(1)

$$\begin{pmatrix} -3 & -2 & 9 & 6 \\ 5 & 1 & -1 & -3 \\ 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 5 & 1 & -1 & -3 \\ -3 & -2 & 9 & 6 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -6 & -3 \\ -3 & -2 & 9 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } += \text{ line1}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & -2 & 12 & 6 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(2)

$$\begin{pmatrix} 0 & 1 & 1 & -1 \\ -3 & 3 & 9 & -9 \\ 1 & -1 & -3 & 3 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -1 & -3 & 3 \\ -3 & 3 & 9 & -9 \\ 0 & 1 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(3)} \begin{pmatrix} 1 & -1 & -3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -1 & -3 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}} \begin{pmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(3)

$$\begin{pmatrix} 0 & 1 & 3 & -2 \\ 1 & 2 & 8 & -3 \\ 1 & 1 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & 2 & 8 & -3 \\ 0 & 1 & 3 & -2 \\ 1 & 1 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line3 } -= \text{ line1}} \begin{pmatrix} 1 & 2 & 8 & -3 \\ 0 & 1 & 3 & -2 \\ 0 & -1 & -3 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } -= \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & -1 & -3 & 2 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}} \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(4)

$$\begin{pmatrix} 0 & 2 & 0 & -2 \\ 1 & 7 & -5 & -5 \\ 0 & 2 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & 7 & -5 & -5 \\ 0 & 2 & 0 & -2 \\ 0 & 2 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 7 & -5 & -5 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & 0 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } -= \text{ line2}\times(7)} \begin{pmatrix} 1 & 0 & -5 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line3 } -= \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & -5 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(5)

$$\begin{pmatrix} 1 & 3 & 3 & 6 \\ -1 & -3 & -3 & -5 \\ 2 & 6 & 6 & -5 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}} \begin{pmatrix} 1 & 3 & 3 & 6 \\ 0 & 0 & 0 & 1 \\ 2 & 6 & 6 & -5 \end{pmatrix} \xrightarrow{\text{line3 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & 3 & 3 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -17 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } -= \text{ line2}\times(6)} \begin{pmatrix} 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -17 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line2}\times(17)} \begin{pmatrix} 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(6)

$$\begin{pmatrix} -6 & -6 & 5 & -5 \\ -8 & -8 & 8 & -8 \\ 1 & 1 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -8 & -8 & 8 & -8 \\ -6 & -6 & 5 & -5 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line1}\times(8)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -8 \\ -6 & -6 & 5 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } += \text{ line1}\times(6)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 5 & -5 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{8}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 5 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } -= \text{ line2}\times(5)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(7)

