311,3221,2222	7: 2221,2221,2221	6: 51,222,222,222
6: 42,33,321,2211	6: 42,33,3111,222	6: 411,33,321,222	5: 32,221,221,221
5: 41,32,32,32,221			
18: 99,666,333321111	16: 88,5551,3333211	15: 771,555,333321	14: 77,44411,333311
13: 661,4441,33331	12: 66,3333,3321111	12: 66,33321,333111	12: b1,66,444,3333
11: 551,3332,33311	10: 4411,3331,3331	9: 333,333,321111	9: 333,3321,33111
9: 81,441,333,333	6: 33,33,33,21111	6: 33,33,321,3111	6: 51,411,33,33,33
16: 88,664,222222211	14: 77,5531,2222222	12: 66,4422,2222211	12: 66,44211,222222
10: 64,22222,222211	10: 442,442,222211	10: 442,4411,22222	8: 422,2222,22211
8: 4211,2222,2222	8: 62,44,44,22211	8: 611,44,44,2222	6: 42,42,222,2211
6: 42,411,222,222	4: 22,22,22,22,211		
20: aa,776,33333311	18: 99,6651,333333	14: 77,4433,333311	13: 76,33331,33331
12: 651,3333,3333	11: 443,443,33311	10: 433,3331,3331	8: 53,44,44,3311
7: 43,43,331,331	6: 51,33,33,33,33		
18: 99,666,33322221	14: 77,4442,333221	12: 66,3333,322221	12: 66,33321,33222
12: 552,444,33321	10: 442,3331,3322	9: 333,333,22221	9: 333,3321,3222
7: 43,43,43,2221	6: 33,33,321,222		
12: 75,444,111111111111	10: 64,3331,1111111111	9: 531,333,111111111	8: 44,311111,11111111
7: 331,31111,1111111	6: 51,33,3111,111111	5: 11111,11111,11111	4: 31,1111,1111,1111
4: 31,31,31,31,31,1111			
16: 88,655,22222222	12: 75,3333,222222	12: 66,4332,222222	10: 55,22222,22222
10: 442,433,22222	8: 332,2222,2222	6: 42,33,222,222	5: 32,32,32,32,32
24: cc,888,44444211	22: bb,7771,444442	20: aa,66611,44444	16: 88,4444,444211
12: 444,444,44211	8: 44,44,44,4211		
24: cc,888,4444332	18: 99,5553,44442	16: 88,4444,44332	12: 444,444,4332
8: 44,44,44,332			
21: ba,777,3333333	15: 87,4443,33333	12: 543,444,3333	

291 fundamental types with idx=-12 (Pidx=7)

18: $9^2, 8^2 2, 1^i$	16: $8^2, 7^2 1^2, 1^g$	15: $7^2 1, 7^2 1, 1^f$	14: $d1, 7^2, 7^2, 1^e$	10: $82, 1^a, 1^a$	9: $71^2, 1^9, 1^9$
8: $71, 71, 1^8, 1^8$	2: $[1^2]_{10}$				
42: $l^2, e^3, 7^5 61$	28: $e^2, 7^4, 7^3 61$	21: $7^3, 7^3, 7^2 61$	14: $7^2, 7^2, 7^2, 761$		
