(1)

$$\begin{pmatrix}1&-7&5&1\\0&-8&8&8\\0&4&-4&-4\\0&9&-9&-9\end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix}1&-7&5&1\\0&4&-4&-4\\0&-8&8&8\\0&9&-9&-9\end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{4}\right)} \begin{pmatrix}1&-7&5&1\\0&1&-1&-1\\0&-8&8&8\\0&9&-9&-9\end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times(7)} \begin{pmatrix}1&0&-2&-6\\0&1&-1&-1\\0&-8&8&8\\0&9&-9&-9\end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(8)} \begin{pmatrix}1&0&-2&-6\\0&1&-1&-1\\0&0&0&0\\0&9&-9&-9\end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}\times(9)} \begin{pmatrix}1&0&-2&-6\\0&1&-1&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

(2)

$$\begin{pmatrix}-2&-6&0&4\\-3&-9&3&-3\\3&9&0&-6\\0&0&-2&6\end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix}3&9&0&-6\\-3&-9&3&-3\\-2&-6&0&4\\0&0&-2&6\end{pmatrix} \xrightarrow{\text{line1 } \times= \left(\frac{1}{3}\right)} \begin{pmatrix}1&3&0&-2\\-3&-9&3&-3\\-2&-6&0&4\\0&0&-2&6\end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(3)} \begin{pmatrix}1&3&0&-2\\0&0&3&-9\\-2&-6&0&4\\0&0&-2&6\end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(2)} \begin{pmatrix}1&3&0&-2\\0&0&3&-9\\0&0&0&0\\0&0&-2&6\end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{3}\right)} \begin{pmatrix}1&3&0&-2\\0&0&1&-3\\0&0&0&0\\0&0&-2&6\end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}\times(2)} \begin{pmatrix}1&3&0&-2\\0&0&1&-3\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

(3)

$$\begin{pmatrix}-1&-5&-5&-3\\1&5&5&4\\0&0&0&7\\-1&-5&-5&-5\end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix}1&5&5&4\\-1&-5&-5&-3\\0&0&0&7\\-1&-5&-5&-5\end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}} \begin{pmatrix}1&5&5&4\\0&0&0&1\\0&0&0&7\\-1&-5&-5&-5\end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}} \begin{pmatrix}1&5&5&4\\0&0&0&1\\0&0&0&7\\0&0&0&-1\end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}\times(4)} \begin{pmatrix}1&5&5&0\\0&0&0&1\\0&0&0&7\\0&0&0&-1\end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line2}\times(7)} \begin{pmatrix}1&5&5&0\\0&0&0&1\\0&0&0&0\\0&0&0&-1\end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}} \begin{pmatrix}1&5&5&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

(4)

$$\begin{pmatrix}-1&3&9&1\\0&-1&-3&1\\-1&2&6&2\\0&-1&-3&1\end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix}1&-3&-9&-1\\0&-1&-3&1\\-1&2&6&2\\0&-1&-3&1\end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}} \begin{pmatrix}1&-3&-9&-1\\0&-1&-3&1\\0&-1&-3&1\\0&-1&-3&1\end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix}1&-3&-9&-1\\0&1&3&-1\\0&-1&-3&1\\0&-1&-3&1\end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}\times(3)} \begin{pmatrix}1&0&0&-4\\0&1&3&-1\\0&-1&-3&1\\0&-1&-3&1\end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}} \begin{pmatrix}1&0&0&-4\\0&1&3&-1\\0&0&0&0\\0&-1&-3&1\end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}} \begin{pmatrix}1&0&0&-4\\0&1&3&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

(5)

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 2 & 2 & -2 & -6 \\ -3 & -1 & -3 & 7 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 2 & 2 & -2 & -6 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ -3 & -1 & -3 & 7 \end{pmatrix} \xrightarrow{\text{line1} \times = \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 1 & -1 & -3 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ -3 & -1 & -3 & 7 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line1} \times (3)} \begin{pmatrix} 1 & 1 & -1 & -3 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 2 & -6 & -2 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 2 & -6 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -6 & -2 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(6)

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 3 & 4 & -1 & 2 \\ 3 & 1 & -7 & 5 \\ 1 & 4 & 5 & -2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1} \times (3)} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 3 & 1 & -7 & 5 \\ 1 & 4 & 5 & -2 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1} \times (3)} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \\ 1 & 4 & 5 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \\ 0 & 3 & 6 & -3 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \\ 0 & 3 & 6 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 6 & -3 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(7)

$$\begin{pmatrix} 1 & 1 & 1 & -6 \\ 1 & 1 & 1 & -6 \\ 1 & 5 & 1 & -3 \\ -1 & -1 & -1 & 6 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 1 & 5 & 1 & -3 \\ -1 & -1 & -1 & 6 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 3 \\ -1 & -1 & -1 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & 1 & -6 \\ 0 & 4 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times = \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & 1 & -6 \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & 1 & -\frac{27}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(8)

$$\begin{pmatrix} 2 & 0 & 4 & -4 \\ 4 & -1 & 9 & -5 \\ 1 & 0 & 2 & -2 \\ 2 & -2 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 4 & -1 & 9 & -5 \\ 2 & 0 & 4 & -4 \\ 2 & -2 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1} \times (4)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & 1 & 3 \\ 2 & 0 & 4 & -4 \\ 2 & -2 & 6 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & -2 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times = (-1)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(9)

$$\begin{pmatrix} 4 & -1 & -4 & 1 \\ -8 & 1 & 8 & -1 \\ -5 & 4 & 5 & -4 \\ 3 & -1 & -3 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line4}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ -8 & 1 & 8 & -1 \\ -5 & 4 & 5 & -4 \\ 3 & -1 & -3 & 1 \end{pmatrix} \xrightarrow{\text{line2 += line1×(8)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -5 & 4 & 5 & -4 \\ 3 & -1 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(5)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 4 & 0 & -4 \\ 3 & -1 & -3 & 1 \end{pmatrix} \xrightarrow{\text{line4 -= line1×(3)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 4 & 0 & -4 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line2×(4)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line4 += line2}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(10)

$$\begin{pmatrix} -3 & 9 & -3 & 0 \\ 0 & 0 & -9 & -9 \\ -2 & 6 & -5 & -3 \\ 3 & -9 & 0 & -3 \end{pmatrix} \xrightarrow{\text{line1 ↔ line4}} \begin{pmatrix} 3 & -9 & 0 & -3 \\ 0 & 0 & -9 & -9 \\ -2 & 6 & -5 & -3 \\ -3 & 9 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line1 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & -9 & -9 \\ -2 & 6 & -5 & -3 \\ -3 & 9 & -3 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & -9 & -9 \\ 0 & 0 & -5 & -5 \\ -3 & 9 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line4 += line1×(3)}} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & -9 & -9 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -3 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{9}\right)} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -3 & -3 \end{pmatrix} \xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2×(3)}} \begin{pmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(11)

$$\begin{pmatrix} -2 & 2 & -4 & -8 \\ 1 & -1 & 3 & 3 \\ 0 & 0 & 6 & -6 \\ -2 & 2 & -5 & -7 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & -1 & 3 & 3 \\ -2 & 2 & -4 & -8 \\ 0 & 0 & 6 & -6 \\ -2 & 2 & -5 & -7 \end{pmatrix} \xrightarrow{\text{line2 += line1×(2)}} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 6 & -6 \\ -2 & 2 & -5 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(2)}} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 6 & -6 \\ 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line2 ↔ line4}} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 6 & -6 \\ 0 & 0 & 2 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2×(3)}} \begin{pmatrix} 1 & -1 & 0 & 6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 6 & -6 \\ 0 & 0 & 2 & -2 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(6)}} \begin{pmatrix} 1 & -1 & 0 & 6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(2)}} \begin{pmatrix} 1 & -1 & 0 & 6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(12)

$$\begin{pmatrix} -1 & -3 & -5 & -3 \\ -1 & -3 & -5 & -3 \\ -1 & 1 & -5 & 9 \\ 0 & 3 & 0 & 9 \end{pmatrix} \xrightarrow{\text{line1 ×= } (-1)} \begin{pmatrix} 1 & 3 & 5 & 3 \\ -1 & -3 & -5 & -3 \\ -1 & 1 & -5 & 9 \\ 0 & 3 & 0 & 9 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & -5 & 9 \\ 0 & 3 & 0 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 12 \\ 0 & 3 & 0 & 9 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & 4 & 0 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 9 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(3)}} \begin{pmatrix} 1 & 0 & 5 & -6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(3)}} \begin{pmatrix} 1 & 0 & 5 & -6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(13)

$$\begin{pmatrix} -1 & -3 & -1 & -4 \\ 0 & 0 & 4 & -1 \\ -2 & -6 & 2 & -9 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 4 & -1 \\ -2 & -6 & 2 & -9 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } -= \text{ line3}} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line4}} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}} \begin{pmatrix} 1 & 3 & 0 & \frac{17}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(14)

$$\begin{pmatrix} -1 & -1 & -4 & -2 \\ 0 & 4 & 1 & 3 \\ -1 & 3 & -3 & 1 \\ 0 & 4 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 4 & 1 & 3 \\ -1 & 3 & -3 & 1 \\ 0 & 4 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 4 & 1 & 3 \\ 0 & 4 & 1 & 3 \\ 0 & 4 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } -= \text{ line3}} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 4 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line3 } -= \text{ line4}} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}} \begin{pmatrix} 1 & 0 & \frac{15}{4} & \frac{5}{4} \\ 0 & 1 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(15)

$$\begin{pmatrix} 0 & -6 & -6 & 0 \\ -1 & -5 & -5 & -3 \\ 2 & -1 & -1 & 6 \\ 1 & 2 & 2 & 3 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 2 & 2 & 3 \\ -1 & -5 & -5 & -3 \\ 2 & -1 & -1 & 6 \\ 0 & -6 & -6 & 0 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}} \begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & -3 & -3 & 0 \\ 2 & -1 & -1 & 6 \\ 0 & -6 & -6 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & -3 & -3 & 0 \\ 0 & -5 & -5 & 0 \\ 0 & -6 & -6 & 0 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -5 & 0 \\ 0 & -6 & -6 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -5 & 0 \\ 0 & -6 & -6 & 0 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -6 & -6 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line2}\times(6)} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(16)

$$\begin{pmatrix} 1 & -5 & 5 & 9 \\ 0 & 0 & -1 & -2 \\ -1 & 5 & 3 & 7 \\ -1 & 5 & 2 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & -5 & 5 & 9 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 8 & 16 \\ -1 & 5 & 2 & 5 \end{pmatrix} \xrightarrow{\text{line4 += line1}} \begin{pmatrix} 1 & -5 & 5 & 9 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 7 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -5 & 5 & 9 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 7 & 14 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{8}\right)} \begin{pmatrix} 1 & -5 & 5 & 9 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 7 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}\times(5)} \begin{pmatrix} 1 & -5 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 7 & 14 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & -5 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(7)} \begin{pmatrix} 1 & -5 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(17)

$$\begin{pmatrix} 1 & -1 & -3 & -3 \\ -1 & 1 & 3 & 5 \\ 2 & -2 & -6 & 4 \\ 2 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 2 \\ 2 & -2 & -6 & 4 \\ 2 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line3 -= line1}\times(2)} \begin{pmatrix} 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 10 \\ 2 & -2 & -6 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1}\times(2)} \begin{pmatrix} 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 5 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2}\times(3)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 5 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(10)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(5)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(18)

$$\begin{pmatrix} 1 & -1 & 5 & -6 \\ 1 & -1 & 5 & 7 \\ 1 & -1 & 5 & -2 \\ 1 & -1 & 5 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line1}} \begin{pmatrix} 1 & -1 & 5 & -6 \\ 0 & 0 & 0 & 13 \\ 1 & -1 & 5 & -2 \\ 1 & -1 & 5 & 1 \end{pmatrix} \xrightarrow{\text{line3 -= line1}} \begin{pmatrix} 1 & -1 & 5 & -6 \\ 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & 4 \\ 1 & -1 & 5 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1}} \begin{pmatrix} 1 & -1 & 5 & -6 \\ 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 7 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{13}\right)} \begin{pmatrix} 1 & -1 & 5 & -6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2}\times(6)} \begin{pmatrix} 1 & -1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 7 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(4)} \begin{pmatrix} 1 & -1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(7)} \begin{pmatrix} 1 & -1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(19)

$$\begin{pmatrix} 1 & 1 & -5 & -3 \\ -4 & -1 & 8 & 9 \\ 3 & -2 & 5 & -4 \\ -4 & 2 & -4 & 6 \end{pmatrix} \xrightarrow{\text{line2 += line1×(4)}} \begin{pmatrix} 1 & 1 & -5 & -3 \\ 0 & 3 & -12 & -3 \\ 3 & -2 & 5 & -4 \\ -4 & 2 & -4 & 6 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(3)}} \begin{pmatrix} 1 & 1 & -5 & -3 \\ 0 & 3 & -12 & -3 \\ 0 & -5 & 20 & 5 \\ -4 & 2 & -4 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(4)}} \begin{pmatrix} 1 & 1 & -5 & -3 \\ 0 & 3 & -12 & -3 \\ 0 & -5 & 20 & 5 \\ 0 & 6 & -24 & -6 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 1 & -5 & -3 \\ 0 & 1 & -4 & -1 \\ 0 & -5 & 20 & 5 \\ 0 & 6 & -24 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -4 & -1 \\ 0 & -5 & 20 & 5 \\ 0 & 6 & -24 & -6 \end{pmatrix} \xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -4 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 6 & -24 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(6)}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -4 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 & -3 & 6 \\ -2 & -2 & 6 & -6 \\ -3 & -3 & 9 & 6 \\ -3 & -3 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line2 += line1×(2)}} \begin{pmatrix} 1 & 1 & -3 & 6 \\ 0 & 0 & 0 & 6 \\ -3 & -3 & 9 & 6 \\ -3 & -3 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & 1 & -3 & 6 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 24 \\ -3 & -3 & 9 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(3)}} \begin{pmatrix} 1 & 1 & -3 & 6 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 23 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 1 & -3 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 23 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2×(6)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 24 \\ 0 & 0 & 0 & 23 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(24)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(23)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 6 & 3 & 9 \\ -1 & -7 & -5 & -7 \\ -4 & -6 & -9 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 6 & 3 & 9 \\ 0 & -4 & -2 & -6 \\ -4 & -6 & -9 & 5 \end{pmatrix} \xrightarrow{\text{line4 += line1×(4)}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 6 & 3 & 9 \\ 0 & -4 & -2 & -6 \\ 0 & 6 & 3 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line4}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & -2 & -6 \\ 0 & 6 & 3 & 9 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & -4 & -2 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 6 & 3 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 6 & 3 & 9 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(3)}} \begin{pmatrix} 1 & 0 & \frac{3}{2} & -\frac{7}{2} \\ 0 & 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 6 & 3 & 9 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(6)}} \begin{pmatrix} 1 & 0 & \frac{3}{2} & -\frac{7}{2} \\ 0 & 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 6 & -2 & -2 \\ -3 & -9 & 4 & 7 \\ -2 & -6 & 1 & -2 \\ 3 & 9 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line1 } \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 3 & -1 & -1 \\ -3 & -9 & 4 & 7 \\ -2 & -6 & 1 & -2 \\ 3 & 9 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(3)} \begin{pmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 1 & 4 \\ -2 & -6 & 1 & -2 \\ 3 & 9 & -2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -4 \\ 3 & 9 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line1}\times(3)} \begin{pmatrix} 1 & 3 & -1 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}} \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 4 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}} \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}} \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -9 & 1 & -1 & -7 \\ 6 & -2 & 2 & 2 \\ -2 & -3 & 3 & -8 \\ -8 & 0 & 0 & -8 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -8 & 0 & 0 & -8 \\ 6 & -2 & 2 & 2 \\ -2 & -3 & 3 & -8 \\ -9 & 1 & -1 & -7 \end{pmatrix} \xrightarrow{\text{line1 } \times= \left(-\frac{1}{8}\right)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 6 & -2 & 2 & 2 \\ -2 & -3 & 3 & -8 \\ -9 & 1 & -1 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } -= \text{ line1}\times(6)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & 2 & -4 \\ -2 & -3 & 3 & -8 \\ -9 & 1 & -1 & -7 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & 2 & -4 \\ 0 & -3 & 3 & -6 \\ -9 & 1 & -1 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}\times(9)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & 2 & -4 \\ 0 & -3 & 3 & -6 \\ 0 & 1 & -1 & 2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & 3 & -6 \\ 0 & -2 & 2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -5 & 2 & 4 & -4 \\ -6 & 3 & 4 & -7 \\ 0 & -5 & 1 & 7 \\ 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}} \begin{pmatrix} 1 & -1 & 0 & 3 \\ -6 & 3 & 4 & -7 \\ 0 & -5 & 1 & 7 \\ 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(6)} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & -3 & 4 & 11 \\ 0 & -5 & 1 & 7 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & -1 & 0 & 1 \\ 0 & -5 & 1 & 7 \\ 0 & -3 & 4 & 11 \end{pmatrix} \xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & -5 & 1 & 7 \\ 0 & -3 & 4 & 11 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & -5 & 1 & 7 \\ 0 & -3 & 4 & 11 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & -3 & 4 & 11 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 8 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line3}\times(4)} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix}-4&-2&0&-2\\6&6&6&6\\1&-1&-3&-1\\8&5&2&5\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&-1&-3&-1\\6&6&6&6\\-4&-2&0&-2\\8&5&2&5\end{pmatrix}\xrightarrow{\text{line2 }-=\text{ line1}\times(6)}\begin{pmatrix}1&-1&-3&-1\\0&12&24&12\\-4&-2&0&-2\\8&5&2&5\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line1}\times(4)}\begin{pmatrix}1&-1&-3&-1\\0&12&24&12\\0&-6&-12&-6\\8&5&2&5\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line1}\times(8)}\begin{pmatrix}1&-1&-3&-1\\0&12&24&12\\0&-6&-12&-6\\0&13&26&13\end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{12}\right)}\begin{pmatrix}1&-1&-3&-1\\0&1&2&1\\0&-6&-12&-6\\0&13&26&13\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line2}}\begin{pmatrix}1&0&-1&0\\0&1&2&1\\0&-6&-12&-6\\0&13&26&13\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line2}\times(6)}\begin{pmatrix}1&0&-1&0\\0&1&2&1\\0&0&0&0\\0&13&26&13\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line2}\times(13)}\begin{pmatrix}1&0&-1&0\\0&1&2&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}-3&-2&-7&0\\-2&0&-2&4\\-4&-1&-6&5\\-3&1&-1&9\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}-2&0&-2&4\\-3&-2&-7&0\\-4&-1&-6&5\\-3&1&-1&9\end{pmatrix}\xrightarrow{\text{line1 }\times=\left(-\frac{1}{2}\right)}\begin{pmatrix}1&0&1&-2\\-3&-2&-7&0\\-4&-1&-6&5\\-3&1&-1&9\end{pmatrix}$$