$$\begin{pmatrix} -2 & -8 & 2 & -2 \\ 3 & 9 & -4 & 5 \\ 0 & 9 & 3 & -6 \end{pmatrix} \xrightarrow{\text{line1 } \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 3 & 9 & -4 & 5 \\ 0 & 9 & 3 & -6 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(3)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & -3 & -1 & 2 \\ 0 & 9 & 3 & -6 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 4 & -1 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 9 & 3 & -6 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & \frac{11}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 9 & 3 & -6 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } -= \text{ line2}\times(9)} \begin{pmatrix} 1 & 0 & -\frac{7}{3} & \frac{11}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(8)
$$\begin{pmatrix} -2 & 1 & 3 & 4 \\ 0 & -1 & 1 & -2 \\ -1 & 0 & 2 & 1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ -2 & 1 & 3 & 4 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & -1 & 1 & -2 \\ -2 & 1 & 3 & 4 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & -1 & 2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(9)
$$\begin{pmatrix} 0 & 4 & 3 & 0 \\ 1 & 1 & -2 & -3 \\ 0 & 4 & 3 & 0 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 1 & -2 & -3 \\ 0 & 4 & 3 & 0 \\ 0 & 4 & 3 & 0 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line3}} \begin{pmatrix} 1 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & -2 & -3 \\ 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & -2 & -3 \\ 0 & 1 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -\frac{11}{4} & -3 \\ 0 & 1 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(10)
$$\begin{pmatrix} 1 & -5 & 1 & -1 \\ 1 & -5 & 0 & -3 \\ -1 & 5 & -3 & -3 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & -5 & 1 & -1 \\ 0 & 0 & -1 & -2 \\ -1 & 5 & -3 & -3 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & -5 & 1 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -2 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & -5 & 1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & -5 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2} \times (2)} \begin{pmatrix} 1 & -5 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(11)
$$\begin{pmatrix} 1 & -3 & 5 & -2 \\ -1 & 3 & -5 & -1 \\ 1 & -3 & 5 & -9 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & -3 & 5 & -2 \\ 0 & 0 & 0 & -3 \\ 1 & -3 & 5 & -9 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1}} \begin{pmatrix} 1 & -3 & 5 & -2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & -3 & 5 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -7 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2} \times (2)} \begin{pmatrix} 1 & -3 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2} \times (7)} \begin{pmatrix} 1 & -3 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(12)
$$\begin{pmatrix} 1 & -1 & 3 & 1 \\ -2 & 4 & -8 & -8 \\ -2 & -1 & -3 & 7 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & 2 & -2 & -6 \\ -2 & -1 & -3 & 7 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & 2 & -2 & -6 \\ 0 & -3 & 3 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & -3 & 3 & 9 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2}} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & -3 & 3 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(13)
$$\begin{pmatrix} 1 & 1 & -2 & 1 \\ 2 & 2 & -4 & 2 \\ -1 & -5 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1} \times (2)} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & -5 & 5 & -1 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & 3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -\frac{5}{4} & 1 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(14)
$$\begin{pmatrix} 1 & 3 & -5 & -9 \\ 1 & 3 & -5 & -6 \\ -1 & -3 & 5 & 4 \end{pmatrix} \xrightarrow{\text{line2 -= line1}} \begin{pmatrix} 1 & 3 & -5 & -9 \\ 0 & 0 & 0 & 3 \\ -1 & -3 & 5 & 4 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 3 & -5 & -9 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 3 & -5 & -9 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line1 += line2×(9)}} \begin{pmatrix} 1 & 3 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 3 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(15)
$$\begin{pmatrix} 1 & 4 & 2 & 5 \\ -1 & -5 & -3 & -5 \\ 0 & 5 & 5 & 0 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 4 & 2 & 5 \\ 0 & -1 & -1 & 0 \\ 0 & 5 & 5 & 0 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 4 & 2 & 5 \\ 0 & 5 & 5 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 4 & 2 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2×(4)}} \begin{pmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(16)
$$\begin{pmatrix} 3 & 6 & 9 & -6 \\ 0 & 2 & 4 & 2 \\ 2 & -1 & -4 & -9 \end{pmatrix} \xrightarrow{\text{line1 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 2 \\ 2 & -1 & -4 & -9 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(2)}} \begin{pmatrix} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 2 \\ 0 & -5 & -10 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 2 & 3 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & -10 & -5 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(2)}} \begin{pmatrix} 1 & 0 & -1 & -4 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & -10 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 0 & -1 & -4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(17)
$$\begin{pmatrix} -8 & 2 & -3 & -8 \\ 2 & 5 & -5 & 2 \\ 9 & 0 & 1 & 9 \end{pmatrix} \xrightarrow{\text{line1 += line3}} \begin{pmatrix} 1 & 2 & -2 & 1 \\ 2 & 5 & -5 & 2 \\ 9 & 0 & 1 & 9 \end{pmatrix} \xrightarrow{\text{line2 -= line1×(2)}} \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & 1 & -1 & 0 \\ 9 & 0 & 1 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1×(9)}} \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -18 & 19 & 0 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(2)}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -18 & 19 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(18)}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(18)
$$\begin{pmatrix} -7 & -3 & 5 & -5 \\ 6 & 3 & -6 & 6 \\ -3 & -2 & 5 & -5 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} -1 & 0 & -1 & 1 \\ 6 & 3 & -6 & 6 \\ -3 & -2 & 5 & -5 \end{pmatrix} \xrightarrow{\text{line1 ×= } (-1)} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 6 & 3 & -6 & 6 \\ -3 & -2 & 5 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1×(6)}} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 3 & -12 & 12 \\ -3 & -2 & 5 & -5 \end{pmatrix} \xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 3 & -12 & 12 \\ 0 & -2 & 8 & -8 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -4 & 4 \\ 0 & -2 & 8 & -8 \end{pmatrix} \xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(19)
$$\begin{pmatrix} -7 & 4 & 1 & 1 \\ 1 & 4 & 9 & 9 \\ -3 & 3 & 3 & 3 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 4 & 9 & 9 \\ -7 & 4 & 1 & 1 \\ -3 & 3 & 3 & 3 \end{pmatrix} \xrightarrow{\text{line2 += line1×(7)}} \begin{pmatrix} 1 & 4 & 9 & 9 \\ 0 & 32 & 64 & 64 \\ -3 & 3 & 3 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & 4 & 9 & 9 \\ 0 & 32 & 64 & 64 \\ 0 & 15 & 30 & 30 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{32}\right)} \begin{pmatrix} 1 & 4 & 9 & 9 \\ 0 & 1 & 2 & 2 \\ 0 & 15 & 30 & 30 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2×(4)}} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 15 & 30 & 30 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(15)}} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(20)
$$\begin{pmatrix}-3&-3&-9&6\\3&-3&5&0\\1&-8&-3&7\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&-8&-3&7\\3&-3&5&0\\-3&-3&-9&6\end{pmatrix}\xrightarrow{\text{line2 }-=\text{ line1}\times(3)}\begin{pmatrix}1&-8&-3&7\\0&21&14&-21\\-3&-3&-9&6\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line1}\times(3)}\begin{pmatrix}1&-8&-3&7\\0&21&14&-21\\0&-27&-18&27\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(\frac{1}{21}\right)}\begin{pmatrix}1&-8&-3&7\\0&1&\frac{2}{3}&-1\\0&-27&-18&27\end{pmatrix}$$
$$\xrightarrow{\text{line1 }+=\text{ line2}\times(8)}\begin{pmatrix}1&0&\frac{7}{3}&-1\\0&1&\frac{2}{3}&-1\\0&-27&-18&27\end{pmatrix}\xrightarrow{\text{line3 }+=\text{ line2}\times(27)}\begin{pmatrix}1&0&\frac{7}{3}&-1\\0&1&\frac{2}{3}&-1\\0&0&0&0\end{pmatrix}$$