14: $7^2, 5^2 4, 2^3 1^8$	12: $6^2, 4^2 31, 2^3 1^6$	11: $5^2 1, 4^2 3, 2^3 1^5$	10: $5^2, 3^2 2^2, 21^8$	10: $5^2, 3^2 1^2, 2^2 1^6$	10: $5^2, 3^2 1^4, 2^3 1^4$
9: $54, 2^4 1, 21^7$	9: $54, 2^3 1^3, 2^2 1^5$	9: $4^2 1, 3^2 21, 2^2 1^5$	9: $4^2 1, 3^2 1^3, 2^3 1^3$	8: $431, 2^4, 21^6$	8: $431, 2^3 1^2, 2^2 1^4$
8: $3^2 2, 3^2 2, 1^8$	8: $3^2 2, 3^2 1^2, 21^6$	8: $3^2 1^2, 3^2 1^2, 2^2 1^4$	8: $71, 4^2, 3^2 2, 2^2 1^4$	8: $71, 4^2, 3^2 1^2, 2^3 1^2$	7: $32^2, 2^3 1, 1^7$
7: $32^2, 2^2 1^3, 21^5$	7: $321^2, 2^3 1, 21^5$	7: $321^2, 2^2 1^3, 2^2 1^3$	7: $31^4, 2^3 1, 2^2 1^3$	7: $61, 43, 2^3 1, 2^2 1^3$	7: $61, 3^2 1, 3^2 1, 2^2 1^3$
7: $52, 43, 43, 1^7$	6: $51, 321, 2^3, 21^4$	6: $51, 321, [2^2 1^2]_2$	6: $51, 31^3, 2^3, 2^2 1^2$	6: $42, 42, 321, 1^6$	6: $41^2, 3^2, 3^2, 1^6$
6: $51, 51, 3^2, 3^2, 21^4$	5: $32, 32, 21^3, 1^5$	5: $32, 31^2, 2^2 1, 1^5$	5: $32, 31^2, 21^3, 21^3$	5: $31^2, 31^2, 2^2 1, 21^3$	5: $41, 41, 32, 2^2 1, 21^3$
5: $[41]_2, 31^2, [2^2 1]_2$	4: $31, 2^2, 2^2, 21^2, 1^4$	4: $31, 2^2, [21^2]_3$	4: $[31]_3, [2^2]_2, 21^2$		
12: $6^2, 4^3, 1^c$	10: $64, 3^2 2^2, 1^a$	10: $5^2, 42^3, 1^a$	10: $5^2, 3^3 1, 1^a$	9: $54, 32^2 1^2, 1^9$	9: $52^2, 3^3, 1^9$
9: $4^2 1, 3^3, 1^9$	8: $4^2, 2^2 1^4, 1^8$	8: $4^2, 21^6, 21^6$	8: $431, 32^2 1, 1^8$	8: $42^2, 3^2 1^2, 1^8$	8: $421^2, 3^2 2, 1^8$
7: $3^2 1, 2^2 1^3, 1^7$	7: $3^2 1, 21^5, 21^5$	7: $32^2, 31^4, 1^7$	7: $321^2, 321^2, 1^7$	7: $61, 43, 32^2, 1^7$	6: $2^3, 1^6, 1^6$
6: $2^2 1^2, 21^4, 1^6$	6: $21^4, 21^4, 21^4$	6: $51, 3^2, 2^2 1^2, 1^6$	6: $51, 3^2, 21^4, 21^4$	6: $51, 321, 321, 1^6$	6: $41^2, 41^2, 3^2, 1^6$
6: $51, 51, 42, 3^2, 1^6$	5: $41, 2^2 1, 21^3, 1^5$	5: $41, 21^3, 21^3, 21^3$	5: $31^2, 31^2, 31^2, 1^5$	5: $41, 41, 32, 31^2, 1^5$	4: $2^2, 1^4, 1^4, 1^4$
4: $21^2, 21^2, 1^4, 1^4$	4: $31, 31, 2^2, 1^4, 1^4$	4: $[31]_2, [21^2]_2, 1^4$	4 : $[31]_4, 2^2, 1^4$	4: $[31]_4, 21^2, 21^2$	4: $[31]_6, 2^2$
15: $87, 5^3, 2^5 1^5$	13: $76, 4^3 1, 2^5 1^3$	12: $651, 4^3, 2^5 1^2$	11: $65, 3^3 2, 2^3 1^5$	11: $65, 3^3 1^2, 2^4 1^3$	10: $5^2, 32^3 1, 2^3 1^4$