$$\xrightarrow{\text{line2 }+=\text{ line1}\times(3)}\begin{pmatrix}1&0&1&-2\\0&-2&-4&-6\\-4&-1&-6&5\\-3&1&-1&9\end{pmatrix}\xrightarrow{\text{line3 }+=\text{ line1}\times(4)}\begin{pmatrix}1&0&1&-2\\0&-2&-4&-6\\0&-1&-2&-3\\-3&1&-1&9\end{pmatrix}$$

$$\xrightarrow{\text{line4 }+=\text{ line1}\times(3)}\begin{pmatrix}1&0&1&-2\\0&-2&-4&-6\\0&-1&-2&-3\\0&1&2&3\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line4}}\begin{pmatrix}1&0&1&-2\\0&1&2&3\\0&-1&-2&-3\\0&-2&-4&-6\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line2}}\begin{pmatrix}1&0&1&-2\\0&1&2&3\\0&0&0&0\\0&-2&-4&-6\end{pmatrix}\xrightarrow{\text{line4 }+=\text{ line2}\times(2)}\begin{pmatrix}1&0&1&-2\\0&1&2&3\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}-2&4&2&-4\\-1&0&-3&0\\2&1&8&-1\\7&-9&3&9\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}-1&0&-3&0\\-2&4&2&-4\\2&1&8&-1\\7&-9&3&9\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&0&3&0\\-2&4&2&-4\\2&1&8&-1\\7&-9&3&9\end{pmatrix}$$

$$\xrightarrow{\text{line2 }+=\text{ line1}\times(2)}\begin{pmatrix}1&0&3&0\\0&4&8&-4\\2&1&8&-1\\7&-9&3&9\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line1}\times(2)}\begin{pmatrix}1&0&3&0\\0&4&8&-4\\0&1&2&-1\\7&-9&3&9\end{pmatrix}$$

$$\xrightarrow{\text{line4 }-=\text{ line1}\times(7)}\begin{pmatrix}1&0&3&0\\0&4&8&-4\\0&1&2&-1\\0&-9&-18&9\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&0&3&0\\0&1&2&-1\\0&4&8&-4\\0&-9&-18&9\end{pmatrix}$$