(21)
$$\begin{pmatrix}-3&0&9&0\\-2&1&6&-2\\2&3&-6&-7\end{pmatrix}\xrightarrow{\text{line1 }\times=\left(-\frac{1}{3}\right)}\begin{pmatrix}1&0&-3&0\\-2&1&6&-2\\2&3&-6&-7\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(2)}\begin{pmatrix}1&0&-3&0\\0&1&0&-2\\2&3&-6&-7\end{pmatrix}$$
$$\xrightarrow{\text{line3 }-=\text{ line1}\times(2)}\begin{pmatrix}1&0&-3&0\\0&1&0&-2\\0&3&0&-7\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}\times(3)}\begin{pmatrix}1&0&-3&0\\0&1&0&-2\\0&0&0&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3 }\times=(-1)}\begin{pmatrix}1&0&-3&0\\0&1&0&-2\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line3}\times(2)}\begin{pmatrix}1&0&-3&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$$

(22)
$$\begin{pmatrix}-2&-2&2&-6\\1&2&0&4\\-2&-1&3&-5\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}1&2&0&4\\-2&-2&2&-6\\-2&-1&3&-5\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(2)}\begin{pmatrix}1&2&0&4\\0&2&2&2\\-2&-1&3&-5\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line1}\times(2)}\begin{pmatrix}1&2&0&4\\0&2&2&2\\0&3&3&3\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(\frac{1}{2}\right)}\begin{pmatrix}1&2&0&4\\0&1&1&1\\0&3&3&3\end{pmatrix}$$
$$\xrightarrow{\text{line1 }-=\text{ line2}\times(2)}\begin{pmatrix}1&0&-2&2\\0&1&1&1\\0&3&3&3\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}\times(3)}\begin{pmatrix}1&0&-2&2\\0&1&1&1\\0&0&0&0\end{pmatrix}$$

(23)
$$\begin{pmatrix}-1&-2&3&-1\\-2&-5&8&0\\1&4&-7&-3\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&4&-7&-3\\-2&-5&8&0\\-1&-2&3&-1\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(2)}\begin{pmatrix}1&4&-7&-3\\0&3&-6&-6\\-1&-2&3&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line1}}\begin{pmatrix}1&4&-7&-3\\0&3&-6&-6\\0&2&-4&-4\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(\frac{1}{3}\right)}\begin{pmatrix}1&4&-7&-3\\0&1&-2&-2\\0&2&-4&-4\end{pmatrix}$$
$$\xrightarrow{\text{line1 }-=\text{ line2}\times(4)}\begin{pmatrix}1&0&1&5\\0&1&-2&-2\\0&2&-4&-4\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}\times(2)}\begin{pmatrix}1&0&1&5\\0&1&-2&-2\\0&0&0&0\end{pmatrix}$$

(24)
$$\begin{pmatrix}-1&0&4&6\\-4&1&7&5\\7&-2&-9&-2\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&0&-4&-6\\-4&1&7&5\\7&-2&-9&-2\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(4)}\begin{pmatrix}1&0&-4&-6\\0&1&-9&-19\\7&-2&-9&-2\end{pmatrix}$$
$$\xrightarrow{\text{line3 }-=\text{ line1}\times(7)}\begin{pmatrix}1&0&-4&-6\\0&1&-9&-19\\0&-2&19&40\end{pmatrix}\xrightarrow{\text{line3 }+=\text{ line2}\times(2)}\begin{pmatrix}1&0&-4&-6\\0&1&-9&-19\\0&0&1&2\end{pmatrix}$$
$$\xrightarrow{\text{line1 }+=\text{ line3}\times(4)}\begin{pmatrix}1&0&0&2\\0&1&-9&-19\\0&0&1&2\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line3}\times(9)}\begin{pmatrix}1&0&0&2\\0&1&0&-1\\0&0&1&2\end{pmatrix}$$

(25)
$$\begin{pmatrix}0&0&-1&1\\1&5&-8&-1\\1&5&-7&-1\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}1&5&-8&-1\\0&0&-1&1\\1&5&-7&-1\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line1}}\begin{pmatrix}1&5&-8&-1\\0&0&-1&1\\0&0&1&0\end{pmatrix}$$
$$\xrightarrow{\text{line2 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&5&-8&-1\\0&0&1&0\\0&0&-1&1\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line2}\times(8)}\begin{pmatrix}1&5&0&-1\\0&0&1&0\\0&0&-1&1\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line2}}\begin{pmatrix}1&5&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line3}}\begin{pmatrix}1&5&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