10: $5^2, 32^2 1^3, 2^4 1^2$	10: $5^2, 321^5, 2^5$	10: $541, 3^3 1, 2^4 1^2$	9: $4^2 1, 32^3, 2^3 1^3$	9: $4^2 1, 32^2 1^2, 2^4 1$	9: $432, 3^3, 2^2 1^5$
9: $431^2, 3^3, 2^3 1^3$	9: $81, 54, 3^3, 2^4 1$	8: $3^2 2, 32^2 1, 2^2 1^4$	8: $3^2 2, 321^3, 2^3 1^2$	8: $3^2 2, 31^5, 2^4$	8: $3^2 1^2, 32^2 1, 2^3 1^2$
8: $3^2 1^2, 321^3, 2^4$	8: $71, 4^2, 32^2 1, 2^4$	7: $2^3 1, 2^3 1, 2^2 1^3$	7: $61, 3^2 1, 32^2, 2^3 1$	6: $51, 2^3, 2^3, 2^2 1^2$	6: $42, 3^2, 321, 21^4$
6: $42, 3^2, 31^3, 2^2 1^2$	6: $41^2, 3^2, 321, 2^2 1^2$	6: $41^2, 3^2, 31^3, 2^3$	6 : $51, 51, 3^2, 321, 2^3$	5: $32, 2^2 1, 2^2 1, 21^3$	5: $31^2, 2^2 1, 2^2 1, 2^2 1$
5: $41, [32]_3, 21^3$	5: $41, [32]_2, 31^2, 2^2 1$				
16: $8^2, 6^2 4, 2^6 1^4$	14: $7^2, 5^2 31, 2^6 1^2$	13: $6^2 1, 5^2 3, 2^6 1$	12: $6^2, 4^2 2^2, 2^4 1^4$	12: $6^2, 4^2 21^2, 2^5 1^2$	12: $6^2, 4^2 1^4, 2^6$
11: $5^2 1, 4^2 21, 2^5 1$	10: $64, 2^5, 2^3 1^4$	10: $64, 2^4 1^2, 2^4 1^2$	10: $4^2 2, 4^2 2, 2^3 1^4$	10: $4^2 2, 4^2 1^2, 2^4 1^2$	10: $4^2 1^2, 4^2 1^2, 2^5$
10: $91, 5^2, 4^2 2, 2^5$	9: $531, 2^4 1, 2^4 1$	8: $42^2, 2^4, 2^2 1^4$	8: $42^2, 2^3 1^2, 2^3 1^2$	8: $421^2, 2^4, 2^3 1^2$	8: $41^4, 2^4, 2^4$
8: $71, 53, 2^4, 2^4$	8: $62, 4^2, 4^2, 2^2 1^4$	8: $61^2, 4^2, 4^2, 2^3 1^2$	8: $71, 71, 4^2, 4^2, 2^4$	6: $42, 42, 2^3, 21^4$	6: $42, 42, 2^2 1^2, 2^2 1^2$
6: $42, 41^2, 2^3, 2^2 1^2$	6: $41^2, 41^2, 2^3, 2^3$	6: $51, 51, 42, 2^3, 2^3$	4: $2^2, 2^2, 2^2, 2^2, 1^4$	4: $2^2, 2^2, 2^2, 21^2, 21^2$	4: $31, 31, [2^2]_4$
18: $9^2, 6^3, 3^3 2^3 1^3$	16: $8^2, 5^3 1, 3^3 2^3 1$	15: $7^2 1, 5^3, 3^3 2^3$	14: $7^2, 4^3 2, 3^3 21^3$	14: $7^2, 4^3 1^2, 3^3 2^2 1$	13: $6^2 1, 4^3 1, 3^3 2^2$
12: $6^2, 3^4, 32^3 1^3$	12: $6^2, 3^3 21, 3^2 2^2 1^2$	12: $6^2, 3^3 1^3, 3^2 2^3$	12: $5^2 2, 4^3, 3^3 1^3$	12: $5^2 1^2, 4^3, 3^3 21$	11: $5^2 1, 3^3 2, 3^2 2^2 1$
10: $4^2 2, 3^3 1, 3^2 21^2$	10: $4^2 1^2, 3^3 1, 3^2 2^2$	9: $3^3, 3^3, 2^3 1^3$	9: $3^3, 3^2 21, 32^2 1^2$	9: $3^3, 3^2 1^3, 32^3$	9: $3^2 21, 3^2 21, 3^2 21$
8: $71, 3^2 2, 3^2 2, 3^2 2$	7: $52, 3^2 1, 3^2 1, 3^2 1$	7: $43, 43, 43, 2^2 1^3$	6: $3^2, 3^2, 321, 2^2 1^2$	6: $3^2, 3^2, 31^3, 2^3$	6: $3^2, 321, 321, 321$