$$\xrightarrow{\text{line3 }-=\text{ line2}\times(4)}\begin{pmatrix}1&0&3&0\\0&1&2&-1\\0&0&0&0\\0&-9&-18&9\end{pmatrix}\xrightarrow{\text{line4 }+=\text{ line2}\times(9)}\begin{pmatrix}1&0&3&0\\0&1&2&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix} -1 & -3 & -7 & -5 \\ -1 & -3 & -7 & 1 \\ 1 & 3 & 7 & 2 \\ 1 & 3 & 7 & -3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & 7 & 2 \\ -1 & -3 & -7 & 1 \\ -1 & -3 & -7 & -5 \\ 1 & 3 & 7 & -3 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 3 & 7 & 2 \\ 0 & 0 & 0 & 3 \\ -1 & -3 & -7 & -5 \\ 1 & 3 & 7 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 3 & 7 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -3 \\ 1 & 3 & 7 & -3 \end{pmatrix} \xrightarrow{\text{line4 -= line1}} \begin{pmatrix} 1 & 3 & 7 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 3 & 7 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times(2)} \begin{pmatrix} 1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times(3)} \begin{pmatrix} 1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line4 += line2}\times(5)} \begin{pmatrix} 1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & -1 & 5 & -9 \\ -1 & -1 & 5 & 4 \\ 1 & 1 & -5 & 6 \\ -1 & -1 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & -5 & 6 \\ -1 & -1 & 5 & 4 \\ -1 & -1 & 5 & -9 \\ -1 & -1 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 1 & -5 & 6 \\ 0 & 0 & 0 & 10 \\ -1 & -1 & 5 & -9 \\ -1 & -1 & 5 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 1 & -5 & 6 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & -3 \\ -1 & -1 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line4 += line1}} \begin{pmatrix} 1 & 1 & -5 & 6 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{10}\right)} \begin{pmatrix} 1 & 1 & -5 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 5 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times(6)} \begin{pmatrix} 1 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times(3)} \begin{pmatrix} 1 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix} \xrightarrow{\text{line4 -= line2}\times(5)} \begin{pmatrix} 1 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & 1 & 3 & -8 \\ -1 & 1 & 3 & 9 \\ -2 & 2 & 6 & -2 \\ 2 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line1 } \times = (-1)} \begin{pmatrix} 1 & -1 & -3 & 8 \\ -1 & 1 & 3 & 9 \\ -2 & 2 & 6 & -2 \\ 2 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & -1 & -3 & 8 \\ 0 & 0 & 0 & 17 \\ -2 & 2 & 6 & -2 \\ 2 & -2 & -6 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1}\times(2)} \begin{pmatrix} 1 & -1 & -3 & 8 \\ 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 14 \\ 2 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line4 -= line1}\times(2)} \begin{pmatrix} 1 & -1 & -3 & 8 \\ 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & -17 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{17}\right)} \begin{pmatrix} 1 & -1 & -3 & 8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & -17 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times(8)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & -17 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line2}\times(14)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -17 \end{pmatrix} \xrightarrow{\text{line4 += line2}\times(17)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & 3 & -1 & 1 \\ -3 & 9 & -5 & 1 \\ 1 & -3 & 0 & -2 \\ -3 & 9 & -4 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -3 & 0 & -2 \\ -3 & 9 & -5 & 1 \\ -1 & 3 & -1 & 1 \\ -3 & 9 & -4 & 2 \end{pmatrix} \xrightarrow{\text{line2 += line1}\times(3)} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & -5 & -5 \\ -1 & 3 & -1 & 1 \\ -3 & 9 & -4 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -1 & -1 \\ -3 & 9 & -4 & 2 \end{pmatrix} \xrightarrow{\text{line4 += line1}\times(3)} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -4 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -4 & -4 \end{pmatrix} \xrightarrow{\text{line2 }\times= (-1)} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -4 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times(5)} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \end{pmatrix} \xrightarrow{\text{line4 += line2}\times(4)} \begin{pmatrix} 1 & -3 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & -2 & 8 & -4 \\ -1 & -2 & 5 & -7 \\ 2 & 0 & 6 & 6 \\ -1 & -1 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -1 & -1 & 1 & -5 \\ -1 & -2 & 5 & -7 \\ 2 & 0 & 6 & 6 \\ 0 & -2 & 8 & -4 \end{pmatrix} \xrightarrow{\text{line1 }\times= (-1)} \begin{pmatrix} 1 & 1 & -1 & 5 \\ -1 & -2 & 5 & -7 \\ 2 & 0 & 6 & 6 \\ 0 & -2 & 8 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 1 & -1 & 5 \\ 0 & -1 & 4 & -2 \\ 2 & 0 & 6 & 6 \\ 0 & -2 & 8 & -4 \end{pmatrix} \xrightarrow{\text{line3 -= line1}\times(2)} \begin{pmatrix} 1 & 1 & -1 & 5 \\ 0 & -1 & 4 & -2 \\ 0 & -2 & 8 & -4 \\ 0 & -2 & 8 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times= (-1)} \begin{pmatrix} 1 & 1 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & -2 & 8 & -4 \\ 0 & -2 & 8 & -4 \end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -4 & 2 \\ 0 & -2 & 8 & -4 \\ 0 & -2 & 8 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times(2)} \begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 8 & -4 \end{pmatrix} \xrightarrow{\text{line4 += line2}\times(2)} \begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 5 & -9 & 9 & 3 \\ -4 & 9 & -9 & -6 \\ 3 & -8 & 8 & 7 \\ 1 & -3 & 3 & 3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & -3 & 3 & 3 \\ -4 & 9 & -9 & -6 \\ 3 & -8 & 8 & 7 \\ 5 & -9 & 9 & 3 \end{pmatrix} \xrightarrow{\text{line2 += line1}\times(4)} \begin{pmatrix} 1 & -3 & 3 & 3 \\ 0 & -3 & 3 & 6 \\ 3 & -8 & 8 & 7 \\ 5 & -9 & 9 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line1}\times(3)} \begin{pmatrix} 1 & -3 & 3 & 3 \\ 0 & -3 & 3 & 6 \\ 0 & 1 & -1 & -2 \\ 5 & -9 & 9 & 3 \end{pmatrix} \xrightarrow{\text{line4 -= line1}\times(5)} \begin{pmatrix} 1 & -3 & 3 & 3 \\ 0 & -3 & 3 & 6 \\ 0 & 1 & -1 & -2 \\ 0 & 6 & -6 & -12 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -3 & 3 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 3 & 6 \\ 0 & 6 & -6 & -12 \end{pmatrix} \xrightarrow{\text{line1 += line2}\times(3)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 3 & 6 \\ 0 & 6 & -6 & -12 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times(3)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 6 & -6 & -12 \end{pmatrix} \xrightarrow{\text{line4 -= line2}\times(6)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 5 & -2 & 9 & 8 \\ 1 & 0 & 1 & 2 \\ -3 & 2 & -7 & -4 \\ 1 & -3 & 7 & -1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 5 & -2 & 9 & 8 \\ -3 & 2 & -7 & -4 \\ 1 & -3 & 7 & -1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & -2 & 4 & -2 \\ -3 & 2 & -7 & -4 \\ 1 & -3 & 7 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & -2 & 4 & -2 \\ 0 & 2 & -4 & 2 \\ 1 & -3 & 7 & -1 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & -2 & 4 & -2 \\ 0 & 2 & -4 & 2 \\ 0 & -3 & 6 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & -4 & 2 \\ 0 & -2 & 4 & -2 \\ 0 & -3 & 6 & -3 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \\ 0 & -3 & 6 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & 6 & -3 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -7 & -5 & 1 & -9 \\ -2 & -4 & -1 & 0 \\ 1 & 1 & 0 & 1 \\ -1 & 3 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ -2 & -4 & -1 & 0 \\ -7 & -5 & 1 & -9 \\ -1 & 3 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & 2 \\ -7 & -5 & 1 & -9 \\ -1 & 3 & 2 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(7)} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & 2 \\ 0 & 2 & 1 & -2 \\ -1 & 3 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & 2 \\ 0 & 2 & 1 & -2 \\ 0 & 4 & 2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & -2 \\ 0 & 4 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line2}\times(4)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -7 & 7 & -7 & 7 \\ -5 & 1 & 3 & 9 \\ -2 & 1 & 0 & 3 \\ -2 & 1 & 0 & 3 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{\times}= \left(-\frac{1}{7}\right)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ -5 & 1 & 3 & 9 \\ -2 & 1 & 0 & 3 \\ -2 & 1 & 0 & 3 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(5)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -4 & 8 & 4 \\ -2 & 1 & 0 & 3 \\ -2 & 1 & 0 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -4 & 8 & 4 \\ 0 & -1 & 2 & 1 \\ -2 & 1 & 0 & 3 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -4 & 8 & 4 \\ 0 & -1 & 2 & 1 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & -4 & 8 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= (-1)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & -4 & 8 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & -4 & 8 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(4)} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line2}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix}-5&-1&-3&-9\\-2&1&-4&-5\\-6&-4&2&-8\\-4&-4&4&-4\end{pmatrix}\xrightarrow{\text{line1 } \leftrightarrow \text{ line4}}\begin{pmatrix}-4&-4&4&-4\\-2&1&-4&-5\\-6&-4&2&-8\\-5&-1&-3&-9\end{pmatrix}\xrightarrow{\text{line1 } \times= \left(-\frac{1}{4}\right)}\begin{pmatrix}1&1&-1&1\\-2&1&-4&-5\\-6&-4&2&-8\\-5&-1&-3&-9\end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(2)}\begin{pmatrix}1&1&-1&1\\0&3&-6&-3\\-6&-4&2&-8\\-5&-1&-3&-9\end{pmatrix}\xrightarrow{\text{line3 } += \text{ line1}\times(6)}\begin{pmatrix}1&1&-1&1\\0&3&-6&-3\\0&2&-4&-2\\-5&-1&-3&-9\end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}\times(5)}\begin{pmatrix}1&1&-1&1\\0&3&-6&-3\\0&2&-4&-2\\0&4&-8&-4\end{pmatrix}\xrightarrow{\text{line2 } \times= \left(\frac{1}{3}\right)}\begin{pmatrix}1&1&-1&1\\0&1&-2&-1\\0&2&-4&-2\\0&4&-8&-4\end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}}\begin{pmatrix}1&0&1&2\\0&1&-2&-1\\0&2&-4&-2\\0&4&-8&-4\end{pmatrix}\xrightarrow{\text{line3 } -= \text{ line2}\times(2)}\begin{pmatrix}1&0&1&2\\0&1&-2&-1\\0&0&0&0\\0&4&-8&-4\end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}\times(4)}\begin{pmatrix}1&0&1&2\\0&1&-2&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}-5&5&-6&4\\-2&2&-7&-3\\3&-3&9&3\\-1&1&-7&-5\end{pmatrix}\xrightarrow{\text{line1 } \leftrightarrow \text{ line4}}\begin{pmatrix}-1&1&-7&-5\\-2&2&-7&-3\\3&-3&9&3\\-5&5&-6&4\end{pmatrix}\xrightarrow{\text{line1 } \times= (-1)}\begin{pmatrix}1&-1&7&5\\-2&2&-7&-3\\3&-3&9&3\\-5&5&-6&4\end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(2)}\begin{pmatrix}1&-1&7&5\\0&0&7&7\\3&-3&9&3\\-5&5&-6&4\end{pmatrix}\xrightarrow{\text{line3 } -= \text{ line1}\times(3)}\begin{pmatrix}1&-1&7&5\\0&0&7&7\\0&0&-12&-12\\-5&5&-6&4\end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}\times(5)}\begin{pmatrix}1&-1&7&5\\0&0&7&7\\0&0&-12&-12\\0&0&29&29\end{pmatrix}\xrightarrow{\text{line2 } \times= \left(\frac{1}{7}\right)}\begin{pmatrix}1&-1&7&5\\0&0&1&1\\0&0&-12&-12\\0&0&29&29\end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}\times(7)}\begin{pmatrix}1&-1&0&-2\\0&0&1&1\\0&0&-12&-12\\0&0&29&29\end{pmatrix}\xrightarrow{\text{line3 } += \text{ line2}\times(12)}\begin{pmatrix}1&-1&0&-2\\0&0&1&1\\0&0&0&0\\0&0&29&29\end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}\times(29)}\begin{pmatrix}1&-1&0&-2\\0&0&1&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}-2&-6&-5&-2\\-2&-6&-6&-2\\2&6&-7&2\\1&3&-5&1\end{pmatrix}\xrightarrow{\text{line1 } \leftrightarrow \text{ line4}}\begin{pmatrix}1&3&-5&1\\-2&-6&-6&-2\\2&6&-7&2\\-2&-6&-5&-2\end{pmatrix}\xrightarrow{\text{line2 } += \text{ line1}\times(2)}\begin{pmatrix}1&3&-5&1\\0&0&-16&0\\2&6&-7&2\\-2&-6&-5&-2\end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line1}\times(2)}\begin{pmatrix}1&3&-5&1\\0&0&-16&0\\0&0&3&0\\-2&-6&-5&-2\end{pmatrix}\xrightarrow{\text{line4 } += \text{ line1}\times(2)}\begin{pmatrix}1&3&-5&1\\0&0&-16&0\\0&0&3&0\\0&0&-15&0\end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}}\begin{pmatrix}1&3&-5&1\\0&0&3&0\\0&0&-16&0\\0&0&-15&0\end{pmatrix}\xrightarrow{\text{line2 } \times= \left(\frac{1}{3}\right)}\begin{pmatrix}1&3&-5&1\\0&0&1&0\\0&0&-16&0\\0&0&-15&0\end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times(5)}\begin{pmatrix}1&3&0&1\\0&0&1&0\\0&0&-16&0\\0&0&-15&0\end{pmatrix}\xrightarrow{\text{line3 } += \text{ line2}\times(16)}\begin{pmatrix}1&3&0&1\\0&0&1&0\\0&0&0&0\\0&0&-15&0\end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line2}\times(15)}\begin{pmatrix}1&3&0&1\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix} 0 & 1 & 4 & -9 \\ 1 & 0 & -1 & -5 \\ 2 & -1 & -6 & 1 \\ -4 & 1 & 8 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 2 & -1 & -6 & 1 \\ -4 & 1 & 8 & 2 \end{pmatrix} \xrightarrow{\text{line3 -= line1}\times(2)} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 0 & -1 & -4 & 11 \\ -4 & 1 & 8 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1}\times(4)} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 0 & -1 & -4 & 11 \\ 0 & 1 & 4 & -18 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 4 & -18 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{\text{line3 }\times=\left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3}\times(5)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{\text{line2 += line3}\times(9)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3}\times(9)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 5 & 7 & -6 \\ 0 & 0 & 4 & -5 \\ -1 & -5 & -7 & 9 \\ -1 & -5 & 3 & -8 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 5 & 7 & -6 \\ 0 & 0 & 4 & -5 \\ 0 & 0 & 0 & 3 \\ -1 & -5 & 3 & -8 \end{pmatrix} \xrightarrow{\text{line4 += line1}} \begin{pmatrix} 1 & 5 & 7 & -6 \\ 0 & 0 & 4 & -5 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 10 & -14 \end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 5 & 7 & -6 \\ 0 & 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 10 & -14 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times(7)} \begin{pmatrix} 1 & 5 & 0 & \frac{11}{4} \\ 0 & 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 10 & -14 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(10)} \begin{pmatrix} 1 & 5 & 0 & \frac{11}{4} \\ 0 & 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \xrightarrow{\text{line3 }\times=\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 5 & 0 & \frac{11}{4} \\ 0 & 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line3}\times\left(\frac{11}{4}\right)} \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \xrightarrow{\text{line2 += line3}\times\left(\frac{5}{4}\right)} \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3}\times\left(\frac{3}{2}\right)} \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 1 & 0 & -4 \\ 3 & 4 & 5 & -1 \\ -2 & -5 & -8 & -4 \\ -4 & -5 & -6 & 2 \end{pmatrix} \xrightarrow{\text{line2 -= line1}} \begin{pmatrix} 2 & 1 & 0 & -4 \\ 1 & 3 & 5 & 3 \\ -2 & -5 & -8 & -4 \\ -4 & -5 & -6 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 2 & 1 & 0 & -4 \\ -2 & -5 & -8 & -4 \\ -4 & -5 & -6 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line1}\times(2)} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & -5 & -10 & -10 \\ -2 & -5 & -8 & -4 \\ -4 & -5 & -6 & 2 \end{pmatrix} \xrightarrow{\text{line3 += line1}\times(2)} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & -5 & -10 & -10 \\ 0 & 1 & 2 & 2 \\ -4 & -5 & -6 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1}\times(4)} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & -5 & -10 & -10 \\ 0 & 1 & 2 & 2 \\ 0 & 7 & 14 & 14 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & 5 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & -10 & -10 \\ 0 & 7 & 14 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}\times(3)} \begin{pmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & -10 & -10 \\ 0 & 7 & 14 & 14 \end{pmatrix} \xrightarrow{\text{line3 += line2}\times(5)} \begin{pmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & 14 & 14 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(7)} \begin{pmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 5 & 6 & 4 \\ -2 & -3 & -6 & 0 \\ -2 & -3 & -6 & 0 \\ 3 & 7 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} 0 & 2 & 0 & 4 \\ -2 & -3 & -6 & 0 \\ -2 & -3 & -6 & 0 \\ 3 & 7 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ -2 & -3 & -6 & 0 \\ 3 & 7 & 9 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line4}} \begin{pmatrix} 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 1 & 4 & 3 & 5 \\ 3 & 7 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 4 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \\ 3 & 7 & 9 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1}\times(3)} \begin{pmatrix} 1 & 4 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & -5 & 0 & -10 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 4 & 3 & 5 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & -10 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 4 & 3 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & -10 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times(4)} \begin{pmatrix} 1 & 0 & 3 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & -10 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2}\times(5)} \begin{pmatrix} 1 & 0 & 3 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -8 & 8 & 8 \\ -4 & 1 & -1 & -1 \\ 8 & -9 & 9 & 9 \\ 6 & -5 & 5 & 5 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} -1 & -7 & 7 & 7 \\ -4 & 1 & -1 & -1 \\ 8 & -9 & 9 & 9 \\ 6 & -5 & 5 & 5 \end{pmatrix} \xrightarrow{\text{line1 } \times = (-1)} \begin{pmatrix} 1 & 7 & -7 & -7 \\ -4 & 1 & -1 & -1 \\ 8 & -9 & 9 & 9 \\ 6 & -5 & 5 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1}\times(4)} \begin{pmatrix} 1 & 7 & -7 & -7 \\ 0 & 29 & -29 & -29 \\ 8 & -9 & 9 & 9 \\ 6 & -5 & 5 & 5 \end{pmatrix} \xrightarrow{\text{line3 -= line1}\times(8)} \begin{pmatrix} 1 & 7 & -7 & -7 \\ 0 & 29 & -29 & -29 \\ 0 & -65 & 65 & 65 \\ 6 & -5 & 5 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1}\times(6)} \begin{pmatrix} 1 & 7 & -7 & -7 \\ 0 & 29 & -29 & -29 \\ 0 & -65 & 65 & 65 \\ 0 & -47 & 47 & 47 \end{pmatrix} \xrightarrow{\text{line2 } \times = \left(\frac{1}{29}\right)} \begin{pmatrix} 1 & 7 & -7 & -7 \\ 0 & 1 & -1 & -1 \\ 0 & -65 & 65 & 65 \\ 0 & -47 & 47 & 47 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}\times(7)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -65 & 65 & 65 \\ 0 & -47 & 47 & 47 \end{pmatrix} \xrightarrow{\text{line3 += line2}\times(65)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -47 & 47 & 47 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2}\times(47)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -3 & -3 & -9 \\ -3 & 3 & 2 & 4 \\ -1 & 1 & 1 & 3 \\ 6 & -6 & -4 & -8 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} -1 & 1 & 1 & 3 \\ -3 & 3 & 2 & 4 \\ 3 & -3 & -3 & -9 \\ 6 & -6 & -4 & -8 \end{pmatrix} \xrightarrow{\text{line1 } \times = (-1)} \begin{pmatrix} 1 & -1 & -1 & -3 \\ -3 & 3 & 2 & 4 \\ 3 & -3 & -3 & -9 \\ 6 & -6 & -4 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1}\times(3)} \begin{pmatrix} 1 & -1 & -1 & -3 \\ 0 & 0 & -1 & -5 \\ 3 & -3 & -3 & -9 \\ 6 & -6 & -4 & -8 \end{pmatrix} \xrightarrow{\text{line3 -= line1}\times(3)} \begin{pmatrix} 1 & -1 & -1 & -3 \\ 0 & 0 & -1 & -5 \\ 0 & 0 & 0 & 0 \\ 6 & -6 & -4 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1}\times(6)} \begin{pmatrix} 1 & -1 & -1 & -3 \\ 0 & 0 & -1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 10 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & -1 & -1 & -3 \\ 0 & 0 & 2 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & -1 & -3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -5 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2}} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & 3 & 3 & 9 \\ 1 & 5 & 1 & 3 \\ -2 & -5 & -2 & -6 \\ 2 & -3 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 3 & 3 & 3 & 9 \\ -2 & -5 & -2 & -6 \\ 2 & -3 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 0 & -12 & 0 & 0 \\ -2 & -5 & -2 & -6 \\ 2 & -3 & 2 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 0 & -12 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 2 & -3 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line1}\times(2)} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 0 & -12 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & -13 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 0 & 5 & 0 & 0 \\ 0 & -12 & 0 & 0 \\ 0 & -13 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 5 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -12 & 0 & 0 \\ 0 & -13 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -12 & 0 & 0 \\ 0 & -13 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(12)} \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -13 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(13)} \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 4 & -4 & -2 & 2 \\ 4 & -8 & -3 & 3 \\ 7 & 3 & -1 & 1 \\ -7 & -3 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 0 & 4 & 1 & -1 \\ 4 & -8 & -3 & 3 \\ 7 & 3 & -1 & 1 \\ -7 & -3 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line4}} \begin{pmatrix} 0 & 4 & 1 & -1 \\ 4 & -8 & -3 & 3 \\ 0 & 0 & 0 & 0 \\ -7 & -3 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -7 & -3 & 1 & -1 \\ 4 & -8 & -3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line1} \times= \left(-\frac{1}{7}\right)} \begin{pmatrix} 1 & \frac{3}{7} & -\frac{1}{7} & \frac{1}{7} \\ 4 & -8 & -3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(4)} \begin{pmatrix} 1 & \frac{3}{7} & -\frac{1}{7} & \frac{1}{7} \\ 0 & -\frac{68}{7} & -\frac{17}{7} & \frac{17}{7} \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & \frac{3}{7} & -\frac{1}{7} & \frac{1}{7} \\ 0 & 4 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{68}{7} & -\frac{17}{7} & \frac{17}{7} \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & \frac{3}{7} & -\frac{1}{7} & \frac{1}{7} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{68}{7} & -\frac{17}{7} & \frac{17}{7} \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times\left(\frac{3}{7}\right)} \begin{pmatrix} 1 & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{68}{7} & -\frac{17}{7} & \frac{17}{7} \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times\left(\frac{68}{7}\right)} \begin{pmatrix} 1 & 0 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -4 & 7 & 6 & -4 \\ -4 & 5 & 7 & -1 \\ 3 & -5 & -5 & 3 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 0 & 2 & -1 & -3 \\ -4 & 5 & 7 & -1 \\ 3 & -5 & -5 & 3 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line3}} \begin{pmatrix} 0 & 2 & -1 & -3 \\ -1 & 0 & 2 & 2 \\ 3 & -5 & -5 & 3 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} -1 & 0 & 2 & 2 \\ 0 & 2 & -1 & -3 \\ 3 & -5 & -5 & 3 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 0 & -2 & -2 \\ 0 & 2 & -1 & -3 \\ 3 & -5 & -5 & 3 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & 0 & -2 & -2 \\ 0 & 2 & -1 & -3 \\ 0 & -5 & 1 & 9 \\ 0 & 1 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & -5 & 1 & 9 \\ 0 & 2 & -1 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 2 & -1 & -3 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line3}} \begin{pmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -3 & 3 & 9 & 5 \\ 3 & -3 & -9 & -7 \\ -2 & 2 & 6 & -3 \\ -1 & 1 & 3 & 1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -1 & 1 & 3 & 1 \\ 3 & -3 & -9 & -7 \\ -2 & 2 & 6 & -3 \\ -3 & 3 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 3 & -3 & -9 & -7 \\ -2 & 2 & 6 & -3 \\ -3 & 3 & 9 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2} -= \text{line1} \times (3)} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 0 & 0 & 0 & -4 \\ -2 & 2 & 6 & -3 \\ -3 & 3 & 9 & 5 \end{pmatrix} \xrightarrow{\text{line3} += \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -5 \\ -3 & 3 & 9 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4} += \text{line1} \times (3)} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & -3 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3} += \text{line2} \times (5)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line4} += \text{line2} \times (4)} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & -3 & -3 & -3 \\ 1 & 3 & 3 & 1 \\ -1 & -3 & -3 & -3 \\ -1 & -1 & -5 & -1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ -1 & -3 & -3 & -3 \\ -1 & -3 & -3 & -3 \\ -1 & -1 & -5 & -1 \end{pmatrix} \xrightarrow{\text{line2} += \text{line1}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & -2 \\ -1 & -3 & -3 & -3 \\ -1 & -1 & -5 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} += \text{line1}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \\ -1 & -1 & -5 & -1 \end{pmatrix} \xrightarrow{\text{line4} += \text{line1}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & 2 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 3 & 3 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1} -= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line3} \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1} -= \text{line3}} \begin{pmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line4} += \text{line3} \times (2)} \begin{pmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & -3 & 1 & 0 \\ 8 & 9 & 7 & 2 \\ 0 & 1 & -1 & 0 \\ 7 & 8 & 6 & -1 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 3 & -1 & 0 \\ 8 & 9 & 7 & 2 \\ 0 & 1 & -1 & 0 \\ 7 & 8 & 6 & -1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (8)} \begin{pmatrix} 1 & 3 & -1 & 0 \\ 0 & -15 & 15 & 2 \\ 0 & 1 & -1 & 0 \\ 7 & 8 & 6 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4} -= \text{line1} \times (7)} \begin{pmatrix} 1 & 3 & -1 & 0 \\ 0 & -15 & 15 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & -13 & 13 & -1 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -15 & 15 & 2 \\ 0 & -13 & 13 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} -= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -15 & 15 & 2 \\ 0 & -13 & 13 & -1 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (15)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & -13 & 13 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4} += \text{line2} \times (13)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line3} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= (-1)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line3} \times (2)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -1 & 0 & -1 & -4 \\ -2 & -4 & 6 & 8 \\ 3 & 1 & 1 & 7 \\ 0 & -1 & 2 & 0 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & 0 & 1 & 4 \\ -2 & -4 & 6 & 8 \\ 3 & 1 & 1 & 7 \\ 0 & -1 & 2 & 0 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & -4 & 8 & 16 \\ 3 & 1 & 1 & 7 \\ 0 & -1 & 2 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line1}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & -4 & 8 & 16 \\ 0 & 1 & -2 & -5 \\ 0 & -1 & 2 & 0 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & -5 \\ 0 & -4 & 8 & 16 \\ 0 & -1 & 2 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & -4 \\ 0 & -1 & 2 & 0 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line3}\times(4)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line3}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & -5 & 6 & -6 \\ 1 & 6 & 4 & -2 \\ 0 & 9 & -9 & 9 \\ -2 & -5 & 2 & -6 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 6 & 4 & -2 \\ 0 & -5 & 6 & -6 \\ 0 & 9 & -9 & 9 \\ -2 & -5 & 2 & -6 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & 6 & 4 & -2 \\ 0 & -5 & 6 & -6 \\ 0 & 9 & -9 & 9 \\ 0 & 7 & 10 & -10 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 6 & 4 & -2 \\ 0 & 9 & -9 & 9 \\ 0 & -5 & 6 & -6 \\ 0 & 7 & 10 & -10 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{9}\right)} \begin{pmatrix} 1 & 6 & 4 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & -5 & 6 & -6 \\ 0 & 7 & 10 & -10 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line2}\times(6)} \begin{pmatrix} 1 & 0 & 10 & -8 \\ 0 & 1 & -1 & 1 \\ 0 & -5 & 6 & -6 \\ 0 & 7 & 10 & -10 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(5)} \begin{pmatrix} 1 & 0 & 10 & -8 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 7 & 10 & -10 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}\times(7)} \begin{pmatrix} 1 & 0 & 10 & -8 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 17 & -17 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line3}\times(10)} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 17 & -17 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 17 & -17 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line3}\times(17)} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & 4 & -2 & -8 \\ 1 & -2 & 1 & 2 \\ -6 & 9 & -3 & -3 \\ -1 & 3 & -1 & -3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & -2 & 1 & 2 \\ 0 & 4 & -2 & -8 \\ -6 & 9 & -3 & -3 \\ -1 & 3 & -1 & -3 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(6)} \begin{pmatrix} 1 & -2 & 1 & 2 \\ 0 & 4 & -2 & -8 \\ 0 & -3 & 3 & 9 \\ -1 & 3 & -1 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}} \begin{pmatrix} 1 & -2 & 1 & 2 \\ 0 & 4 & -2 & -8 \\ 0 & -3 & 3 & 9 \\ 0 & 1 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & -2 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 3 & 9 \\ 0 & 4 & -2 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 3 & 9 \\ 0 & 4 & -2 & -8 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 3 & 6 \\ 0 & 4 & -2 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & -2 & -4 \end{pmatrix} \xrightarrow{\text{line3 } \times= \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix}2&-2&-2&-8\\0&2&-2&1\\0&-4&4&-9\\1&-9&7&-7\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line4}}\begin{pmatrix}1&-9&7&-7\\0&2&-2&1\\0&-4&4&-9\\2&-2&-2&-8\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line1}\times(2)}\begin{pmatrix}1&-9&7&-7\\0&2&-2&1\\0&-4&4&-9\\0&16&-16&6\end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{2}\right)}\begin{pmatrix}1&-9&7&-7\\0&1&-1&\frac{1}{2}\\0&-4&4&-9\\0&16&-16&6\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line2}\times(9)}\begin{pmatrix}1&0&-2&-\frac{5}{2}\\0&1&-1&\frac{1}{2}\\0&-4&4&-9\\0&16&-16&6\end{pmatrix}\xrightarrow{\text{line3 }+=\text{ line2}\times(4)}\begin{pmatrix}1&0&-2&-\frac{5}{2}\\0&1&-1&\frac{1}{2}\\0&0&0&-7\\0&16&-16&6\end{pmatrix}$$