(26)
$$\begin{pmatrix} 0 & 0 & 1 & 2 \\ 3 & -3 & 7 & -1 \\ -2 & 2 & -2 & 6 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -2 & 2 & -2 & 6 \\ 3 & -3 & 7 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line1} \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & -1 & 1 & -3 \\ 3 & -3 & 7 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line2} -= \text{line1} \times (3)} \begin{pmatrix} 1 & -1 & 1 & -3 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 1 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 8 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2}} \begin{pmatrix} 1 & -1 & 0 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 8 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line2} \times (4)} \begin{pmatrix} 1 & -1 & 0 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(27)
$$\begin{pmatrix} 0 & 6 & -6 & 0 \\ -1 & -7 & 5 & -3 \\ -3 & -4 & -2 & -9 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} -1 & -7 & 5 & -3 \\ 0 & 6 & -6 & 0 \\ -3 & -4 & -2 & -9 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 7 & -5 & 3 \\ 0 & 6 & -6 & 0 \\ -3 & -4 & -2 & -9 \end{pmatrix}$$
$$\xrightarrow{\text{line3} += \text{line1} \times (3)} \begin{pmatrix} 1 & 7 & -5 & 3 \\ 0 & 6 & -6 & 0 \\ 0 & 17 & -17 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 7 & -5 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 17 & -17 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2} \times (7)} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 17 & -17 & 0 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line2} \times (17)} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(28)
$$\begin{pmatrix} 1 & -4 & -8 & -3 \\ -2 & 4 & 4 & 6 \\ 3 & -7 & -9 & -9 \end{pmatrix} \xrightarrow{\text{line2} += \text{line1} \times (2)} \begin{pmatrix} 1 & -4 & -8 & -3 \\ 0 & -4 & -12 & 0 \\ 3 & -7 & -9 & -9 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line1} \times (3)} \begin{pmatrix} 1 & -4 & -8 & -3 \\ 0 & -4 & -12 & 0 \\ 0 & 5 & 15 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -4 & -8 & -3 \\ 0 & 5 & 15 & 0 \\ 0 & -4 & -12 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & -4 & -8 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & -4 & -12 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} += \text{line2} \times (4)} \begin{pmatrix} 1 & 0 & 4 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & -4 & -12 & 0 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (4)} \begin{pmatrix} 1 & 0 & 4 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(29)
$$\begin{pmatrix} 1 & -1 & -9 & 7 \\ 1 & -1 & -9 & -2 \\ -1 & 1 & 9 & -1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1}} \begin{pmatrix} 1 & -1 & -9 & 7 \\ 0 & 0 & 0 & -9 \\ -1 & 1 & 9 & -1 \end{pmatrix} \xrightarrow{\text{line3} += \text{line1}} \begin{pmatrix} 1 & -1 & -9 & 7 \\ 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & -9 & 7 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & -1 & -9 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2} \times (7)} \begin{pmatrix} 1 & -1 & -9 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (9)} \begin{pmatrix} 1 & -1 & -9 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(30)
$$\begin{pmatrix} 2 & 6 & -7 & 6 \\ 2 & 6 & -3 & 6 \\ 1 & 3 & 6 & 3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & 6 & 3 \\ 2 & 6 & -3 & 6 \\ 2 & 6 & -7 & 6 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 3 & 6 & 3 \\ 0 & 0 & -15 & 0 \\ 2 & 6 & -7 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 3 & 6 & 3 \\ 0 & 0 & -15 & 0 \\ 0 & 0 & -19 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(-\frac{1}{15}\right)} \begin{pmatrix} 1 & 3 & 6 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -19 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2} \times (6)} \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -19 & 0 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (19)} \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(31)
$$\begin{pmatrix} 3 & -3 & 4 & 1 \\ 1 & -1 & -1 & 5 \\ 1 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & -1 & -1 & 5 \\ 3 & -3 & 4 & 1 \\ 1 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (3)} \begin{pmatrix} 1 & -1 & -1 & 5 \\ 0 & 0 & 7 & -14 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1}} \begin{pmatrix} 1 & -1 & -1 & 5 \\ 0 & 0 & 7 & -14 \\ 0 & 0 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{7}\right)} \begin{pmatrix} 1 & -1 & -1 & 5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line2} \times (2)} \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(32)
$$\begin{pmatrix} 3 & -2 & 8 & 9 \\ 0 & 3 & -3 & 0 \\ -1 & 1 & -3 & -3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -1 & 1 & -3 & -3 \\ 0 & 3 & -3 & 0 \\ 3 & -2 & 8 & 9 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 3 & -3 & 0 \\ 3 & -2 & 8 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1} \times (3)} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 3 & -3 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & -3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(33)