6: $51, 42, [3^2]_2, 321$					
13: $76, 5^2 3, 1^d$	11: $65, 4^2 21, 1^b$	10: $73, 2^5, 1^a$	10: $5^2, 431^3, 1^a$	10: $541, 4^2 2, 1^a$	9: $63, 2^3 1^3, 1^9$
9: $4^2 1, 431^2, 1^9$	8: $53, 21^6, 1^8$	8: $521, 2^3 1^2, 1^8$	8: $51^3, 2^4, 1^8$	8: $71, 4^2, 431, 1^8$	7 : $421, 21^5, 1^7$
7: $41^3, 2^2 1^3, 1^7$	7: $61, 52, 2^3 1, 1^7$	6: $31^3, 1^6, 1^6$	6: $51, 42, 21^4, 1^6$	6: $51, 41^2, 2^2 1^2, 1^6$	6: $51, 51, 51, 2^3, 1^6$
5: $41, 31^2, 1^5, 1^5$	5: $41, 41, 41, 21^3, 1^5$	3: $[1^3]_4$	3: $[21]_3, [1^3]_3$	3: $[21]_6, 1^3$	
16: $8^2, 65^2, 2^7 1^2$	14: $7^2, 54^2 1, 2^7$	12: $75, 3^4, 2^5 1^2$	12: $6^2, 43^2 2, 2^5 1^2$	12: $6^2, 43^2 1^2, 2^6$	11: $65, 3^2 2^2 1, 2^5 1$
10: $5^2, 2^5, 2^4 1^2$	10: $541, 3^2 2^2, 2^5$	10: $4^2 2, 43^2, 2^4 1^2$	10: $4^2 1^2, 43^2, 2^5$	9: $432, 3^2 21, 2^4 1$	9: $42^2 1, 3^3, 2^4 1$
8: $3^2 2, 2^4, 2^3 1^2$	8: $3^2 1^2, 2^4, 2^4$	8: $32^2 1, 32^2 1, 2^4$	7: $52, 43, 3^2 1, 2^3 1$	6: $42, 3^2, 2^3, 2^2 1^2$	6: $42, 321, 321, 2^3$
6: $41^2, 3^2, 2^3, 2^3$	6: $51, 42, 42, 3^2, 2^3$	5: $[32]_4, 31^2$			
24: $c^2, 8^3, 4^4 3^2 1^2$	22: $b^2, 7^3 1, 4^4 3^2$	18: $9^2, 5^3 3, 4^4 1^2$	16: $8^2, 4^4, 4^2 3^2 1^2$	16: $8^2, 4^3 31, 4^3 31$	15: $7^2 1, 4^3 3, 4^3 3$
13: $5^2 3, 4^3 1, 4^3 1$	12: $4^3, 4^3, 43^2 1^2$	12: $4^3, 4^2 31, 4^2 31$	8: $4^2, 4^2, 4^2, 3^2 1^2$	8: $4^2, 4^2, 431, 431$	8: $71, 53, 4^2, 4^2, 4^2$
20: $a^2, 7^2 6, 3^5 2^2 1$	16: $8^2, 5^2 42, 3^5 1$	14: $7^2, 4^2 3^2, 3^3 2^2 1$	14: $7^2, 4^2 321, 3^4 2$	13: $76, 3^4 1, 3^3 2^2$	12: $5^2 2, 4^2 31, 3^4$
11: $4^2 3, 4^2 3, 3^2 2^2 1$	11: $4^2 3, 4^2 21, 3^3 2$	10: $43^2, 3^3 1, 3^2 2^2$	8: $53, 4^2, 4^2, 32^2 1$	8: $521, 4^2, 4^2, 3^2 2$	7: $43, 43, 3^2 1, 32^2$
18: $9^2, 6^3, 3^4 1^6$	16: $8^2, 5^3 1, 3^4 1^4$	15: $7^2 1, 5^3, 3^4 1^3$	12: $6^2, 3^4, 3^2 1^6$	12: $6^2, 3^3 1^3, 3^3 1^3$	11: $5^2 1, 3^3 1^2, 3^3 1^2$
10: $91, 5^2, 3^3 1, 3^3 1$	9: $3^3, 3^3, 31^6$	9: $3^3, 3^2 1^3, 3^2 1^3$	6: $3^2, 3^2, 3^2, 1^6$	6: $3^2, 3^2, 31^3, 31^3$	6: $[51]_3, [3^2]_3$
18: $9^2, 6^3, 3^2 2^6$	14: $7^2, 4^3 2, 3^2 2^4$	12: $6^2, 3^4, 2^6$	12: $6^2, 3^2 2^3, 3^2 2^3$	12: $5^2 2, 4^3, 3^2 2^3$	10: $4^2 2, 3^2 2^2, 3^2 2^2$