$$\xrightarrow{\text{line4 }-=\text{ line2}\times(16)}\begin{pmatrix}1&0&-2&-\frac{5}{2}\\0&1&-1&\frac{1}{2}\\0&0&0&-7\\0&0&0&-2\end{pmatrix}\xrightarrow{\text{line3 }\times=\left(-\frac{1}{7}\right)}\begin{pmatrix}1&0&-2&-\frac{5}{2}\\0&1&-1&\frac{1}{2}\\0&0&0&1\\0&0&0&-2\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line3}\times\left(\frac{5}{2}\right)}\begin{pmatrix}1&0&-2&0\\0&1&-1&\frac{1}{2}\\0&0&0&1\\0&0&0&-2\end{pmatrix}$$

$$\xrightarrow{\text{line2 }-=\text{ line3}\times\left(\frac{1}{2}\right)}\begin{pmatrix}1&0&-2&0\\0&1&-1&0\\0&0&0&1\\0&0&0&-2\end{pmatrix}\xrightarrow{\text{line4 }+=\text{ line3}\times(2)}\begin{pmatrix}1&0&-2&0\\0&1&-1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}3&7&-9&-3\\-3&-7&9&3\\-1&-1&3&0\\3&-1&-9&3\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}-1&-1&3&0\\-3&-7&9&3\\3&7&-9&-3\\3&-1&-9&3\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&1&-3&0\\-3&-7&9&3\\3&7&-9&-3\\3&-1&-9&3\end{pmatrix}$$

$$\xrightarrow{\text{line2 }+=\text{ line1}\times(3)}\begin{pmatrix}1&1&-3&0\\0&-4&0&3\\3&7&-9&-3\\3&-1&-9&3\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line1}\times(3)}\begin{pmatrix}1&1&-3&0\\0&-4&0&3\\0&4&0&-3\\3&-1&-9&3\end{pmatrix}$$

$$\xrightarrow{\text{line4 }-=\text{ line1}\times(3)}\begin{pmatrix}1&1&-3&0\\0&-4&0&3\\0&4&0&-3\\0&-4&0&3\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line3}}\begin{pmatrix}1&1&-3&0\\0&0&0&0\\0&4&0&-3\\0&-4&0&3\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line4}}\begin{pmatrix}1&1&-3&0\\0&0&0&0\\0&0&0&0\\0&-4&0&3\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line4}}\begin{pmatrix}1&1&-3&0\\0&-4&0&3\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(-\frac{1}{4}\right)}\begin{pmatrix}1&1&-3&0\\0&1&0&-\frac{3}{4}\\0&0&0&0\\0&0&0&0\end{pmatrix}\xrightarrow{\text{line1 }-=\text{ line2}}\begin{pmatrix}1&0&-3&\frac{3}{4}\\0&1&0&-\frac{3}{4}\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}9&8&-8&8\\-8&-7&7&-1\\-9&-8&8&-4\\2&2&-2&3\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line2}}\begin{pmatrix}1&1&-1&7\\-8&-7&7&-1\\-9&-8&8&-4\\2&2&-2&3\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(8)}\begin{pmatrix}1&1&-1&7\\0&1&-1&55\\-9&-8&8&-4\\2&2&-2&3\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line1}\times(9)}\begin{pmatrix}1&1&-1&7\\0&1&-1&55\\0&1&-1&59\\2&2&-2&3\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line1}\times(2)}\begin{pmatrix}1&1&-1&7\\0&1&-1&55\\0&1&-1&59\\0&0&0&-11\end{pmatrix}$$

$$\xrightarrow{\text{line1 }-=\text{ line2}}\begin{pmatrix}1&0&0&-48\\0&1&-1&55\\0&1&-1&59\\0&0&0&-11\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}}\begin{pmatrix}1&0&0&-48\\0&1&-1&55\\0&0&0&4\\0&0&0&-11\end{pmatrix}$$

$$\xrightarrow{\text{line3 }\times=\left(\frac{1}{4}\right)}\begin{pmatrix}1&0&0&-48\\0&1&-1&55\\0&0&0&1\\0&0&0&-11\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line3}\times(48)}\begin{pmatrix}1&0&0&0\\0&1&-1&55\\0&0&0&1\\0&0&0&-11\end{pmatrix}$$

$$\xrightarrow{\text{line2 }-=\text{ line3}\times(55)}\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&1\\0&0&0&-11\end{pmatrix}\xrightarrow{\text{line4 }+=\text{ line3}\times(11)}\begin{pmatrix}1&0&0&0\\0&1&-1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix} -6 & 0 & 0 & 7 \\ -9 & -3 & 9 & 9 \\ -2 & 1 & -3 & 3 \\ 0 & 1 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line3}\times(5)} \begin{pmatrix} -6 & 0 & 0 & 7 \\ 1 & -8 & 24 & -6 \\ -2 & 1 & -3 & 3 \\ 0 & 1 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & -8 & 24 & -6 \\ -6 & 0 & 0 & 7 \\ -2 & 1 & -3 & 3 \\ 0 & 1 & -3 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(6)} \begin{pmatrix} 1 & -8 & 24 & -6 \\ 0 & -48 & 144 & -29 \\ -2 & 1 & -3 & 3 \\ 0 & 1 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & -8 & 24 & -6 \\ 0 & -48 & 144 & -29 \\ 0 & -15 & 45 & -9 \\ 0 & 1 & -3 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & -8 & 24 & -6 \\ 0 & 1 & -3 & 0 \\ 0 & -15 & 45 & -9 \\ 0 & -48 & 144 & -29 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}\times(8)} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & -3 & 0 \\ 0 & -15 & 45 & -9 \\ 0 & -48 & 144 & -29 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times(15)} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & -9 \\ 0 & -48 & 144 & -29 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}\times(48)} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & -29 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= \left(-\frac{1}{9}\right)} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -29 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line3}\times(6)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -29 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line3}\times(29)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & 9 & -1 & -1 \\ 4 & 6 & -3 & 5 \\ 1 & 4 & -1 & 1 \\ -3 & -5 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 4 & 6 & -3 & 5 \\ 0 & 9 & -1 & -1 \\ -3 & -5 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(4)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & -10 & 1 & 1 \\ 0 & 9 & -1 & -1 \\ -3 & -5 & 2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}\times(3)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & -10 & 1 & 1 \\ 0 & 9 & -1 & -1 \\ 0 & 7 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 9 & -1 & -1 \\ 0 & 7 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 9 & -1 & -1 \\ 0 & 7 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 9 & -1 & -1 \\ 0 & 7 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line2}\times(9)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 7 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line2}\times(7)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= (-1)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & -1 & 5 & -3 \\ -6 & 1 & -5 & 9 \\ -3 & -1 & 5 & 4 \\ -2 & 1 & -5 & 6 \end{pmatrix} \xrightarrow{\text{line2 += line1×(6)}} \begin{pmatrix} 1 & -1 & 5 & -3 \\ 0 & -5 & 25 & -9 \\ -3 & -1 & 5 & 4 \\ -2 & 1 & -5 & 6 \end{pmatrix} \xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & -1 & 5 & -3 \\ 0 & -5 & 25 & -9 \\ 0 & -4 & 20 & -5 \\ -2 & 1 & -5 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(2)}} \begin{pmatrix} 1 & -1 & 5 & -3 \\ 0 & -5 & 25 & -9 \\ 0 & -4 & 20 & -5 \\ 0 & -1 & 5 & 0 \end{pmatrix} \xrightarrow{\text{line2 ↔ line4}} \begin{pmatrix} 1 & -1 & 5 & -3 \\ 0 & -1 & 5 & 0 \\ 0 & -4 & 20 & -5 \\ 0 & -5 & 25 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix} 1 & -1 & 5 & -3 \\ 0 & 1 & -5 & 0 \\ 0 & -4 & 20 & -5 \\ 0 & -5 & 25 & -9 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -5 & 0 \\ 0 & -4 & 20 & -5 \\ 0 & -5 & 25 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(4)}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & -5 \\ 0 & -5 & 25 & -9 \end{pmatrix} \xrightarrow{\text{line4 += line2×(5)}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= } \left(-\frac{1}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{\text{line1 += line3×(3)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(9)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & 6 & 2 & 0 \\ -1 & -7 & 0 & -9 \\ 3 & 0 & -7 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & 6 & 2 & 0 \\ 0 & -6 & -2 & -2 \\ 3 & 0 & -7 & 5 \end{pmatrix} \xrightarrow{\text{line4 −= line1×(3)}} \begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & 6 & 2 & 0 \\ 0 & -6 & -2 & -2 \\ 0 & -3 & -1 & -16 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & 0 & 0 & -2 \\ 0 & -6 & -2 & -2 \\ 0 & -3 & -1 & -16 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & -6 & -2 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & -3 & -1 & -16 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{6}\right)} \begin{pmatrix} 1 & 1 & -2 & 7 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -2 \\ 0 & -3 & -1 & -16 \end{pmatrix} \xrightarrow{\text{line1 −= line2}} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & \frac{20}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -2 \\ 0 & -3 & -1 & -16 \end{pmatrix} \xrightarrow{\text{line4 += line2×(3)}} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & \frac{20}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -15 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= } \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & \frac{20}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -15 \end{pmatrix} \xrightarrow{\text{line1 −= line3×}\left(\frac{20}{3}\right)} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -15 \end{pmatrix} \xrightarrow{\text{line2 −= line3×}\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -15 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(15)}} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 8 & 1 & 4 & 5 \\ 2 & 2 & -6 & 6 \\ -6 & 0 & -6 & 9 \\ -1 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -1 & 0 & -1 & 2 \\ 2 & 2 & -6 & 6 \\ -6 & 0 & -6 & 9 \\ 8 & 1 & 4 & 5 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 2 & 2 & -6 & 6 \\ -6 & 0 & -6 & 9 \\ 8 & 1 & 4 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 2 & -8 & 10 \\ -6 & 0 & -6 & 9 \\ 8 & 1 & 4 & 5 \end{pmatrix} \xrightarrow{\text{line3} += \text{line1} \times (6)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 2 & -8 & 10 \\ 0 & 0 & 0 & -3 \\ 8 & 1 & 4 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4} -= \text{line1} \times (8)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 2 & -8 & 10 \\ 0 & 0 & 0 & -3 \\ 0 & 1 & -4 & 21 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -4 & 21 \\ 0 & 0 & 0 & -3 \\ 0 & 2 & -8 & 10 \end{pmatrix}$$

$$\xrightarrow{\text{line4} -= \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -4 & 21 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -32 \end{pmatrix} \xrightarrow{\text{line3} \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -4 & 21 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -32 \end{pmatrix}$$

$$\xrightarrow{\text{line1} += \text{line3} \times (2)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 21 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -32 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line3} \times (21)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -32 \end{pmatrix}$$

$$\xrightarrow{\text{line4} += \text{line3} \times (32)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -7 & -7 & 9 & -2 \\ 2 & 2 & 3 & 5 \\ 5 & 5 & -1 & 5 \\ -3 & -3 & 5 & 1 \end{pmatrix} \xrightarrow{\text{line1} += \text{line2} \times (4)} \begin{pmatrix} 1 & 1 & 21 & 18 \\ 2 & 2 & 3 & 5 \\ 5 & 5 & -1 & 5 \\ -3 & -3 & 5 & 1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 1 & 21 & 18 \\ 0 & 0 & -39 & -31 \\ 5 & 5 & -1 & 5 \\ -3 & -3 & 5 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} -= \text{line1} \times (5)} \begin{pmatrix} 1 & 1 & 21 & 18 \\ 0 & 0 & -39 & -31 \\ 0 & 0 & -106 & -85 \\ -3 & -3 & 5 & 1 \end{pmatrix} \xrightarrow{\text{line4} += \text{line1} \times (3)} \begin{pmatrix} 1 & 1 & 21 & 18 \\ 0 & 0 & -39 & -31 \\ 0 & 0 & -106 & -85 \\ 0 & 0 & 68 & 55 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times= \left(-\frac{1}{39}\right)} \begin{pmatrix} 1 & 1 & 21 & 18 \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & -106 & -85 \\ 0 & 0 & 68 & 55 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line2} \times (21)} \begin{pmatrix} 1 & 1 & 0 & \frac{17}{13} \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & -106 & -85 \\ 0 & 0 & 68 & 55 \end{pmatrix}$$

$$\xrightarrow{\text{line3} += \text{line2} \times (106)} \begin{pmatrix} 1 & 1 & 0 & \frac{17}{13} \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & 0 & -\frac{29}{39} \\ 0 & 0 & 68 & 55 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line2} \times (68)} \begin{pmatrix} 1 & 1 & 0 & \frac{17}{13} \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & 0 & -\frac{29}{39} \\ 0 & 0 & 0 & \frac{37}{39} \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(-\frac{39}{29}\right)} \begin{pmatrix} 1 & 1 & 0 & \frac{17}{13} \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{37}{39} \end{pmatrix} \xrightarrow{\text{line1} -= \text{line3} \times \left(\frac{17}{13}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{31}{39} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{37}{39} \end{pmatrix}$$