$$\begin{pmatrix} 3 & 3 & 6 & 0 \\ -2 & 4 & -7 & 3 \\ 4 & 6 & 7 & 1 \end{pmatrix} \xrightarrow{\text{line1} \times= \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 1 & 2 & 0 \\ -2 & 4 & -7 & 3 \\ 4 & 6 & 7 & 1 \end{pmatrix} \xrightarrow{\text{line2} += \text{line1} \times (2)} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 6 & -3 & 3 \\ 4 & 6 & 7 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1} \times (4)} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 6 & -3 & 3 \\ 0 & 2 & -1 & 1 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 2 & -1 & 1 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line2}} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 2 & -1 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line2} \times (2)} \begin{pmatrix} 1 & 0 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(34)
$$\begin{pmatrix} 3 & 3 & 9 & -9 \\ 1 & 6 & -2 & -8 \\ 0 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 6 & -2 & -8 \\ 3 & 3 & 9 & -9 \\ 0 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (3)} \begin{pmatrix} 1 & 6 & -2 & -8 \\ 0 & -15 & 15 & 15 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 6 & -2 & -8 \\ 0 & -1 & 1 & 1 \\ 0 & -15 & 15 & 15 \end{pmatrix} \xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & 6 & -2 & -8 \\ 0 & 1 & -1 & -1 \\ 0 & -15 & 15 & 15 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2} \times (6)} \begin{pmatrix} 1 & 0 & 4 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -15 & 15 & 15 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (15)} \begin{pmatrix} 1 & 0 & 4 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(35)
$$\begin{pmatrix} 4 & -9 & 9 & 7 \\ 1 & 0 & 0 & 4 \\ -3 & 5 & -5 & -7 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 4 & -9 & 9 & 7 \\ -3 & 5 & -5 & -7 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (4)} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & -9 & 9 & -9 \\ -3 & 5 & -5 & -7 \end{pmatrix}$$
$$\xrightarrow{\text{line3} += \text{line1} \times (3)} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & -9 & 9 & -9 \\ 0 & 5 & -5 & 5 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 5 & -5 & 5 \\ 0 & -9 & 9 & -9 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -9 & 9 & -9 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (9)} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(36)
$$\begin{pmatrix} 4 & 0 & -4 & 0 \\ -9 & -9 & 9 & 9 \\ 2 & 5 & -2 & -5 \end{pmatrix} \xrightarrow{\text{line1} \times= \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ -9 & -9 & 9 & 9 \\ 2 & 5 & -2 & -5 \end{pmatrix} \xrightarrow{\text{line2} += \text{line1} \times (9)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -9 & 0 & 9 \\ 2 & 5 & -2 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -9 & 0 & 9 \\ 0 & 5 & 0 & -5 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 5 & 0 & -5 \\ 0 & -9 & 0 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -9 & 0 & 9 \end{pmatrix} \xrightarrow{\text{line3} += \text{line2} \times (9)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(37)
$$\begin{pmatrix} -9 & -9 & -5 & -2 \\ 9 & 9 & -3 & 6 \\ 2 & 2 & -4 & 3 \end{pmatrix} \xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 0 & 0 & -8 & 4 \\ 9 & 9 & -3 & 6 \\ 2 & 2 & -4 & 3 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line3} \times (4)} \begin{pmatrix} 0 & 0 & -8 & 4 \\ 1 & 1 & 13 & -6 \\ 2 & 2 & -4 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 1 & 13 & -6 \\ 0 & 0 & -8 & 4 \\ 2 & 2 & -4 & 3 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 1 & 13 & -6 \\ 0 & 0 & -8 & 4 \\ 0 & 0 & -30 & 15 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(-\frac{1}{8}\right)} \begin{pmatrix} 1 & 1 & 13 & -6 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -30 & 15 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line2} \times (13)} \begin{pmatrix} 1 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -30 & 15 \end{pmatrix}$$
$$\xrightarrow{\text{line3} += \text{line2} \times (30)} \begin{pmatrix} 1 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(38)
$$\begin{pmatrix}-9&9&-1&7\\0&1&1&3\\1&2&3&8\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&2&3&8\\0&1&1&3\\-9&9&-1&7\end{pmatrix}\xrightarrow{\text{line3 += line1}\times(9)}\begin{pmatrix}1&2&3&8\\0&1&1&3\\0&27&26&79\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}\times(2)}\begin{pmatrix}1&0&1&2\\0&1&1&3\\0&27&26&79\end{pmatrix}\xrightarrow{\text{line3 -= line2}\times(27)}\begin{pmatrix}1&0&1&2\\0&1&1&3\\0&0&-1&-2\end{pmatrix}$$
$$\xrightarrow{\text{line3 }\times=(-1)}\begin{pmatrix}1&0&1&2\\0&1&1&3\\0&0&1&2\end{pmatrix}\xrightarrow{\text{line1 -= line3}}\begin{pmatrix}1&0&0&0\\0&1&1&3\\0&0&1&2\end{pmatrix}\xrightarrow{\text{line2 -= line3}}\begin{pmatrix}1&0&0&0\\0&1&0&1\\0&0&1&2\end{pmatrix}$$