10: $43^2, 43^2, 2^5$	9: $3^3, 32^3, 32^3$	8: $62, 4^2, 3^2 2, 3^2 2$	8: $53, 53, 4^2, 2^4$	6: $3^2, 3^2, 2^3, 2^3$	6: $42, 42, 42, 3^2, 3^2$
24: $c^2, 8^3, 4^5 1^4$	22: $b^2, 7^3 1, 4^5 1^2$	21: $a^2 1, 7^3, 4^5 1$	16: $8^2, 4^4, 4^3 1^4$	12: $4^3, 4^3, 4^2 1^4$	8: $4^2, 4^2, 4^2, 41^4$
24: $c^2, 8^3, 4^3 3^4$	18: $9^2, 5^3 3, 4^3 3^2$	16: $8^2, 4^4, 43^4$	15: $6^2 3, 5^3, 4^3 3$	12: $4^3, 4^3, 3^4$	9: $54, 54, 54, 3^3$
30: $f^2, a^3, 5^5 32$	20: $a^2, 5^4, 5^3 32$	15: $5^3, 5^3, 5^2 32$	10: $5^2, 5^2, 5^2, 532$		
26: $d^2, 9^2 8, 4^6 2$	18: $9^2, 5^2 4^2, 4^4 2$	14: $5^2 4, 5^2 4, 4^3 2$	10: $64, 5^2, 5^2, 4^2 2$		
18: $9^2, 7^2 4, 2^9$	14: $7^2, 5^2 2^2, 2^7$	12: $5^2 2, 5^2 2, 2^6$	10: $82, 5^2, 5^2, 2^5$		

a: 10 b: 11 c: 12 d: 13 e: 14 f: 15 g: 16 h: 17 i: 18 j: 19 k: 20 l: 21 m: 22
n: 23 o: 24 p: 25 q: 26 r: 27 s: 28 t: 29 u: 30 v: 31 w: 32 x: 33 y: 34 z: 35

E_8-fundamental spectral types with a given index of rigidity

0	6: $3^2, 2^3, 1^6$				
-2	8: $4^2, 3^2 2, 1^8$	12: $6^2, 4^3, 2^5 1^2$			
-4	9: $54, 3^3, 1^9$	10: $5^2, 4^2 2, 1^a$	12: $6^2, 4^3, 2^4 1^4$	14: $7^2, 5^2 4, 2^7$	18: $9^2, 6^3, 3^5 21$
-6	10: $5^2, 43^2, 1^a$	12: $6^2, 4^3, 2^3 1^6$	12: $6^2, 5^2 2, 1^c$	14: $7^2, 5^2 4, 2^6 1^2$	18: $9^2, 6^3, 3^4 2^3$
	18: $9^2, 6^3, 3^5 1^3$	24: $c^2, 8^3, 4^5 31$			
-8	11: $65, 4^2 3, 1^b$	12: $6^2, 4^3, 2^2 1^8$	14: $7^2, 5^2 4, 2^5 1^4$	14: $7^2, 6^2 2, 1^e$	15: $87, 5^3, 2^7 1$
	16: $8^2, 6^2 4, 2^8$	18: $9^2, 6^3, 3^4 2^2 1^2$	20: $a^2, 7^2 6, 3^6 2$	24: $c^2, 8^3, 4^5 2^2$	30: $f^2, a^3, 5^5 41$
-10	12: $6^2, 4^3, 21^a$	12: $6^2, 543, 1^c$	12: $75, 4^3, 1^c$	14: $7^2, 5^2 4, 2^4 1^6$	15: $87, 5^3, 2^6 1^3$
	16: $8^2, 65^2, 2^8$	16: $8^2, 6^2 4, 2^7 1^2$	16: $8^2, 7^2 2, 1^g$	18: $9^2, 6^3, 3^3 2^4 1$	18: $9^2, 6^3, 3^4 21^4$
	20: $a^2, 7^2 6, 3^6 1^2$	21: $ba, 7^3, 3^7$	24: $c^2, 8^3, 4^4 3^2 2$	24: $c^2, 8^3, 4^5 21^2$	36: $i^2, c^3, 6^5 51$
-12	12: $6^2, 4^3, 1^c$	13: $76, 5^2 3, 1^d$	14: $7^2, 5^2 4, 2^3 1^8$	15: $87, 5^3, 2^5 1^5$	16: $8^2, 65^2, 2^7 1^2$