$$\xrightarrow{\text{line2} -= \text{line3} \times \left(\frac{31}{39}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{37}{39} \end{pmatrix} \xrightarrow{\text{line4} -= \text{line3} \times \left(\frac{37}{39}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & -3 & -7 & -9 \\ 3 & 1 & -1 & -5 \\ -4 & -4 & -7 & -7 \\ -4 & -2 & -1 & 3 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} 1 & -2 & -8 & -14 \\ 3 & 1 & -1 & -5 \\ -4 & -4 & -7 & -7 \\ -4 & -2 & -1 & 3 \end{pmatrix} \xrightarrow{\text{line2 -= line1×(3)}} \begin{pmatrix} 1 & -2 & -8 & -14 \\ 0 & 7 & 23 & 37 \\ -4 & -4 & -7 & -7 \\ -4 & -2 & -1 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(4)}} \begin{pmatrix} 1 & -2 & -8 & -14 \\ 0 & 7 & 23 & 37 \\ 0 & -12 & -39 & -63 \\ -4 & -2 & -1 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(4)}} \begin{pmatrix} 1 & -2 & -8 & -14 \\ 0 & 7 & 23 & 37 \\ 0 & -12 & -39 & -63 \\ 0 & -10 & -33 & -53 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{7}\right)} \begin{pmatrix} 1 & -2 & -8 & -14 \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & -12 & -39 & -63 \\ 0 & -10 & -33 & -53 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & -\frac{10}{7} & -\frac{24}{7} \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & -12 & -39 & -63 \\ 0 & -10 & -33 & -53 \end{pmatrix} \xrightarrow{\text{line3 += line2×(12)}} \begin{pmatrix} 1 & 0 & -\frac{10}{7} & -\frac{24}{7} \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & 0 & \frac{3}{7} & \frac{3}{7} \\ 0 & -10 & -33 & -53 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2×(10)}} \begin{pmatrix} 1 & 0 & -\frac{10}{7} & -\frac{24}{7} \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & 0 & \frac{3}{7} & \frac{3}{7} \\ 0 & 0 & -\frac{1}{7} & -\frac{1}{7} \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{7}{3}\right)} \begin{pmatrix} 1 & 0 & -\frac{10}{7} & -\frac{24}{7} \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{1}{7} & -\frac{1}{7} \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×}\left(\frac{10}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & \frac{23}{7} & \frac{37}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{1}{7} & -\frac{1}{7} \end{pmatrix} \xrightarrow{\text{line2 -= line3×}\left(\frac{23}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{1}{7} & -\frac{1}{7} \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×}\left(\frac{1}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & -3 & -1 & -4 \\ -2 & -4 & 1 & 1 \\ -7 & -9 & -2 & -8 \\ -1 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 ↔ line4}} \begin{pmatrix} -1 & 0 & 0 & 1 \\ -2 & -4 & 1 & 1 \\ -7 & -9 & -2 & -8 \\ -2 & -3 & -1 & -4 \end{pmatrix} \xrightarrow{\text{line1 ×= } (-1)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ -2 & -4 & 1 & 1 \\ -7 & -9 & -2 & -8 \\ -2 & -3 & -1 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1×(2)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -4 & 1 & -1 \\ -7 & -9 & -2 & -8 \\ -2 & -3 & -1 & -4 \end{pmatrix} \xrightarrow{\text{line3 += line1×(7)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -4 & 1 & -1 \\ 0 & -9 & -2 & -15 \\ -2 & -3 & -1 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(2)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -4 & 1 & -1 \\ 0 & -9 & -2 & -15 \\ 0 & -3 & -1 & -6 \end{pmatrix} \xrightarrow{\text{line4 -= line2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -4 & 1 & -1 \\ 0 & -9 & -2 & -15 \\ 0 & 1 & -2 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ↔ line4}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & -9 & -2 & -15 \\ 0 & -4 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line3 += line2×(9)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & -20 & -60 \\ 0 & -4 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2×(4)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & -20 & -60 \\ 0 & 0 & -7 & -21 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(-\frac{1}{20}\right)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -7 & -21 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line3×(2)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -7 & -21 \end{pmatrix} \xrightarrow{\text{line4 += line3×(7)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & -2 & 1 & 3 \\ -6 & -6 & 5 & -6 \\ -3 & -3 & 4 & 0 \\ 3 & 3 & -9 & -7 \end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix} 1 & 1 & -3 & 3 \\ -6 & -6 & 5 & -6 \\ -3 & -3 & 4 & 0 \\ 3 & 3 & -9 & -7 \end{pmatrix} \xrightarrow{\text{line2 += line1×(6)}} \begin{pmatrix} 1 & 1 & -3 & 3 \\ 0 & 0 & -13 & 12 \\ -3 & -3 & 4 & 0 \\ 3 & 3 & -9 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & 1 & -3 & 3 \\ 0 & 0 & -13 & 12 \\ 0 & 0 & -5 & 9 \\ 3 & 3 & -9 & -7 \end{pmatrix} \xrightarrow{\text{line4 -= line1×(3)}} \begin{pmatrix} 1 & 1 & -3 & 3 \\ 0 & 0 & -13 & 12 \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 0 & -16 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{13}\right)} \begin{pmatrix} 1 & 1 & -3 & 3 \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 0 & -16 \end{pmatrix} \xrightarrow{\text{line1 += line2×(3)}} \begin{pmatrix} 1 & 1 & 0 & \frac{3}{13} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 0 & -16 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 1 & 0 & \frac{3}{13} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & 0 & \frac{57}{13} \\ 0 & 0 & 0 & -16 \end{pmatrix} \xrightarrow{\text{line3 ↔ line4}} \begin{pmatrix} 1 & 1 & 0 & \frac{3}{13} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & 0 & -16 \\ 0 & 0 & 0 & \frac{57}{13} \end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= } \left(-\frac{1}{16}\right)} \begin{pmatrix} 1 & 1 & 0 & \frac{3}{13} \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{57}{13} \end{pmatrix} \xrightarrow{\text{line1 -= line3×}\left(\frac{3}{13}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{12}{13} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{57}{13} \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line3×}\left(\frac{12}{13}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{57}{13} \end{pmatrix} \xrightarrow{\text{line4 -= line3×}\left(\frac{57}{13}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & 0 & 2 & -5 \\ -1 & -3 & 0 & 2 \\ -2 & -6 & 1 & 3 \\ 1 & 3 & -5 & 7 \end{pmatrix} \xrightarrow{\text{line1 ↔ line4}} \begin{pmatrix} 1 & 3 & -5 & 7 \\ -1 & -3 & 0 & 2 \\ -2 & -6 & 1 & 3 \\ 0 & 0 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 0 & -5 & 9 \\ -2 & -6 & 1 & 3 \\ 0 & 0 & 2 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 0 & -5 & 9 \\ 0 & 0 & -9 & 17 \\ 0 & 0 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(2)}} \begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line1 += line2×(5)}} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -5 & 9 \\ 0 & 0 & 2 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & -5 \end{pmatrix} \xrightarrow{\text{line4 -= line2×(2)}} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= } \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix} \xrightarrow{\text{line1 -= line3×(2)}} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix} \xrightarrow{\text{line4 += line3×(3)}} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & -6 & 8 & 6 \\ 0 & -2 & 1 & 1 \\ -1 & 9 & -7 & 1 \\ 2 & 8 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -1 & 9 & -7 & 1 \\ 0 & -2 & 1 & 1 \\ 2 & -6 & 8 & 6 \\ 2 & 8 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -9 & 7 & -1 \\ 0 & -2 & 1 & 1 \\ 2 & -6 & 8 & 6 \\ 2 & 8 & 1 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & -9 & 7 & -1 \\ 0 & -2 & 1 & 1 \\ 0 & 12 & -6 & 8 \\ 2 & 8 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line1} \times (2)} \begin{pmatrix} 1 & -9 & 7 & -1 \\ 0 & -2 & 1 & 1 \\ 0 & 12 & -6 & 8 \\ 0 & 26 & -13 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & -9 & 7 & -1 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 12 & -6 & 8 \\ 0 & 26 & -13 & 5 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2} \times (9)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{11}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 12 & -6 & 8 \\ 0 & 26 & -13 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line3} -= \text{line2} \times (12)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{11}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 14 \\ 0 & 26 & -13 & 5 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line2} \times (26)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{11}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 14 \\ 0 & 0 & 0 & 18 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(\frac{1}{14}\right)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{11}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 18 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3} \times \left(\frac{11}{2}\right)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 18 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3} \times \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 18 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line3} \times (18)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -9 & -1 & 2 \\ -7 & 8 & 6 & 7 \\ 1 & 7 & -3 & -8 \\ 5 & -7 & -4 & -4 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 7 & -3 & -8 \\ -7 & 8 & 6 & 7 \\ 3 & -9 & -1 & 2 \\ 5 & -7 & -4 & -4 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1} \times (7)} \begin{pmatrix} 1 & 7 & -3 & -8 \\ 0 & 57 & -15 & -49 \\ 3 & -9 & -1 & 2 \\ 5 & -7 & -4 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3} -= \text{line1} \times (3)} \begin{pmatrix} 1 & 7 & -3 & -8 \\ 0 & 57 & -15 & -49 \\ 0 & -30 & 8 & 26 \\ 5 & -7 & -4 & -4 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line1} \times (5)} \begin{pmatrix} 1 & 7 & -3 & -8 \\ 0 & 57 & -15 & -49 \\ 0 & -30 & 8 & 26 \\ 0 & -42 & 11 & 36 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times= \left(\frac{1}{57}\right)} \begin{pmatrix} 1 & 7 & -3 & -8 \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & -30 & 8 & 26 \\ 0 & -42 & 11 & 36 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line2} \times (7)} \begin{pmatrix} 1 & 0 & -\frac{22}{19} & -\frac{113}{57} \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & -30 & 8 & 26 \\ 0 & -42 & 11 & 36 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2} \times (30)} \begin{pmatrix} 1 & 0 & -\frac{22}{19} & -\frac{113}{57} \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & 0 & \frac{2}{19} & \frac{4}{19} \\ 0 & -42 & 11 & 36 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line2} \times (42)} \begin{pmatrix} 1 & 0 & -\frac{22}{19} & -\frac{113}{57} \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & 0 & \frac{2}{19} & \frac{4}{19} \\ 0 & 0 & -\frac{1}{19} & -\frac{2}{19} \end{pmatrix} \xrightarrow{\text{line3} \times= \left(\frac{19}{2}\right)} \begin{pmatrix} 1 & 0 & -\frac{22}{19} & -\frac{113}{57} \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{19} & -\frac{2}{19} \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{+}= \text{line3} \times \left(\frac{22}{19}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{5}{19} & -\frac{49}{57} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{19} & -\frac{2}{19} \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line3} \times \left(\frac{5}{19}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -\frac{1}{19} & -\frac{2}{19} \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line3} \times \left(\frac{1}{19}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -3 & -3 & -3 \\ 3 & -3 & -3 & -4 \\ -7 & -1 & -9 & 3 \\ 6 & -7 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line1 } \times= \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 3 & -3 & -3 & -4 \\ -7 & -1 & -9 & 3 \\ 6 & -7 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(3)} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ -7 & -1 & -9 & 3 \\ 6 & -7 & -8 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1}\times(7)} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & -8 & -16 & -4 \\ 6 & -7 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line1}\times(6)} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & -8 & -16 & -4 \\ 0 & -1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & -1 & -2 & 0 \\ 0 & -8 & -16 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & -8 & -16 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & -8 & -16 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(8)} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line3 } \times= (-1)} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line3}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line3}\times(4)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 4 & -1 & 1 & 2 \\ 7 & 1 & -1 & 6 \\ -8 & -3 & 3 & 2 \\ -5 & 1 & -1 & -7 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line4}} \begin{pmatrix} -1 & 0 & 0 & -5 \\ 7 & 1 & -1 & 6 \\ -8 & -3 & 3 & 2 \\ -5 & 1 & -1 & -7 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 7 & 1 & -1 & 6 \\ -8 & -3 & 3 & 2 \\ -5 & 1 & -1 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } -= \text{ line1}\times(7)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ -8 & -3 & 3 & 2 \\ -5 & 1 & -1 & -7 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(8)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & -3 & 3 & 42 \\ -5 & 1 & -1 & -7 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line1}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & -3 & 3 & 42 \\ 0 & 1 & -1 & 18 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & 0 & 0 & -45 \\ 0 & 1 & -1 & 18 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line2}} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & 0 & 0 & -45 \\ 0 & 0 & 0 & 47 \end{pmatrix} \xrightarrow{\text{line3 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & 0 & 0 & 47 \\ 0 & 0 & 0 & -45 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= \left(\frac{1}{47}\right)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -29 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -45 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line3}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -29 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -45 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times(29)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -45 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line3}\times(45)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 4 & 2 & 2 & -8 \\ -2 & -8 & 3 & 1 \\ 1 & -3 & 2 & -4 \\ 2 & 2 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -3 & 2 & -4 \\ -2 & -8 & 3 & 1 \\ 4 & 2 & 2 & -8 \\ 2 & 2 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(2)} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & -14 & 7 & -7 \\ 4 & 2 & 2 & -8 \\ 2 & 2 & 0 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line1}\times(4)} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & -14 & 7 & -7 \\ 0 & 14 & -6 & 8 \\ 2 & 2 & 0 & -4 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & -14 & 7 & -7 \\ 0 & 14 & -6 & 8 \\ 0 & 8 & -4 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 14 & -6 & 8 \\ 0 & 8 & -4 & 4 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & 14 & -6 & 8 \\ 0 & 0 & 1 & 1 \\ 0 & 8 & -4 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{14}\right)} \begin{pmatrix} 1 & -3 & 2 & -4 \\ 0 & 1 & -\frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 8 & -4 & 4 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & \frac{5}{7} & -\frac{16}{7} \\ 0 & 1 & -\frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 8 & -4 & 4 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line2}\times(8)} \begin{pmatrix} 1 & 0 & \frac{5}{7} & -\frac{16}{7} \\ 0 & 1 & -\frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{4}{7} & -\frac{4}{7} \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line3}\times\left(\frac{5}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -\frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{4}{7} & -\frac{4}{7} \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{3}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{4}{7} & -\frac{4}{7} \end{pmatrix}$$

$$\xrightarrow{\text{line4 } += \text{ line3}\times\left(\frac{4}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 7 & -1 & -2 & -3 \\ -4 & 0 & 2 & 0 \\ 7 & 0 & -4 & 1 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line3}} \begin{pmatrix} 0 & -1 & 2 & -4 \\ -4 & 0 & 2 & 0 \\ 7 & 0 & -4 & 1 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(2)} \begin{pmatrix} 0 & -1 & 2 & -4 \\ -4 & 0 & 2 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} -1 & 0 & 0 & 1 \\ -4 & 0 & 2 & 0 \\ 0 & -1 & 2 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ -4 & 0 & 2 & 0 \\ 0 & -1 & 2 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(4)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 2 & -4 \\ 0 & -1 & 2 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 2 & -4 \\ 0 & 0 & 2 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 2 & -4 \\ 0 & -1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line3 } \times= (-1)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -3 & 2 & 2 & 2 \\ -6 & 8 & 8 & 6 \\ -2 & 9 & 9 & 6 \\ -5 & 7 & 7 & 5 \end{pmatrix} \xrightarrow{\text{line3 -= line1}} \begin{pmatrix} -3 & 2 & 2 & 2 \\ -6 & 8 & 8 & 6 \\ 1 & 7 & 7 & 4 \\ -5 & 7 & 7 & 5 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 7 & 7 & 4 \\ -6 & 8 & 8 & 6 \\ -3 & 2 & 2 & 2 \\ -5 & 7 & 7 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1}\times(6)} \begin{pmatrix} 1 & 7 & 7 & 4 \\ 0 & 50 & 50 & 30 \\ -3 & 2 & 2 & 2 \\ -5 & 7 & 7 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1}\times(3)} \begin{pmatrix} 1 & 7 & 7 & 4 \\ 0 & 50 & 50 & 30 \\ 0 & 23 & 23 & 14 \\ -5 & 7 & 7 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1}\times(5)} \begin{pmatrix} 1 & 7 & 7 & 4 \\ 0 & 50 & 50 & 30 \\ 0 & 23 & 23 & 14 \\ 0 & 42 & 42 & 25 \end{pmatrix} \xrightarrow{\text{line2 }\times=\left(\frac{1}{50}\right)} \begin{pmatrix} 1 & 7 & 7 & 4 \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 23 & 23 & 14 \\ 0 & 42 & 42 & 25 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}\times(7)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 23 & 23 & 14 \\ 0 & 42 & 42 & 25 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(23)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 0 & 0 & -\frac{1}{5} \\ 0 & 42 & 42 & 25 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2}\times(42)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 0 & 0 & -\frac{1}{5} \\ 0 & 0 & 0 & -\frac{1}{5} \end{pmatrix} \xrightarrow{\text{line3 }\times=(5)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{5} \end{pmatrix} \xrightarrow{\text{line1 += line3}\times\left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \frac{3}{5} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{5} \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line3}\times\left(\frac{3}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{5} \end{pmatrix} \xrightarrow{\text{line4 += line3}\times\left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & 9 & 6 & 5 \\ 1 & 3 & -3 & -5 \\ 2 & -4 & -6 & -4 \\ 2 & -5 & -6 & 3 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & 3 & -3 & -5 \\ -2 & 9 & 6 & 5 \\ 2 & -4 & -6 & -4 \\ 2 & -5 & -6 & 3 \end{pmatrix} \xrightarrow{\text{line2 += line1}\times(2)} \begin{pmatrix} 1 & 3 & -3 & -5 \\ 0 & 15 & 0 & -5 \\ 2 & -4 & -6 & -4 \\ 2 & -5 & -6 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line1}\times(2)} \begin{pmatrix} 1 & 3 & -3 & -5 \\ 0 & 15 & 0 & -5 \\ 0 & -10 & 0 & 6 \\ 2 & -5 & -6 & 3 \end{pmatrix} \xrightarrow{\text{line4 -= line1}\times(2)} \begin{pmatrix} 1 & 3 & -3 & -5 \\ 0 & 15 & 0 & -5 \\ 0 & -10 & 0 & 6 \\ 0 & -11 & 0 & 13 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line4}} \begin{pmatrix} 1 & 3 & -3 & -5 \\ 0 & 15 & 0 & -5 \\ 0 & 1 & 0 & -7 \\ 0 & -11 & 0 & 13 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 3 & -3 & -5 \\ 0 & 1 & 0 & -7 \\ 0 & 15 & 0 & -5 \\ 0 & -11 & 0 & 13 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}\times(3)} \begin{pmatrix} 1 & 0 & -3 & 16 \\ 0 & 1 & 0 & -7 \\ 0 & 15 & 0 & -5 \\ 0 & -11 & 0 & 13 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(15)} \begin{pmatrix} 1 & 0 & -3 & 16 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 100 \\ 0 & -11 & 0 & 13 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2}\times(11)} \begin{pmatrix} 1 & 0 & -3 & 16 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & -64 \end{pmatrix} \xrightarrow{\text{line3 }\times=\left(\frac{1}{100}\right)} \begin{pmatrix} 1 & 0 & -3 & 16 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -64 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line3}\times(16)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -64 \end{pmatrix} \xrightarrow{\text{line2 += line3}\times(7)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -64 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3}\times(64)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix}-1&-7&0&9\\2&2&-1&-4\\0&2&-4&6\\-2&2&1&0\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&7&0&-9\\2&2&-1&-4\\0&2&-4&6\\-2&2&1&0\end{pmatrix}\xrightarrow{\text{line2 }-=\text{ line1}\times(2)}\begin{pmatrix}1&7&0&-9\\0&-12&-1&14\\0&2&-4&6\\-2&2&1&0\end{pmatrix}$$

$$\xrightarrow{\text{line4 }+=\text{ line1}\times(2)}\begin{pmatrix}1&7&0&-9\\0&-12&-1&14\\0&2&-4&6\\0&16&1&-18\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&7&0&-9\\0&2&-4&6\\0&-12&-1&14\\0&16&1&-18\end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{2}\right)}\begin{pmatrix}1&7&0&-9\\0&1&-2&3\\0&-12&-1&14\\0&16&1&-18\end{pmatrix}\xrightarrow{\text{line1 }-=\text{ line2}\times(7)}\begin{pmatrix}1&0&14&-30\\0&1&-2&3\\0&-12&-1&14\\0&16&1&-18\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line2}\times(12)}\begin{pmatrix}1&0&14&-30\\0&1&-2&3\\0&0&-25&50\\0&16&1&-18\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line2}\times(16)}\begin{pmatrix}1&0&14&-30\\0&1&-2&3\\0&0&-25&50\\0&0&33&-66\end{pmatrix}$$

$$\xrightarrow{\text{line3 }\leftrightarrow\text{ line4}}\begin{pmatrix}1&0&14&-30\\0&1&-2&3\\0&0&33&-66\\0&0&-25&50\end{pmatrix}\xrightarrow{\text{line3 }\times=\left(\frac{1}{33}\right)}\begin{pmatrix}1&0&14&-30\\0&1&-2&3\\0&0&1&-2\\0&0&-25&50\end{pmatrix}$$

$$\xrightarrow{\text{line1 }-=\text{ line3}\times(14)}\begin{pmatrix}1&0&0&-2\\0&1&-2&3\\0&0&1&-2\\0&0&-25&50\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line3}\times(2)}\begin{pmatrix}1&0&0&-2\\0&1&0&-1\\0&0&1&-2\\0&0&-25&50\end{pmatrix}$$

$$\xrightarrow{\text{line4 }+=\text{ line3}\times(25)}\begin{pmatrix}1&0&0&-2\\0&1&0&-1\\0&0&1&-2\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}-1&8&-4&7\\2&5&-4&-7\\1&7&-5&-2\\-1&8&2&7\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&7&-5&-2\\2&5&-4&-7\\-1&8&-4&7\\-1&8&2&7\end{pmatrix}\xrightarrow{\text{line2 }-=\text{ line1}\times(2)}\begin{pmatrix}1&7&-5&-2\\0&-9&6&-3\\-1&8&-4&7\\-1&8&2&7\end{pmatrix}$$

$$\xrightarrow{\text{line3 }+=\text{ line1}}\begin{pmatrix}1&7&-5&-2\\0&-9&6&-3\\0&15&-9&5\\-1&8&2&7\end{pmatrix}\xrightarrow{\text{line4 }+=\text{ line1}}\begin{pmatrix}1&7&-5&-2\\0&-9&6&-3\\0&15&-9&5\\0&15&-3&5\end{pmatrix}$$

$$\xrightarrow{\text{line3 }-=\text{ line4}}\begin{pmatrix}1&7&-5&-2\\0&-9&6&-3\\0&0&-6&0\\0&15&-3&5\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line4}}\begin{pmatrix}1&7&-5&-2\\0&15&-3&5\\0&0&-6&0\\0&-9&6&-3\end{pmatrix}$$