(39)
$$\begin{pmatrix}-8&-8&8&8\\3&3&-3&-5\\-2&-2&2&1\end{pmatrix}\xrightarrow{\text{line1 }\times=\left(-\frac{1}{8}\right)}\begin{pmatrix}1&1&-1&-1\\3&3&-3&-5\\-2&-2&2&1\end{pmatrix}\xrightarrow{\text{line2 -= line1}\times(3)}\begin{pmatrix}1&1&-1&-1\\0&0&0&-2\\-2&-2&2&1\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1}\times(2)}\begin{pmatrix}1&1&-1&-1\\0&0&0&-2\\0&0&0&-1\end{pmatrix}\xrightarrow{\text{line2 }\leftrightarrow\text{ line3}}\begin{pmatrix}1&1&-1&-1\\0&0&0&-1\\0&0&0&-2\end{pmatrix}$$
$$\xrightarrow{\text{line2 }\times=(-1)}\begin{pmatrix}1&1&-1&-1\\0&0&0&1\\0&0&0&-2\end{pmatrix}\xrightarrow{\text{line1 += line2}}\begin{pmatrix}1&1&-1&0\\0&0&0&1\\0&0&0&-2\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2}\times(2)}\begin{pmatrix}1&1&-1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$$

(40)
$$\begin{pmatrix}-7&-6&5&-7\\-4&2&-8&-4\\4&3&-2&4\end{pmatrix}\xrightarrow{\text{line2 += line3}}\begin{pmatrix}-7&-6&5&-7\\0&5&-10&0\\4&3&-2&4\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}4&3&-2&4\\0&5&-10&0\\-7&-6&5&-7\end{pmatrix}$$
$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{4}\right)}\begin{pmatrix}1&\frac{3}{4}&-\frac{1}{2}&1\\0&5&-10&0\\-7&-6&5&-7\end{pmatrix}\xrightarrow{\text{line3 += line1}\times(7)}\begin{pmatrix}1&\frac{3}{4}&-\frac{1}{2}&1\\0&5&-10&0\\0&-\frac{3}{4}&\frac{3}{2}&0\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(\frac{1}{5}\right)}\begin{pmatrix}1&\frac{3}{4}&-\frac{1}{2}&1\\0&1&-2&0\\0&-\frac{3}{4}&\frac{3}{2}&0\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}\times\left(\frac{3}{4}\right)}\begin{pmatrix}1&0&1&1\\0&1&-2&0\\0&-\frac{3}{4}&\frac{3}{2}&0\end{pmatrix}\xrightarrow{\text{line3 += line2}\times\left(\frac{3}{4}\right)}\begin{pmatrix}1&0&1&1\\0&1&-2&0\\0&0&0&0\end{pmatrix}$$

(41)
$$\begin{pmatrix}-3&3&-8&-3\\-5&5&-8&-5\\-6&6&-5&-6\end{pmatrix}\xrightarrow{\text{line2 -= line3}}\begin{pmatrix}-3&3&-8&-3\\1&-1&-3&1\\-6&6&-5&-6\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}1&-1&-3&1\\-3&3&-8&-3\\-6&6&-5&-6\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1}\times(3)}\begin{pmatrix}1&-1&-3&1\\0&0&-17&0\\-6&6&-5&-6\end{pmatrix}\xrightarrow{\text{line3 += line1}\times(6)}\begin{pmatrix}1&-1&-3&1\\0&0&-17&0\\0&0&-23&0\end{pmatrix}$$
$$\xrightarrow{\text{line2 }\times=\left(-\frac{1}{17}\right)}\begin{pmatrix}1&-1&-3&1\\0&0&1&0\\0&0&-23&0\end{pmatrix}\xrightarrow{\text{line1 += line2}\times(3)}\begin{pmatrix}1&-1&0&1\\0&0&1&0\\0&0&-23&0\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2}\times(23)}\begin{pmatrix}1&-1&0&1\\0&0&1&0\\0&0&0&0\end{pmatrix}$$

(42)
$$\begin{pmatrix}-2&-2&-4&-7\\-4&-4&-4&-8\\-1&-1&-5&-8\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line3}}\begin{pmatrix}-1&-1&-5&-8\\-4&-4&-4&-8\\-2&-2&-4&-7\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&1&5&8\\-4&-4&-4&-8\\-2&-2&-4&-7\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1}\times(4)}\begin{pmatrix}1&1&5&8\\0&0&16&24\\-2&-2&-4&-7\end{pmatrix}\xrightarrow{\text{line3 += line1}\times(2)}\begin{pmatrix}1&1&5&8\\0&0&16&24\\0&0&6&9\end{pmatrix}$$
$$\xrightarrow{\text{line2 }\times=\left(\frac{1}{16}\right)}\begin{pmatrix}1&1&5&8\\0&0&1&\frac{3}{2}\\0&0&6&9\end{pmatrix}\xrightarrow{\text{line1 -= line2}\times(5)}\begin{pmatrix}1&1&0&\frac{1}{2}\\0&0&1&\frac{3}{2}\\0&0&6&9\end{pmatrix}\xrightarrow{\text{line3 -= line2}\times(6)}\begin{pmatrix}1&1&0&\frac{1}{2}\\0&0&1&\frac{3}{2}\\0&0&0&0\end{pmatrix}$$

(43)

$$\begin{pmatrix} -2 & 2 & -2 & 8 \\ -1 & 1 & -1 & -6 \\ 8 & -8 & 8 & 9 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} -1 & 1 & -1 & -6 \\ -2 & 2 & -2 & 8 \\ 8 & -8 & 8 & 9 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -1 & 1 & 6 \\ -2 & 2 & -2 & 8 \\ 8 & -8 & 8 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -1 & 1 & 6 \\ 0 & 0 & 0 & 20 \\ 8 & -8 & 8 & 9 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(8)} \begin{pmatrix} 1 & -1 & 1 & 6 \\ 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & -39 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{20}\right)} \begin{pmatrix} 1 & -1 & 1 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -39 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(6)} \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -39 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(39)} \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(44)

$$\begin{pmatrix} -2 & 5 & 6 & 2 \\ -1 & 9 & 3 & 1 \\ -2 & -2 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -2 & -2 & 6 & 2 \\ -1 & 9 & 3 & 1 \\ -2 & 5 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line1} \times= \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 1 & -3 & -1 \\ -1 & 9 & 3 & 1 \\ -2 & 5 & 6 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & 10 & 0 & 0 \\ -2 & 5 & 6 & 2 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & 10 & 0 & 0 \\ 0 & 7 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{10}\right)} \begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 7 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 7 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(7)} \begin{pmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(45)