	16: $8^2, 6^2 4, 2^6 1^4$	18: $9^2, 6^3, 3^2 2^6$	18: $9^2, 6^3, 3^3 2^3 1^3$	18: $9^2, 6^3, 3^4 1^6$	18: $9^2, 7^2 4, 2^9$
	18: $9^2, 8^2 2, 1^i$	20: $a^2, 7^2 6, 3^5 2^2 1$	24: $c^2, 8^3, 4^3 3^4$	24: $c^2, 8^3, 4^4 3^2 1^2$	24: $c^2, 8^3, 4^5 1^4$
	26: $d^2, 9^2 8, 4^6 2$	30: $f^2, a^3, 5^5 32$	42: $l^2, e^3, 7^5 61$		
-14	13: $76, 54^2, 1^d$	14: $7^2, 5^2 4, 2^2 1^a$	14: $7^2, 653, 1^e$	15: $87, 5^3, 2^4 1^7$	16: $8^2, 65^2, 2^6 1^4$
	16: $8^2, 6^2 4, 2^5 1^6$	17: $98, 6^2 5, 2^8 1$	18: $9^2, 6^3, 3^2 2^5 1^2$	18: $9^2, 6^3, 3^3 2^2 1^5$	18: $9^2, 7^2 4, 2^8 1^2$
	20: $a^2, 7^2 6, 3^4 2^4$	20: $a^2, 7^2 6, 3^5 21^3$	20: $a^2, 9^2 2, 1^k$	21: $ba, 7^3, 3^6 21$	22: $b^2, 8^2 6, 3^7 1$
	24: $c^2, 8^3, 4^4 32^2 1$	26: $d^2, 9^2 8, 4^5 3^2$	26: $d^2, 9^2 8, 4^6 1^2$	30: $f^2, a^3, 5^4 4^2 2$	30: $f^2, a^3, 5^5 31^2$
	48: $o^2, g^3, 8^5 71$				
-16	14: $7^2, 5^2 4, 21^c$	14: $7^2, 64^2, 1^e$	14: $86, 5^2 4, 1^e$	15: $87, 5^3, 2^3 1^9$	15: $87, 6^2 3, 1^f$
	16: $8^2, 65^2, 2^5 1^6$	16: $8^2, 6^2 4, 2^4 1^8$	17: $98, 6^2 5, 2^7 1^3$	18: $9^2, 6^3, 32^7 1$	18: $9^2, 6^3, 3^2 2^4 1^4$
	18: $9^2, 6^3, 3^3 21^7$	18: $9^2, 765, 2^9$	18: $9^2, 7^2 4, 2^7 1^4$	18: $a8, 6^3, 2^9$	20: $a^2, 7^2 6, 3^4 2^3 1^2$
	20: $a^2, 7^2 6, 3^5 1^5$	20: $a^2, 8^2 4, 2^a$	21: $ba, 7^3, 3^5 2^3$	21: $ba, 7^3, 3^6 1^3$	22: $b^2, 87^2, 3^7 1$
	22: $b^2, 8^2 6, 3^6 2^2$	22: $b^2, a^2 2, 1^m$	24: $c^2, 8^3, 4^3 3^3 21$	24: $c^2, 8^3, 4^4 2^4$	24: $c^2, 8^3, 4^4 321^3$
	27: $ed, 9^3, 4^6 3$	28: $e^2, a^2 8, 4^7$	30: $f^2, a^3, 5^4 43^2$	30: $f^2, a^3, 5^4 4^2 1^2$	30: $f^2, a^3, 5^5 2^2 1$
	32: $g^2, b^2 a, 5^6 2$	36: $i^2, c^3, 6^5 42$	54: $r^2, i^3, 9^5 81$		
-18	14: $7^2, 5^2 4, 1^e$	15: $87, 5^3, 2^2 1^b$	15: $96, 5^3, 1^f$	16: $8^2, 65^2, 2^4 1^8$	16: $8^2, 6^2 4, 2^3 1^a$
	16: $8^2, 763, 1^g$	17: $98, 6^2 5, 2^6 1^5$	18: $9^2, 6^3, 2^9$	18: $9^2, 6^3, 32^6 1^3$	18: $9^2, 6^3, 3^2 2^3 1^6$
	18: $9^2, 6^3, 3^3 1^9$	18: $9^2, 765, 2^8 1^2$	18: $9^2, 7^2 4, 2^6 1^6$	18: $a8, 6^3, 2^8 1^2$	20: $a^2, 7^2 6, 3^3 2^5 1$