$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{15}\right)}\begin{pmatrix}1&7&-5&-2\\0&1&-\frac{1}{5}&\frac{1}{3}\\0&0&-6&0\\0&-9&6&-3\end{pmatrix}\xrightarrow{\text{line1 }-=\text{ line2}\times(7)}\begin{pmatrix}1&0&-\frac{18}{5}&-\frac{13}{3}\\0&1&-\frac{1}{5}&\frac{1}{3}\\0&0&-6&0\\0&-9&6&-3\end{pmatrix}$$

$$\xrightarrow{\text{line4 }+=\text{ line2}\times(9)}\begin{pmatrix}1&0&-\frac{18}{5}&-\frac{13}{3}\\0&1&-\frac{1}{5}&\frac{1}{3}\\0&0&-6&0\\0&0&\frac{21}{5}&0\end{pmatrix}\xrightarrow{\text{line3 }\times=\left(-\frac{1}{6}\right)}\begin{pmatrix}1&0&-\frac{18}{5}&-\frac{13}{3}\\0&1&-\frac{1}{5}&\frac{1}{3}\\0&0&1&0\\0&0&\frac{21}{5}&0\end{pmatrix}$$

$$\xrightarrow{\text{line1 }+=\text{ line3}\times\left(\frac{18}{5}\right)}\begin{pmatrix}1&0&0&-\frac{13}{3}\\0&1&-\frac{1}{5}&\frac{1}{3}\\0&0&1&0\\0&0&\frac{21}{5}&0\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line3}\times\left(\frac{1}{5}\right)}\begin{pmatrix}1&0&0&-\frac{13}{3}\\0&1&0&\frac{1}{3}\\0&0&1&0\\0&0&\frac{21}{5}&0\end{pmatrix}\xrightarrow{\text{line4 }-=\text{ line3}\times\left(\frac{21}{5}\right)}\begin{pmatrix}1&0&0&-\frac{13}{3}\\0&1&0&\frac{1}{3}\\0&0&1&0\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix} 0 & -2 & 4 & -6 \\ -5 & 7 & -9 & -5 \\ -2 & 5 & -8 & 7 \\ 4 & -6 & 8 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(3)}} \begin{pmatrix} 0 & -2 & 4 & -6 \\ 1 & -8 & 15 & -26 \\ -2 & 5 & -8 & 7 \\ 4 & -6 & 8 & 1 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & -8 & 15 & -26 \\ 0 & -2 & 4 & -6 \\ -2 & 5 & -8 & 7 \\ 4 & -6 & 8 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -8 & 15 & -26 \\ 0 & -2 & 4 & -6 \\ 0 & -11 & 22 & -45 \\ 4 & -6 & 8 & 1 \end{pmatrix} \xrightarrow{\text{line4 -= line1×(4)}} \begin{pmatrix} 1 & -8 & 15 & -26 \\ 0 & -2 & 4 & -6 \\ 0 & -11 & 22 & -45 \\ 0 & 26 & -52 & 105 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & -8 & 15 & -26 \\ 0 & 1 & -2 & 3 \\ 0 & -11 & 22 & -45 \\ 0 & 26 & -52 & 105 \end{pmatrix} \xrightarrow{\text{line1 += line2×(8)}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & -11 & 22 & -45 \\ 0 & 26 & -52 & 105 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(11)}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & -12 \\ 0 & 26 & -52 & 105 \end{pmatrix} \xrightarrow{\text{line4 -= line2×(26)}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & -12 \\ 0 & 0 & 0 & 27 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ↔ line4}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 27 \\ 0 & 0 & 0 & -12 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{27}\right)} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -12 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(2)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -12 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(3)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -12 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(12)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & -2 & 6 & -7 \\ 1 & -2 & 5 & 1 \\ -2 & 1 & -4 & 5 \\ 3 & 0 & 3 & -5 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 2 & -2 & 6 & -7 \\ -2 & 1 & -4 & 5 \\ 3 & 0 & 3 & -5 \end{pmatrix} \xrightarrow{\text{line2 -= line1×(2)}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 0 & 2 & -4 & -9 \\ -2 & 1 & -4 & 5 \\ 3 & 0 & 3 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 0 & 2 & -4 & -9 \\ 0 & -3 & 6 & 7 \\ 3 & 0 & 3 & -5 \end{pmatrix} \xrightarrow{\text{line4 -= line1×(3)}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 0 & 2 & -4 & -9 \\ 0 & -3 & 6 & 7 \\ 0 & 6 & -12 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 0 & 2 & -4 & -9 \\ 0 & 1 & -2 & -11 \\ 0 & 6 & -12 & -8 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & -2 & 5 & 1 \\ 0 & 1 & -2 & -11 \\ 0 & 2 & -4 & -9 \\ 0 & 6 & -12 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & 1 & -21 \\ 0 & 1 & -2 & -11 \\ 0 & 2 & -4 & -9 \\ 0 & 6 & -12 & -8 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(2)}} \begin{pmatrix} 1 & 0 & 1 & -21 \\ 0 & 1 & -2 & -11 \\ 0 & 0 & 0 & 13 \\ 0 & 6 & -12 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(6)}} \begin{pmatrix} 1 & 0 & 1 & -21 \\ 0 & 1 & -2 & -11 \\ 0 & 0 & 0 & 13 \\ 0 & 0 & 0 & 58 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{13}\right)} \begin{pmatrix} 1 & 0 & 1 & -21 \\ 0 & 1 & -2 & -11 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 58 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(21)}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -11 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 58 \end{pmatrix} \xrightarrow{\text{line2 += line3×(11)}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 58 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line3×(58)}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 6 & -1 & 0 \\ -1 & -3 & 1 & -3 \\ -1 & -3 & 1 & 0 \\ 1 & 3 & -5 & -8 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 3 & -5 & -8 \\ -1 & -3 & 1 & -3 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 3 & -5 & -8 \\ 0 & 0 & -4 & -11 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & -1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 3 & -5 & -8 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & -4 & -8 \\ 2 & 6 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line1}\times(2)} \begin{pmatrix} 1 & 3 & -5 & -8 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & -4 & -8 \\ 0 & 0 & 9 & 16 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & -5 & -8 \\ 0 & 0 & -4 & -8 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & 9 & 16 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 3 & -5 & -8 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & 9 & 16 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(5)} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -4 & -11 \\ 0 & 0 & 9 & 16 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(4)} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 9 & 16 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{-}= \text{line2}\times(9)} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{\times}= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times(2)} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line3}\times(2)} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line3}\times(2)} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -1 & 3 & -2 \\ -5 & 3 & -5 & 3 \\ -5 & -8 & -5 & 5 \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line3}} \begin{pmatrix} 3 & -1 & 3 & -2 \\ 0 & 11 & 0 & -2 \\ -5 & -8 & -5 & 5 \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -5 & -8 & -5 & 5 \\ 0 & 11 & 0 & -2 \\ 3 & -1 & 3 & -2 \\ 0 & -6 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{\times}= \left(-\frac{1}{5}\right)} \begin{pmatrix} 1 & \frac{8}{5} & 1 & -1 \\ 0 & 11 & 0 & -2 \\ 3 & -1 & 3 & -2 \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & \frac{8}{5} & 1 & -1 \\ 0 & 11 & 0 & -2 \\ 0 & -\frac{29}{5} & 0 & 1 \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & \frac{8}{5} & 1 & -1 \\ 0 & -\frac{29}{5} & 0 & 1 \\ 0 & 11 & 0 & -2 \\ 0 & -6 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{\times}= \left(-\frac{5}{29}\right)} \begin{pmatrix} 1 & \frac{8}{5} & 1 & -1 \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 11 & 0 & -2 \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times\left(\frac{8}{5}\right)} \begin{pmatrix} 1 & 0 & 1 & -\frac{21}{29} \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 11 & 0 & -2 \\ 0 & -6 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(11)} \begin{pmatrix} 1 & 0 & 1 & -\frac{21}{29} \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 0 & 0 & -\frac{3}{29} \\ 0 & -6 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(6)} \begin{pmatrix} 1 & 0 & 1 & -\frac{21}{29} \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 0 & 0 & -\frac{3}{29} \\ 0 & 0 & 0 & -\frac{1}{29} \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{\times}= \left(-\frac{29}{3}\right)} \begin{pmatrix} 1 & 0 & 1 & -\frac{21}{29} \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{29} \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times\left(\frac{21}{29}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{5}{29} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{29} \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{5}{29}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{29} \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line3}\times\left(\frac{1}{29}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & 4 & 4 & -1 \\ -8 & -9 & -9 & 6 \\ -6 & -8 & -8 & -6 \\ 0 & -9 & -9 & -8 \end{pmatrix} \xrightarrow{\text{line2 += line1×(3)}} \begin{pmatrix} 3 & 4 & 4 & -1 \\ 1 & 3 & 3 & 3 \\ -6 & -8 & -8 & -6 \\ 0 & -9 & -9 & -8 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 3 & 3 & 3 \\ 3 & 4 & 4 & -1 \\ -6 & -8 & -8 & -6 \\ 0 & -9 & -9 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line1×(3)}} \begin{pmatrix} 1 & 3 & 3 & 3 \\ 0 & -5 & -5 & -10 \\ -6 & -8 & -8 & -6 \\ 0 & -9 & -9 & -8 \end{pmatrix} \xrightarrow{\text{line3 += line1×(6)}} \begin{pmatrix} 1 & 3 & 3 & 3 \\ 0 & -5 & -5 & -10 \\ 0 & 10 & 10 & 12 \\ 0 & -9 & -9 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{5}\right)} \begin{pmatrix} 1 & 3 & 3 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 10 & 10 & 12 \\ 0 & -9 & -9 & -8 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(3)}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 10 & 10 & 12 \\ 0 & -9 & -9 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line2×(10)}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -8 \\ 0 & -9 & -9 & -8 \end{pmatrix} \xrightarrow{\text{line4 += line2×(9)}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ↔ line4}} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & -8 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{10}\right)} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(3)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -8 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(2)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(8)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 6 & -9 & 8 & 6 \\ 1 & 5 & 2 & 1 \\ -4 & 7 & -5 & -4 \\ 1 & -3 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 5 & 2 & 1 \\ 6 & -9 & 8 & 6 \\ -4 & 7 & -5 & -4 \\ 1 & -3 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line1×(6)}} \begin{pmatrix} 1 & 5 & 2 & 1 \\ 0 & -39 & -4 & 0 \\ -4 & 7 & -5 & -4 \\ 1 & -3 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(4)}} \begin{pmatrix} 1 & 5 & 2 & 1 \\ 0 & -39 & -4 & 0 \\ 0 & 27 & 3 & 0 \\ 1 & -3 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line4 -= line1}} \begin{pmatrix} 1 & 5 & 2 & 1 \\ 0 & -39 & -4 & 0 \\ 0 & 27 & 3 & 0 \\ 0 & -8 & -1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line4×(5)}} \begin{pmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 27 & 3 & 0 \\ 0 & -8 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(5)}} \begin{pmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 27 & 3 & 0 \\ 0 & -8 & -1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line2×(27)}} \begin{pmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -24 & 0 \\ 0 & -8 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line4 += line2×(8)}} \begin{pmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -24 & 0 \\ 0 & 0 & 7 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ↔ line4}} \begin{pmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & -24 & 0 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{7}\right)} \begin{pmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -24 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(3)}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -24 & 0 \end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -24 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(24)}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 8 & -9 & -8 & 4 \\ -6 & 7 & 6 & 2 \\ -3 & 2 & 3 & 3 \\ -4 & 7 & 4 & -5 \end{pmatrix} \xrightarrow{\text{line3 -= line4}} \begin{pmatrix} 8 & -9 & -8 & 4 \\ -6 & 7 & 6 & 2 \\ 1 & -5 & -1 & 8 \\ -4 & 7 & 4 & -5 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 1 & -5 & -1 & 8 \\ -6 & 7 & 6 & 2 \\ 8 & -9 & -8 & 4 \\ -4 & 7 & 4 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1×(6)}} \begin{pmatrix} 1 & -5 & -1 & 8 \\ 0 & -23 & 0 & 50 \\ 8 & -9 & -8 & 4 \\ -4 & 7 & 4 & -5 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(8)}} \begin{pmatrix} 1 & -5 & -1 & 8 \\ 0 & -23 & 0 & 50 \\ 0 & 31 & 0 & -60 \\ -4 & 7 & 4 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1×(4)}} \begin{pmatrix} 1 & -5 & -1 & 8 \\ 0 & -23 & 0 & 50 \\ 0 & 31 & 0 & -60 \\ 0 & -13 & 0 & 27 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(-\frac{1}{23}\right)} \begin{pmatrix} 1 & -5 & -1 & 8 \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 31 & 0 & -60 \\ 0 & -13 & 0 & 27 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2×(5)}} \begin{pmatrix} 1 & 0 & -1 & -\frac{66}{23} \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 31 & 0 & -60 \\ 0 & -13 & 0 & 27 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(31)}} \begin{pmatrix} 1 & 0 & -1 & -\frac{66}{23} \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 0 & 0 & \frac{170}{23} \\ 0 & -13 & 0 & 27 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2×(13)}} \begin{pmatrix} 1 & 0 & -1 & -\frac{66}{23} \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 0 & 0 & \frac{170}{23} \\ 0 & 0 & 0 & -\frac{29}{23} \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{23}{170}\right)} \begin{pmatrix} 1 & 0 & -1 & -\frac{66}{23} \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{29}{23} \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×}\left(\frac{66}{23}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{50}{23} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{29}{23} \end{pmatrix} \xrightarrow{\text{line2 += line3×}\left(\frac{50}{23}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{29}{23} \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×}\left(\frac{29}{23}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 9 & -3 & -5 & -5 \\ 0 & -1 & 6 & 9 \\ -5 & 3 & 4 & 0 \\ -4 & 0 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line4 -= line3}} \begin{pmatrix} 9 & -3 & -5 & -5 \\ 0 & -1 & 6 & 9 \\ -5 & 3 & 4 & 0 \\ 1 & -3 & -2 & 6 \end{pmatrix} \xrightarrow{\text{line1 ↔ line4}} \begin{pmatrix} 1 & -3 & -2 & 6 \\ 0 & -1 & 6 & 9 \\ -5 & 3 & 4 & 0 \\ 9 & -3 & -5 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(5)}} \begin{pmatrix} 1 & -3 & -2 & 6 \\ 0 & -1 & 6 & 9 \\ 0 & -12 & -6 & 30 \\ 9 & -3 & -5 & -5 \end{pmatrix} \xrightarrow{\text{line4 -= line1×(9)}} \begin{pmatrix} 1 & -3 & -2 & 6 \\ 0 & -1 & 6 & 9 \\ 0 & -12 & -6 & 30 \\ 0 & 24 & 13 & -59 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } (-1)} \begin{pmatrix} 1 & -3 & -2 & 6 \\ 0 & 1 & -6 & -9 \\ 0 & -12 & -6 & 30 \\ 0 & 24 & 13 & -59 \end{pmatrix} \xrightarrow{\text{line1 += line2×(3)}} \begin{pmatrix} 1 & 0 & -20 & -21 \\ 0 & 1 & -6 & -9 \\ 0 & -12 & -6 & 30 \\ 0 & 24 & 13 & -59 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(12)}} \begin{pmatrix} 1 & 0 & -20 & -21 \\ 0 & 1 & -6 & -9 \\ 0 & 0 & -78 & -78 \\ 0 & 24 & 13 & -59 \end{pmatrix} \xrightarrow{\text{line4 -= line2×(24)}} \begin{pmatrix} 1 & 0 & -20 & -21 \\ 0 & 1 & -6 & -9 \\ 0 & 0 & -78 & -78 \\ 0 & 0 & 157 & 157 \end{pmatrix}$$

$$\xrightarrow{\text{line3 ↔ line4}} \begin{pmatrix} 1 & 0 & -20 & -21 \\ 0 & 1 & -6 & -9 \\ 0 & 0 & 157 & 157 \\ 0 & 0 & -78 & -78 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{157}\right)} \begin{pmatrix} 1 & 0 & -20 & -21 \\ 0 & 1 & -6 & -9 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -78 & -78 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(20)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -6 & -9 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -78 & -78 \end{pmatrix} \xrightarrow{\text{line2 += line3×(6)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -78 & -78 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(78)}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}-4&-5&-3&-4\\8&9&2&8\\-6&-3&-4&-8\\-5&-6&-3&-5\end{pmatrix}\xrightarrow{\text{line1 -= line4}}\begin{pmatrix}1&1&0&1\\8&9&2&8\\-6&-3&-4&-8\\-5&-6&-3&-5\end{pmatrix}\xrightarrow{\text{line2 -= line1×(8)}}\begin{pmatrix}1&1&0&1\\0&1&2&0\\-6&-3&-4&-8\\-5&-6&-3&-5\end{pmatrix}$$

$$\xrightarrow{\text{line3 += line1×(6)}}\begin{pmatrix}1&1&0&1\\0&1&2&0\\0&3&-4&-2\\-5&-6&-3&-5\end{pmatrix}\xrightarrow{\text{line4 += line1×(5)}}\begin{pmatrix}1&1&0&1\\0&1&2&0\\0&3&-4&-2\\0&-1&-3&0\end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}}\begin{pmatrix}1&0&-2&1\\0&1&2&0\\0&3&-4&-2\\0&-1&-3&0\end{pmatrix}\xrightarrow{\text{line3 -= line2×(3)}}\begin{pmatrix}1&0&-2&1\\0&1&2&0\\0&0&-10&-2\\0&-1&-3&0\end{pmatrix}$$

$$\xrightarrow{\text{line4 += line2}}\begin{pmatrix}1&0&-2&1\\0&1&2&0\\0&0&-10&-2\\0&0&-1&0\end{pmatrix}\xrightarrow{\text{line3 ↔ line4}}\begin{pmatrix}1&0&-2&1\\0&1&2&0\\0&0&-1&0\\0&0&-10&-2\end{pmatrix}$$