$$\begin{pmatrix} -1 & -1 & 3 & 2 \\ 1 & 2 & -7 & 1 \\ 3 & 3 & -9 & -5 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 2 & -7 & 1 \\ -1 & -1 & 3 & 2 \\ 3 & 3 & -9 & -5 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 2 & -7 & 1 \\ 0 & 1 & -4 & 3 \\ 3 & 3 & -9 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & 2 & -7 & 1 \\ 0 & 1 & -4 & 3 \\ 0 & -3 & 12 & -8 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & -4 & 3 \\ 0 & -3 & 12 & -8 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times(5)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(46)

$$\begin{pmatrix} -1 & -1 & 5 & -1 \\ 3 & 1 & -5 & -1 \\ -4 & 1 & -5 & 6 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 1 & -5 & 1 \\ 3 & 1 & -5 & -1 \\ -4 & 1 & -5 & 6 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & 1 & -5 & 1 \\ 0 & -2 & 10 & -4 \\ -4 & 1 & -5 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(4)} \begin{pmatrix} 1 & 1 & -5 & 1 \\ 0 & -2 & 10 & -4 \\ 0 & 5 & -25 & 10 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & -5 & 1 \\ 0 & 5 & -25 & 10 \\ 0 & -2 & 10 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & 1 & -5 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & -2 & 10 & -4 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -5 & 2 \\ 0 & -2 & 10 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(47)

$$\begin{pmatrix} -1 & 1 & 2 & 6 \\ 3 & -3 & 7 & -5 \\ 1 & -1 & 3 & -1 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 3 & -1 \\ 3 & -3 & 7 & -5 \\ -1 & 1 & 2 & 6 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 0 & -2 & -2 \\ -1 & 1 & 2 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 5 & 5 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & -2 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)} \begin{pmatrix} 1 & -1 & 3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(3)} \begin{pmatrix} 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(2)} \begin{pmatrix} 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(48)

$$\begin{pmatrix} -1 & 4 & -3 & 5 \\ 0 & 5 & -1 & -6 \\ -1 & 5 & -3 & 3 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & -4 & 3 & -5 \\ 0 & 5 & -1 & -6 \\ -1 & 5 & -3 & 3 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & -4 & 3 & -5 \\ 0 & 5 & -1 & -6 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -4 & 3 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & 5 & -1 & -6 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(4)} \begin{pmatrix} 1 & 0 & 3 & -13 \\ 0 & 1 & 0 & -2 \\ 0 & 5 & -1 & -6 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & 3 & -13 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & 4 \end{pmatrix} \xrightarrow{\text{line3} \times= (-1)} \begin{pmatrix} 1 & 0 & 3 & -13 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{pmatrix}$$

(49)

$$\begin{pmatrix} 0 & 3 & 6 & 8 \\ -1 & -1 & -1 & -3 \\ 3 & 1 & -1 & 4 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} -1 & -1 & -1 & -3 \\ 0 & 3 & 6 & 8 \\ 3 & 1 & -1 & 4 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 6 & 8 \\ 3 & 1 & -1 & 4 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(3)} \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 6 & 8 \\ 0 & -2 & -4 & -5 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line3}} \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & -4 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & -4 & -5 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(2)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & -5 & -2 & -2 \\ -2 & -3 & -8 & 3 \\ 0 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -5 & -2 & -2 \\ 0 & -13 & -12 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -5 & -2 & -2 \\ 0 & -1 & -1 & 0 \\ 0 & -13 & -12 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & -5 & -2 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & -13 & -12 & -1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & -13 & -12 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(13)} \begin{pmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{-}= \text{line3}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

(51)

$$\begin{pmatrix}1&-5&1&-2\\1&-8&3&-3\\0&-3&2&0\end{pmatrix} \xrightarrow{\text{line2 -= line1}} \begin{pmatrix}1&-5&1&-2\\0&-3&2&-1\\0&-3&2&0\end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix}1&-5&1&-2\\0&0&0&-1\\0&-3&2&0\end{pmatrix}$$
$$\xrightarrow{\text{line2 <-> line3}} \begin{pmatrix}1&-5&1&-2\\0&-3&2&0\\0&0&0&-1\end{pmatrix} \xrightarrow{\text{line2 *= }\left(-\frac{1}{3}\right)} \begin{pmatrix}1&-5&1&-2\\0&1&-\frac{2}{3}&0\\0&0&0&-1\end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2*(5)}} \begin{pmatrix}1&0&-\frac{7}{3}&-2\\0&1&-\frac{2}{3}&0\\0&0&0&-1\end{pmatrix} \xrightarrow{\text{line3 *= (-1)}} \begin{pmatrix}1&0&-\frac{7}{3}&-2\\0&1&-\frac{2}{3}&0\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line1 += line3*(2)}} \begin{pmatrix}1&0&-\frac{7}{3}&0\\0&1&-\frac{2}{3}&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}1&-2&1&2\\-4&0&-8&-9\\4&1&8&9\end{pmatrix} \xrightarrow{\text{line2 += line1*(4)}} \begin{pmatrix}1&-2&1&2\\0&-8&-4&-1\\4&1&8&9\end{pmatrix} \xrightarrow{\text{line3 -= line1*(4)}} \begin{pmatrix}1&-2&1&2\\0&-8&-4&-1\\0&9&4&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix}1&-2&1&2\\0&1&0&0\\0&9&4&1\end{pmatrix} \xrightarrow{\text{line1 += line2*(2)}} \begin{pmatrix}1&0&1&2\\0&1&0&0\\0&9&4&1\end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line2*(9)}} \begin{pmatrix}1&0&1&2\\0&1&0&0\\0&0&4&1\end{pmatrix} \xrightarrow{\text{line3 *= }\left(\frac{1}{4}\right)} \begin{pmatrix}1&0&1&2\\0&1&0&0\\0&0&1&\frac{1}{4}\end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix}1&0&0&\frac{7}{4}\\0&1&0&0\\0&0&1&\frac{1}{4}\end{pmatrix}$$