	20: $a^2, 7^2 6, 3^4 2^2 1^4$	20: $a^2, 8^2 4, 2^9 1^2$	21: $ba, 7^3, 3^5 2^2 1^2$	22: $b^2, 87^2, 3^6 2^2$	22: $b^2, 8^2 6, 3^6 21^2$
	24: $c^2, 8^3, 4^2 3^5 1$	24: $c^2, 8^3, 4^3 3^2 2^3$	24: $c^2, 8^3, 4^3 3^3 1^3$	24: $c^2, 8^3, 4^4 2^3 1^2$	24: $c^2, 8^3, 4^4 31^5$
	24: $c^2, 9^2 6, 3^8$	24: $c^2, b^2 2, 1^o$	26: $d^2, 9^2 8, 4^5 321$	28: $e^2, a9^2, 4^7$	30: $f^2, a^3, 5^3 4^3 3$
	30: $f^2, a^3, 5^5 21^3$	32: $g^2, b^2 a, 5^6 1^2$	36: $i^2, c^3, 6^4 5^2 2$	36: $i^2, c^3, 6^5 3^2$	36: $i^2, c^3, 6^5 41^2$
	60: $u^2, k^3, a^5 91$				
-20	15: $87, 5^3, 21^d$	15: $87, 654, 1^f$	16: $8^2, 65^2, 2^3 1^a$	16: $8^2, 6^2 4, 2^2 1^c$	17: $98, 6^2 5, 2^5 1^7$
	17: $98, 7^2 3, 1^h$	18: $9^2, 6^3, 2^8 1^2$	18: $9^2, 6^3, 32^5 1^5$	18: $9^2, 6^3, 3^2 2^2 1^8$	18: $9^2, 765, 2^7 1^4$
	18: $9^2, 7^2 4, 2^5 1^8$	18: $a8, 6^3, 2^7 1^4$	19: $a9, 7^2 5, 2^9 1$	20: $a^2, 7^2 6, 3^2 2^7$	20: $a^2, 7^2 6, 3^3 2^4 1^3$
	20: $a^2, 7^2 6, 3^4 21^6$	20: $a^2, 8^2 4, 2^8 1^4$	21: $ba, 7^3, 3^4 2^4 1$	21: $ba, 7^3, 3^5 21^4$	22: $b^2, 87^2, 3^6 21^2$
	22: $b^2, 8^2 6, 3^5 2^3 1$	22: $b^2, 8^2 6, 3^6 1^4$	22: $b^2, 9^2 4, 2^b$	23: $cb, 8^2 7, 3^7 2$	24: $c^2, 8^3, 4^2 3^4 2^2$
	24: $c^2, 8^3, 4^3 3^2 2^2 1^2$	24: $c^2, 8^3, 4^4 2^2 1^4$	26: $d^2, 9^2 8, 4^4 3^3 1$	26: $d^2, 9^2 8, 4^5 2^3$	26: $d^2, 9^2 8, 4^5 31^3$
	26: $d^2, c^2 2, 1^q$	27: $ed, 9^3, 4^6 21$	30: $f^2, a^3, 5^2 4^5$	30: $f^2, a^3, 5^4 4321$	30: $f^2, a^3, 5^5 1^5$
	32: $g^2, b^2 a, 5^5 43$	36: $i^2, c^3, 6^4 5^2 1^2$	38: $j^2, d^2 c, 6^6 2$	42: $l^2, e^3, 7^5 52$	66: $x^2, m^3, b^5 a1$

Numbers of fundamental spectral types and elements in a class (2026.03.24)

idx	total	3 pt	4 pt	5 pt	E_8	E_7	E_6	D_4	max	min
0	4	3	1	0	1	1	1	1	4	4
−2	13	9	3	1	2	3	2	2	8	5
−4	37	25	9	2	5	6	6	4	12	4
−6	69	46	17	4	7	12	10	6	17	4
−8	113	73	29	8	10	15	12	8	23	4
−10	198	127	50	15	15	25	20	9	36	3
−12	291	182	76	24	18	34	28	14	40	4
−14	415	249	115	36	21	39	29	15	53	4
−16	647	395	172	55	33	59	50	19	64	3
−18	884	522	243	83	36	77	57	23	81	4
−20	1186	680	345	111	40	80	67	25	104	4
−22	1682	963	478	164	55	111	92	29	126	3