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$$\xrightarrow{\text{line3 ×= (−1)}}\begin{pmatrix}1&0&-2&1\\0&1&2&0\\0&0&1&0\\0&0&-10&-2\end{pmatrix}\xrightarrow{\text{line1 += line3×(2)}}\begin{pmatrix}1&0&0&1\\0&1&2&0\\0&0&1&0\\0&0&-10&-2\end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line3×(2)}}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&-10&-2\end{pmatrix}\xrightarrow{\text{line4 += line3×(10)}}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&-2\end{pmatrix}$$

$$\xrightarrow{\text{line4 ×= }\left(-\frac{1}{2}\right)}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 -= line4}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$\begin{pmatrix}-2&-2&4&5\\-4&-1&5&7\\-7&7&-9&-1\\2&-1&1&-1\end{pmatrix}\xrightarrow{\text{line1 += line4}}\begin{pmatrix}0&-3&5&4\\-4&-1&5&7\\-7&7&-9&-1\\2&-1&1&-1\end{pmatrix}\xrightarrow{\text{line3 -= line2×(2)}}\begin{pmatrix}0&-3&5&4\\-4&-1&5&7\\1&9&-19&-15\\2&-1&1&-1\end{pmatrix}$$

$$\xrightarrow{\text{line1 ↔ line3}}\begin{pmatrix}1&9&-19&-15\\-4&-1&5&7\\0&-3&5&4\\2&-1&1&-1\end{pmatrix}\xrightarrow{\text{line2 += line1×(4)}}\begin{pmatrix}1&9&-19&-15\\0&35&-71&-53\\0&-3&5&4\\2&-1&1&-1\end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1×(2)}}\begin{pmatrix}1&9&-19&-15\\0&35&-71&-53\\0&-3&5&4\\0&-19&39&29\end{pmatrix}\xrightarrow{\text{line2 += line3×(12)}}\begin{pmatrix}1&9&-19&-15\\0&-1&-11&-5\\0&-3&5&4\\0&-19&39&29\end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= (−1)}}\begin{pmatrix}1&9&-19&-15\\0&1&11&5\\0&-3&5&4\\0&-19&39&29\end{pmatrix}\xrightarrow{\text{line1 -= line2×(9)}}\begin{pmatrix}1&0&-118&-60\\0&1&11&5\\0&-3&5&4\\0&-19&39&29\end{pmatrix}$$

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$$\xrightarrow{\text{line3 += line2×(3)}}\begin{pmatrix}1&0&-118&-60\\0&1&11&5\\0&0&38&19\\0&-19&39&29\end{pmatrix}\xrightarrow{\text{line4 += line2×(19)}}\begin{pmatrix}1&0&-118&-60\\0&1&11&5\\0&0&38&19\\0&0&248&124\end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= }\left(\frac{1}{38}\right)}\begin{pmatrix}1&0&-118&-60\\0&1&11&5\\0&0&1&\frac{1}{2}\\0&0&248&124\end{pmatrix}\xrightarrow{\text{line1 += line3×(118)}}\begin{pmatrix}1&0&0&-1\\0&1&11&5\\0&0&1&\frac{1}{2}\\0&0&248&124\end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line3×(11)}}\begin{pmatrix}1&0&0&-1\\0&1&0&-\frac{1}{2}\\0&0&1&\frac{1}{2}\\0&0&248&124\end{pmatrix}\xrightarrow{\text{line4 -= line3×(248)}}\begin{pmatrix}1&0&0&-1\\0&1&0&-\frac{1}{2}\\0&0&1&\frac{1}{2}\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix} -2 & 2 & -3 & -9 \\ 9 & -9 & -2 & -1 \\ -2 & 2 & -3 & -3 \\ 2 & -2 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix} 0 & 0 & 0 & -6 \\ 9 & -9 & -2 & -1 \\ -2 & 2 & -3 & -3 \\ 2 & -2 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line3 += line4}} \begin{pmatrix} 0 & 0 & 0 & -6 \\ 9 & -9 & -2 & -1 \\ 0 & 0 & -2 & -1 \\ 2 & -2 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line4}} \begin{pmatrix} 2 & -2 & 1 & 2 \\ 9 & -9 & -2 & -1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & -6 \end{pmatrix} \xrightarrow{\text{line1 } \times = \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & \frac{1}{2} & 1 \\ 9 & -9 & -2 & -1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line1}\times(9)} \begin{pmatrix} 1 & -1 & \frac{1}{2} & 1 \\ 0 & 0 & -\frac{13}{2} & -10 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & -6 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -1 & \frac{1}{2} & 1 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -\frac{13}{2} & -10 \\ 0 & 0 & 0 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{13}{2} & -10 \\ 0 & 0 & 0 & -6 \end{pmatrix} \xrightarrow{\text{line1 -= line2}\times\left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & -\frac{13}{2} & -10 \\ 0 & 0 & 0 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2}\times\left(\frac{13}{2}\right)} \begin{pmatrix} 1 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{27}{4} \\ 0 & 0 & 0 & -6 \end{pmatrix} \xrightarrow{\text{line3 } \leftrightarrow \text{ line4}} \begin{pmatrix} 1 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -\frac{27}{4} \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times = \left(-\frac{1}{6}\right)} \begin{pmatrix} 1 & -1 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{27}{4} \end{pmatrix} \xrightarrow{\text{line1 -= line3}\times\left(\frac{3}{4}\right)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{27}{4} \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line3}\times\left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{27}{4} \end{pmatrix} \xrightarrow{\text{line4 += line3}\times\left(\frac{27}{4}\right)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & 2 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 4 \\ -3 & 4 & 1 & 5 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -2 & 2 & -1 & -1 \\ 0 & 1 & -1 & 4 \\ -3 & 4 & 1 & 5 \end{pmatrix} \xrightarrow{\text{line2 += line1}\times(2)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & 4 \\ -3 & 4 & 1 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line1}\times(3)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & 4 \\ 0 & 1 & 1 & 5 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2}} \begin{pmatrix} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 5 \end{pmatrix} \xrightarrow{\text{line4 -= line2}} \begin{pmatrix} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times = (-1)} \begin{pmatrix} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix} \xrightarrow{\text{line1 += line3}} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix} \xrightarrow{\text{line4 -= line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } \times = (-1)} \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line4}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line4}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3 -= line4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & 4 & 8 & 3 \\ -3 & 2 & 8 & -1 \\ -1 & -3 & -1 & -6 \\ -1 & 5 & 7 & 6 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line4}} \begin{pmatrix} -1 & 5 & 7 & 6 \\ -3 & 2 & 8 & -1 \\ -1 & -3 & -1 & -6 \\ -2 & 4 & 8 & 3 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -5 & -7 & -6 \\ -3 & 2 & 8 & -1 \\ -1 & -3 & -1 & -6 \\ -2 & 4 & 8 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(3)} \begin{pmatrix} 1 & -5 & -7 & -6 \\ 0 & -13 & -13 & -19 \\ -1 & -3 & -1 & -6 \\ -2 & 4 & 8 & 3 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & -5 & -7 & -6 \\ 0 & -13 & -13 & -19 \\ 0 & -8 & -8 & -12 \\ -2 & 4 & 8 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -5 & -7 & -6 \\ 0 & -13 & -13 & -19 \\ 0 & -8 & -8 & -12 \\ 0 & -6 & -6 & -9 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line4}\times(2)} \begin{pmatrix} 1 & -5 & -7 & -6 \\ 0 & -1 & -1 & -1 \\ 0 & -8 & -8 & -12 \\ 0 & -6 & -6 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & -5 & -7 & -6 \\ 0 & 1 & 1 & 1 \\ 0 & -8 & -8 & -12 \\ 0 & -6 & -6 & -9 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & -8 & -8 & -12 \\ 0 & -6 & -6 & -9 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(8)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 \\ 0 & -6 & -6 & -9 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(6)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 & 0 & 6 \\ -6 & -7 & -9 & 4 \\ -5 & -5 & -4 & -7 \\ 4 & 2 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(6)} \begin{pmatrix} 1 & 1 & 0 & 6 \\ 0 & -1 & -9 & 40 \\ -5 & -5 & -4 & -7 \\ 4 & 2 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(5)} \begin{pmatrix} 1 & 1 & 0 & 6 \\ 0 & -1 & -9 & 40 \\ 0 & 0 & -4 & 23 \\ 4 & 2 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{-}= \text{line1}\times(4)} \begin{pmatrix} 1 & 1 & 0 & 6 \\ 0 & -1 & -9 & 40 \\ 0 & 0 & -4 & 23 \\ 0 & -2 & 0 & -22 \end{pmatrix} \xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & 1 & 0 & 6 \\ 0 & 1 & 9 & -40 \\ 0 & 0 & -4 & 23 \\ 0 & -2 & 0 & -22 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -9 & 46 \\ 0 & 1 & 9 & -40 \\ 0 & 0 & -4 & 23 \\ 0 & -2 & 0 & -22 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & -9 & 46 \\ 0 & 1 & 9 & -40 \\ 0 & 0 & -4 & 23 \\ 0 & 0 & 18 & -102 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 0 & -9 & 46 \\ 0 & 1 & 9 & -40 \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 18 & -102 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(9)} \begin{pmatrix} 1 & 0 & 0 & -\frac{23}{4} \\ 0 & 1 & 9 & -40 \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 18 & -102 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}\times(9)} \begin{pmatrix} 1 & 0 & 0 & -\frac{23}{4} \\ 0 & 1 & 0 & \frac{47}{4} \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 18 & -102 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line3}\times(18)} \begin{pmatrix} 1 & 0 & 0 & -\frac{23}{4} \\ 0 & 1 & 0 & \frac{47}{4} \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 0 & \frac{3}{2} \end{pmatrix}$$

$$\xrightarrow{\text{line4} \times= \left(\frac{2}{3}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{23}{4} \\ 0 & 1 & 0 & \frac{47}{4} \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line4}\times\left(\frac{23}{4}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{47}{4} \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{-}= \text{line4}\times\left(\frac{47}{4}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{23}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line4}\times\left(\frac{23}{4}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 2 & 4 & 1 \\ 3 & 2 & 0 & 1 \\ -5 & -5 & -8 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(3)} \begin{pmatrix} 1 & 2 & 4 & 1 \\ 0 & -4 & -12 & -2 \\ -5 & -5 & -8 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(5)} \begin{pmatrix} 1 & 2 & 4 & 1 \\ 0 & -4 & -12 & -2 \\ 0 & 5 & 12 & 4 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line1}} \begin{pmatrix} 1 & 2 & 4 & 1 \\ 0 & -4 & -12 & -2 \\ 0 & 5 & 12 & 4 \\ 0 & -1 & -3 & -1 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & 2 & 4 & 1 \\ 0 & -1 & -3 & -1 \\ 0 & 5 & 12 & 4 \\ 0 & -4 & -12 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & 2 & 4 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 5 & 12 & 4 \\ 0 & -4 & -12 & -2 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 5 & 12 & 4 \\ 0 & -4 & -12 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line2}\times(5)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -1 \\ 0 & -4 & -12 & -2 \end{pmatrix} \xrightarrow{\text{line4 } += \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } -= \text{ line3}\times(3)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{\text{line4 } \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line4}\times\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line4}\times\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -3 & -2 & -1 & 8 \\ 2 & 6 & -2 & -9 \\ 4 & -9 & 6 & -1 \\ 2 & 1 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line3}} \begin{pmatrix} 1 & -11 & 5 & 7 \\ 2 & 6 & -2 & -9 \\ 4 & -9 & 6 & -1 \\ 2 & 1 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & -11 & 5 & 7 \\ 0 & 28 & -12 & -23 \\ 4 & -9 & 6 & -1 \\ 2 & 1 & 1 & -5 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line1}\times(4)} \begin{pmatrix} 1 & -11 & 5 & 7 \\ 0 & 28 & -12 & -23 \\ 0 & 35 & -14 & -29 \\ 2 & 1 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & -11 & 5 & 7 \\ 0 & 28 & -12 & -23 \\ 0 & 35 & -14 & -29 \\ 0 & 23 & -9 & -19 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{28}\right)} \begin{pmatrix} 1 & -11 & 5 & 7 \\ 0 & 1 & -\frac{3}{7} & -\frac{23}{28} \\ 0 & 35 & -14 & -29 \\ 0 & 23 & -9 & -19 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}\times(11)} \begin{pmatrix} 1 & 0 & \frac{2}{7} & -\frac{57}{28} \\ 0 & 1 & -\frac{3}{7} & -\frac{23}{28} \\ 0 & 35 & -14 & -29 \\ 0 & 23 & -9 & -19 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } -= \text{ line2}\times(35)} \begin{pmatrix} 1 & 0 & \frac{2}{7} & -\frac{57}{28} \\ 0 & 1 & -\frac{3}{7} & -\frac{23}{28} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 23 & -9 & -19 \end{pmatrix} \xrightarrow{\text{line4 } -= \text{ line2}\times(23)} \begin{pmatrix} 1 & 0 & \frac{2}{7} & -\frac{57}{28} \\ 0 & 1 & -\frac{3}{7} & -\frac{23}{28} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & \frac{6}{7} & -\frac{3}{28} \end{pmatrix}$$

$$\xrightarrow{\text{line1 } -= \text{ line3}\times\left(\frac{2}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{55}{28} \\ 0 & 1 & -\frac{3}{7} & -\frac{23}{28} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & \frac{6}{7} & -\frac{3}{28} \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{3}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{55}{28} \\ 0 & 1 & 0 & -\frac{13}{14} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & \frac{6}{7} & -\frac{3}{28} \end{pmatrix}$$

$$\xrightarrow{\text{line4 } -= \text{ line3}\times\left(\frac{6}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{55}{28} \\ 0 & 1 & 0 & -\frac{13}{14} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{3}{28} \end{pmatrix} \xrightarrow{\text{line4 } \times= \left(\frac{28}{3}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{55}{28} \\ 0 & 1 & 0 & -\frac{13}{14} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line4}\times\left(\frac{55}{28}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{13}{14} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line4}\times\left(\frac{13}{14}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line4}\times\left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix}0&-2&-2&0\\6&4&-4&5\\5&7&1&4\\6&6&-7&6\end{pmatrix}\xrightarrow{\text{line2 -= line3}}\begin{pmatrix}0&-2&-2&0\\1&-3&-5&1\\5&7&1&4\\6&6&-7&6\end{pmatrix}\xrightarrow{\text{line1 <-> line2}}\begin{pmatrix}1&-3&-5&1\\0&-2&-2&0\\5&7&1&4\\6&6&-7&6\end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line1×(5)}}\begin{pmatrix}1&-3&-5&1\\0&-2&-2&0\\0&22&26&-1\\6&6&-7&6\end{pmatrix}\xrightarrow{\text{line4 -= line1×(6)}}\begin{pmatrix}1&-3&-5&1\\0&-2&-2&0\\0&22&26&-1\\0&24&23&0\end{pmatrix}$$