−24	2183	1230	625	229	62	140	107	37	132	4
−26	2889	1577	861	310	70	155	114	37	171	4
−28	3841	2105	1126	416	92	201	166	44	181	3
−30	4831	2570	1430	570	96	234	173	53	230	4
−32	6203	3257	1890	716	114	256	204	57	254	3
−34	8012	4187	2411	940	142	336	261	61	330	3
−36	9789	5009	2982	1214	148	379	282	74	359	4
−38	12290	6144	3832	1546	168	405	312	77	422	3
−40	15379	7731	4721	1939	212	512	410	85	465	3
−42	18683	9153	5785	2491	220	589	424	98	565	4
−44	22910	11072	7227	3046	246	632	482	104	647	4
−46	28096	13493	8763	3805	298	765	592	117	644	3
−48	33387	15768	10484	4675	314	850	631	133	732	3
−50	40617	18878	12953	5700	351	931	701	134	757	4
−52	48703	22594	15392	6916	412	1109	860	146	926	3
−54	57481	26040	18259	8521	429	1210	867	170	1006	3
−56	68622	30819	22056	10114	486	1312	996	174	1122	3
−58	81728	36495	26041	12203	570	1579	1189	189	1271	3
−60	95216	41764	30500	14686	578	1701	1232	213	1477	4
−62	112705	48768	36383	17470	651	1826	1363	226	1434	3
−64	131926	56996	42248	20557	760	2150	1634	240	1801	3
−66	153040	64789	49268	24510	784	2343	1661	263	1981	4
−68	178877	75023	58093	28650	863	2504	1870	275	2054	3
−70	207815	86539	66904	33765	984	2878	2154	300	2267	3
−72	238496	97838	77025	39589	1022	3101	2222	331	2556	3
−74	277376	112560	90143	46109	1138	3369	2467	338	2558	3
−76	318887	128785	102977	53553	1276	3848	2851	361	2744	3
−78	364400	144472	117879	62657	1315	4085	2902	407	2924	3
−80	419160	165023	136352	71851	1456	4386	3251	414	3174	3
−82	479803	187613	155113	83016	1645	5045	3701	439	3444	3
−84	544057	209494	176409	95919	1676	5355	3771	482	3582	4
−86	622572	237324	202485	110093	1852	5707	4162	505	3559	3
−88	706007	267875	228261	125694	2069	6445	4747	530	4164	3
−90	798831	298319	258714	144627	2138	6887	4824	580	4552	3
−92	907339	336327	294936	164356	2328	7370	5346	600	4882	3
−94	1024368	376729	330756	187425	2583	8222	5958	647	5368	3
−96	1150414	417436	372140	213289	2662	8687	6123	691	6905	3
−98	1302028	468441	422261	241590	2924	9346	6736	706	6979	3
−100	1461132	522509	471378	273263	3181	10389	7500	743	7613	3

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