$$\xrightarrow{\text{line2 *= }\left(-\frac{1}{2}\right)}\begin{pmatrix}1&-3&-5&1\\0&1&1&0\\0&22&26&-1\\0&24&23&0\end{pmatrix}\xrightarrow{\text{line1 += line2×(3)}}\begin{pmatrix}1&0&-2&1\\0&1&1&0\\0&22&26&-1\\0&24&23&0\end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line2×(22)}}\begin{pmatrix}1&0&-2&1\\0&1&1&0\\0&0&4&-1\\0&24&23&0\end{pmatrix}\xrightarrow{\text{line4 -= line2×(24)}}\begin{pmatrix}1&0&-2&1\\0&1&1&0\\0&0&4&-1\\0&0&-1&0\end{pmatrix}$$

$$\xrightarrow{\text{line3 <-> line4}}\begin{pmatrix}1&0&-2&1\\0&1&1&0\\0&0&-1&0\\0&0&4&-1\end{pmatrix}\xrightarrow{\text{line3 *= (-1)}}\begin{pmatrix}1&0&-2&1\\0&1&1&0\\0&0&1&0\\0&0&4&-1\end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(2)}}\begin{pmatrix}1&0&0&1\\0&1&1&0\\0&0&1&0\\0&0&4&-1\end{pmatrix}\xrightarrow{\text{line2 -= line3}}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&4&-1\end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line3×(4)}}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\xrightarrow{\text{line4 *= (-1)}}\begin{pmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line4}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix} 1 & -2 & 3 & 1 \\ -9 & -8 & 1 & -2 \\ 1 & -5 & 6 & 2 \\ 2 & 3 & -2 & 0 \end{pmatrix} \xrightarrow{\text{line2 += line1×(9)}} \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -26 & 28 & 7 \\ 1 & -5 & 6 & 2 \\ 2 & 3 & -2 & 0 \end{pmatrix} \xrightarrow{\text{line3 -= line1}} \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -26 & 28 & 7 \\ 0 & -3 & 3 & 1 \\ 2 & 3 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line1×(2)}} \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -26 & 28 & 7 \\ 0 & -3 & 3 & 1 \\ 0 & 7 & -8 & -2 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(9)}} \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & -3 & 3 & 1 \\ 0 & 7 & -8 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & 5 & -3 \\ 0 & 1 & 1 & -2 \\ 0 & -3 & 3 & 1 \\ 0 & 7 & -8 & -2 \end{pmatrix} \xrightarrow{\text{line3 += line2×(3)}} \begin{pmatrix} 1 & 0 & 5 & -3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 6 & -5 \\ 0 & 7 & -8 & -2 \end{pmatrix}$$

$$\xrightarrow{\text{line4 -= line2×(7)}} \begin{pmatrix} 1 & 0 & 5 & -3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -15 & 12 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 0 & 5 & -3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & -15 & 12 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line3×(5)}} \begin{pmatrix} 1 & 0 & 0 & \frac{7}{6} \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & -15 & 12 \end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 1 & 0 & 0 & \frac{7}{6} \\ 0 & 1 & 0 & -\frac{7}{6} \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & -15 & 12 \end{pmatrix}$$

$$\xrightarrow{\text{line4 += line3×(15)}} \begin{pmatrix} 1 & 0 & 0 & \frac{7}{6} \\ 0 & 1 & 0 & -\frac{7}{6} \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \xrightarrow{\text{line4 ×= (−2)}} \begin{pmatrix} 1 & 0 & 0 & \frac{7}{6} \\ 0 & 1 & 0 & -\frac{7}{6} \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line4×}\left(\frac{7}{6}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{7}{6} \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 += line4×}\left(\frac{7}{6}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{5}{6} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line4×}\left(\frac{5}{6}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -6 & -1 & 2 & 6 \\ 2 & 1 & 4 & -3 \\ -1 & 1 & 8 & -1 \\ 0 & -2 & -9 & 3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -1 & 1 & 8 & -1 \\ 2 & 1 & 4 & -3 \\ -6 & -1 & 2 & 6 \\ 0 & -2 & -9 & 3 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -1 & -8 & 1 \\ 2 & 1 & 4 & -3 \\ -6 & -1 & 2 & 6 \\ 0 & -2 & -9 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & -8 & 1 \\ 0 & 3 & 20 & -5 \\ -6 & -1 & 2 & 6 \\ 0 & -2 & -9 & 3 \end{pmatrix} \xrightarrow{\text{line3} += \text{line1} \times (6)} \begin{pmatrix} 1 & -1 & -8 & 1 \\ 0 & 3 & 20 & -5 \\ 0 & -7 & -46 & 12 \\ 0 & -2 & -9 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line2} += \text{line4}} \begin{pmatrix} 1 & -1 & -8 & 1 \\ 0 & 1 & 11 & -2 \\ 0 & -7 & -46 & 12 \\ 0 & -2 & -9 & 3 \end{pmatrix} \xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 11 & -2 \\ 0 & -7 & -46 & 12 \\ 0 & -2 & -9 & 3 \end{pmatrix}$$

$$\xrightarrow{\text{line3} += \text{line2} \times (7)} \begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 11 & -2 \\ 0 & 0 & 31 & -2 \\ 0 & -2 & -9 & 3 \end{pmatrix} \xrightarrow{\text{line4} += \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 11 & -2 \\ 0 & 0 & 31 & -2 \\ 0 & 0 & 13 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(\frac{1}{31}\right)} \begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 11 & -2 \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 13 & -1 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line3} \times (3)} \begin{pmatrix} 1 & 0 & 0 & -\frac{25}{31} \\ 0 & 1 & 11 & -2 \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 13 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} -= \text{line3} \times (11)} \begin{pmatrix} 1 & 0 & 0 & -\frac{25}{31} \\ 0 & 1 & 0 & -\frac{40}{31} \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 13 & -1 \end{pmatrix} \xrightarrow{\text{line4} -= \text{line3} \times (13)} \begin{pmatrix} 1 & 0 & 0 & -\frac{25}{31} \\ 0 & 1 & 0 & -\frac{40}{31} \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 0 & -\frac{5}{31} \end{pmatrix}$$

$$\xrightarrow{\text{line4} \times= \left(-\frac{31}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{25}{31} \\ 0 & 1 & 0 & -\frac{40}{31} \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} += \text{line4} \times \left(\frac{25}{31}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{40}{31} \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} += \text{line4} \times \left(\frac{40}{31}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{2}{31} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3} += \text{line4} \times \left(\frac{2}{31}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -3 & 4 & 0 & -3 \\ -6 & 6 & -1 & -6 \\ 4 & 9 & 9 & 1 \\ -8 & -3 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line1 += line3}} \begin{pmatrix} 1 & 13 & 9 & -2 \\ -6 & 6 & -1 & -6 \\ 4 & 9 & 9 & 1 \\ -8 & -3 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line2 += line1×(6)}} \begin{pmatrix} 1 & 13 & 9 & -2 \\ 0 & 84 & 53 & -18 \\ 4 & 9 & 9 & 1 \\ -8 & -3 & -8 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line3 -= line1×(4)}} \begin{pmatrix} 1 & 13 & 9 & -2 \\ 0 & 84 & 53 & -18 \\ 0 & -43 & -27 & 9 \\ -8 & -3 & -8 & -6 \end{pmatrix} \xrightarrow{\text{line4 += line1×(8)}} \begin{pmatrix} 1 & 13 & 9 & -2 \\ 0 & 84 & 53 & -18 \\ 0 & -43 & -27 & 9 \\ 0 & 101 & 64 & -22 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } \left(\frac{1}{84}\right)} \begin{pmatrix} 1 & 13 & 9 & -2 \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & -43 & -27 & 9 \\ 0 & 101 & 64 & -22 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(13)}} \begin{pmatrix} 1 & 0 & \frac{67}{84} & \frac{11}{14} \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & -43 & -27 & 9 \\ 0 & 101 & 64 & -22 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(43)}} \begin{pmatrix} 1 & 0 & \frac{67}{84} & \frac{11}{14} \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & 0 & \frac{11}{84} & -\frac{3}{14} \\ 0 & 101 & 64 & -22 \end{pmatrix} \xrightarrow{\text{line4 -= line2×(101)}} \begin{pmatrix} 1 & 0 & \frac{67}{84} & \frac{11}{14} \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & 0 & \frac{11}{84} & -\frac{3}{14} \\ 0 & 0 & \frac{23}{84} & -\frac{5}{14} \end{pmatrix}$$

$$\xrightarrow{\text{line3 ×= } \left(\frac{84}{11}\right)} \begin{pmatrix} 1 & 0 & \frac{67}{84} & \frac{11}{14} \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & \frac{23}{84} & -\frac{5}{14} \end{pmatrix} \xrightarrow{\text{line1 -= line3×}\left(\frac{67}{84}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{23}{11} \\ 0 & 1 & \frac{53}{84} & -\frac{3}{14} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & \frac{23}{84} & -\frac{5}{14} \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line3×}\left(\frac{53}{84}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{23}{11} \\ 0 & 1 & 0 & \frac{9}{11} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & \frac{23}{84} & -\frac{5}{14} \end{pmatrix} \xrightarrow{\text{line4 -= line3×}\left(\frac{23}{84}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{23}{11} \\ 0 & 1 & 0 & \frac{9}{11} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & 0 & \frac{1}{11} \end{pmatrix}$$

$$\xrightarrow{\text{line4 ×= (11)}} \begin{pmatrix} 1 & 0 & 0 & \frac{23}{11} \\ 0 & 1 & 0 & \frac{9}{11} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line4×}\left(\frac{23}{11}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{9}{11} \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 -= line4×}\left(\frac{9}{11}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{18}{11} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3 += line4×}\left(\frac{18}{11}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -3 & 6 & 7 & 6 \\ -2 & 3 & 5 & -1 \\ 1 & -3 & -2 & -8 \\ 3 & -4 & -7 & 4 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -3 & -2 & -8 \\ -2 & 3 & 5 & -1 \\ -3 & 6 & 7 & 6 \\ 3 & -4 & -7 & 4 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & -3 & 1 & -17 \\ -3 & 6 & 7 & 6 \\ 3 & -4 & -7 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(3)} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & -3 & 1 & -17 \\ 0 & -3 & 1 & -18 \\ 3 & -4 & -7 & 4 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & -3 & 1 & -17 \\ 0 & -3 & 1 & -18 \\ 0 & 5 & -1 & 28 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & 0 & 0 & 1 \\ 0 & -3 & 1 & -18 \\ 0 & 5 & -1 & 28 \end{pmatrix} \xrightarrow{\text{line4} \mathrel{+}= \text{line3}\times(2)} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & 0 & 0 & 1 \\ 0 & -3 & 1 & -18 \\ 0 & -1 & 1 & -8 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line4}} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & -1 & 1 & -8 \\ 0 & -3 & 1 & -18 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & -3 & -2 & -8 \\ 0 & 1 & -1 & 8 \\ 0 & -3 & 1 & -18 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & -5 & 16 \\ 0 & 1 & -1 & 8 \\ 0 & -3 & 1 & -18 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & -5 & 16 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & -5 & 16 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 8 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{-}= \text{line4}\times(5)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line4}\times(3)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix}-2&-7&1&4\\1&-4&-1&9\\1&0&-2&2\\-2&-5&4&4\end{pmatrix}\xrightarrow{\text{line1} \leftrightarrow \text{line2}}\begin{pmatrix}1&-4&-1&9\\-2&-7&1&4\\1&0&-2&2\\-2&-5&4&4\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&-4&-1&9\\0&-15&-1&22\\1&0&-2&2\\-2&-5&4&4\end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}}\begin{pmatrix}1&-4&-1&9\\0&-15&-1&22\\0&4&-1&-7\\-2&-5&4&4\end{pmatrix}\xrightarrow{\text{line4} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&-4&-1&9\\0&-15&-1&22\\0&4&-1&-7\\0&-13&2&22\end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times(4)}\begin{pmatrix}1&-4&-1&9\\0&1&-5&-6\\0&4&-1&-7\\0&-13&2&22\end{pmatrix}\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(4)}\begin{pmatrix}1&0&-21&-15\\0&1&-5&-6\\0&4&-1&-7\\0&-13&2&22\end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(4)}\begin{pmatrix}1&0&-21&-15\\0&1&-5&-6\\0&0&19&17\\0&-13&2&22\end{pmatrix}\xrightarrow{\text{line4} \mathrel{+}= \text{line2}\times(13)}\begin{pmatrix}1&0&-21&-15\\0&1&-5&-6\\0&0&19&17\\0&0&-63&-56\end{pmatrix}$$

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$$\xrightarrow{\text{line3} \mathrel{\times}= \left(\frac{1}{19}\right)}\begin{pmatrix}1&0&-21&-15\\0&1&-5&-6\\0&0&1&\frac{17}{19}\\0&0&-63&-56\end{pmatrix}\xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(21)}\begin{pmatrix}1&0&0&\frac{72}{19}\\0&1&-5&-6\\0&0&1&\frac{17}{19}\\0&0&-63&-56\end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times(5)}\begin{pmatrix}1&0&0&\frac{72}{19}\\0&1&0&-\frac{29}{19}\\0&0&1&\frac{17}{19}\\0&0&-63&-56\end{pmatrix}\xrightarrow{\text{line4} \mathrel{+}= \text{line3}\times(63)}\begin{pmatrix}1&0&0&\frac{72}{19}\\0&1&0&-\frac{29}{19}\\0&0&1&\frac{17}{19}\\0&0&0&\frac{7}{19}\end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{\times}= \left(\frac{19}{7}\right)}\begin{pmatrix}1&0&0&\frac{72}{19}\\0&1&0&-\frac{29}{19}\\0&0&1&\frac{17}{19}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1} \mathrel{-}= \text{line4}\times\left(\frac{72}{19}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&-\frac{29}{19}\\0&0&1&\frac{17}{19}\\0&0&0&1\end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line4}\times\left(\frac{29}{19}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&\frac{17}{19}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line3} \mathrel{-}= \text{line4}\times\left(\frac{17}{19}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$\begin{pmatrix}-1&-1&8&-2\\-1&-2&2&-3\\3&4&-9&9\\-1&-2&-4&-4\end{pmatrix}\xrightarrow{\text{line1} \mathrel{\times}= (-1)}\begin{pmatrix}1&1&-8&2\\-1&-2&2&-3\\3&4&-9&9\\-1&-2&-4&-4\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line1}}\begin{pmatrix}1&1&-8&2\\0&-1&-6&-1\\3&4&-9&9\\-1&-2&-4&-4\end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(3)}\begin{pmatrix}1&1&-8&2\\0&-1&-6&-1\\0&1&15&3\\-1&-2&-4&-4\end{pmatrix}\xrightarrow{\text{line4} \mathrel{+}= \text{line1}}\begin{pmatrix}1&1&-8&2\\0&-1&-6&-1\\0&1&15&3\\0&-1&-12&-2\end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}}\begin{pmatrix}1&1&-8&2\\0&1&15&3\\0&-1&-6&-1\\0&-1&-12&-2\end{pmatrix}\xrightarrow{\text{line1} \mathrel{-}= \text{line2}}\begin{pmatrix}1&0&-23&-1\\0&1&15&3\\0&-1&-6&-1\\0&-1&-12&-2\end{pmatrix}$$

(100)

$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}}\begin{pmatrix}1&0&-23&-1\\0&1&15&3\\0&0&9&2\\0&-1&-12&-2\end{pmatrix}\xrightarrow{\text{line4} \mathrel{+}= \text{line2}}\begin{pmatrix}1&0&-23&-1\\0&1&15&3\\0&0&9&2\\0&0&3&1\end{pmatrix}$$

$$\xrightarrow{\text{line3} \mathrel{\times}= \left(\frac{1}{9}\right)}\begin{pmatrix}1&0&-23&-1\\0&1&15&3\\0&0&1&\frac{2}{9}\\0&0&3&1\end{pmatrix}\xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(23)}\begin{pmatrix}1&0&0&\frac{37}{9}\\0&1&15&3\\0&0&1&\frac{2}{9}\\0&0&3&1\end{pmatrix}\xrightarrow{\text{line2} \mathrel{-}= \text{line3}\times(15)}\begin{pmatrix}1&0&0&\frac{37}{9}\\0&1&0&-\frac{1}{3}\\0&0&1&\frac{2}{9}\\0&0&3&1\end{pmatrix}$$

$$\xrightarrow{\text{line4} \mathrel{-}= \text{line3}\times(3)}\begin{pmatrix}1&0&0&\frac{37}{9}\\0&1&0&-\frac{1}{3}\\0&0&1&\frac{2}{9}\\0&0&0&\frac{1}{3}\end{pmatrix}\xrightarrow{\text{line4} \mathrel{\times}= (3)}\begin{pmatrix}1&0&0&\frac{37}{9}\\0&1&0&-\frac{1}{3}\\0&0&1&\frac{2}{9}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1} \mathrel{-}= \text{line4}\times\left(\frac{37}{9}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&-\frac{1}{3}\\0&0&1&\frac{2}{9}\\0&0&0&1\end{pmatrix}$$

$$\xrightarrow{\text{line2} \mathrel{+}= \text{line4}\times\left(\frac{1}{3}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&\frac{2}{9}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line3} \mathrel{-}= \text{line4}\times\left(\frac{2}{9}\right)}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$