(53)

$$\begin{pmatrix}2&-1&-9&-6\\-1&-1&-3&3\\0&-1&-5&0\end{pmatrix} \xrightarrow{\text{line1 <-> line2}} \begin{pmatrix}-1&-1&-3&3\\2&-1&-9&-6\\0&-1&-5&0\end{pmatrix} \xrightarrow{\text{line1 *= (-1)}} \begin{pmatrix}1&1&3&-3\\2&-1&-9&-6\\0&-1&-5&0\end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1*(2)}} \begin{pmatrix}1&1&3&-3\\0&-3&-15&0\\0&-1&-5&0\end{pmatrix} \xrightarrow{\text{line2 <-> line3}} \begin{pmatrix}1&1&3&-3\\0&-1&-5&0\\0&-3&-15&0\end{pmatrix}$$
$$\xrightarrow{\text{line2 *= (-1)}} \begin{pmatrix}1&1&3&-3\\0&1&5&0\\0&-3&-15&0\end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix}1&0&-2&-3\\0&1&5&0\\0&-3&-15&0\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2*(3)}} \begin{pmatrix}1&0&-2&-3\\0&1&5&0\\0&0&0&0\end{pmatrix}$$

(54)

$$\begin{pmatrix}3&-1&7&-1\\-1&1&-3&-1\\-2&3&-7&-4\end{pmatrix} \xrightarrow{\text{line1 <-> line2}} \begin{pmatrix}-1&1&-3&-1\\3&-1&7&-1\\-2&3&-7&-4\end{pmatrix} \xrightarrow{\text{line1 *= (-1)}} \begin{pmatrix}1&-1&3&1\\3&-1&7&-1\\-2&3&-7&-4\end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1*(3)}} \begin{pmatrix}1&-1&3&1\\0&2&-2&-4\\-2&3&-7&-4\end{pmatrix} \xrightarrow{\text{line3 += line1*(2)}} \begin{pmatrix}1&-1&3&1\\0&2&-2&-4\\0&1&-1&-2\end{pmatrix}$$
$$\xrightarrow{\text{line2 <-> line3}} \begin{pmatrix}1&-1&3&1\\0&1&-1&-2\\0&2&-2&-4\end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix}1&0&2&-1\\0&1&-1&-2\\0&2&-2&-4\end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line2*(2)}} \begin{pmatrix}1&0&2&-1\\0&1&-1&-2\\0&0&0&0\end{pmatrix}$$

(55)

$$\begin{pmatrix}4&-3&-4&-6\\-7&6&7&9\\-4&4&4&4\end{pmatrix} \xrightarrow{\text{line1 <-> line3}} \begin{pmatrix}-4&4&4&4\\-7&6&7&9\\4&-3&-4&-6\end{pmatrix} \xrightarrow{\text{line1 *= }\left(-\frac{1}{4}\right)} \begin{pmatrix}1&-1&-1&-1\\-7&6&7&9\\4&-3&-4&-6\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1*(7)}} \begin{pmatrix}1&-1&-1&-1\\0&-1&0&2\\4&-3&-4&-6\end{pmatrix} \xrightarrow{\text{line3 -= line1*(4)}} \begin{pmatrix}1&-1&-1&-1\\0&-1&0&2\\0&1&0&-2\end{pmatrix}$$
$$\xrightarrow{\text{line2 <-> line3}} \begin{pmatrix}1&-1&-1&-1\\0&1&0&-2\\0&-1&0&2\end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix}1&0&-1&-3\\0&1&0&-2\\0&-1&0&2\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2}} \begin{pmatrix}1&0&-1&-3\\0&1&0&-2\\0&0&0&0\end{pmatrix}$$

(56)
$$\begin{pmatrix} 5 & -1 & -3 & 2 \\ -8 & -3 & -9 & -6 \\ -2 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line1 += line3×(2)}} \begin{pmatrix} 1 & -1 & -3 & 0 \\ -8 & -3 & -9 & -6 \\ -2 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line2 += line1×(8)}} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & -11 & -33 & -6 \\ -2 & 0 & 0 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & -11 & -33 & -6 \\ 0 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(6)}} \begin{pmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -2 & -6 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -2 & -6 & -1 \end{pmatrix} \xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line3 ×= (−1)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(57)
$$\begin{pmatrix} 6 & -8 & -6 & 6 \\ -7 & 6 & 7 & -7 \\ -4 & 3 & 4 & -4 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(2)}} \begin{pmatrix} 6 & -8 & -6 & 6 \\ 1 & 0 & -1 & 1 \\ -4 & 3 & 4 & -4 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 6 & -8 & -6 & 6 \\ -4 & 3 & 4 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1×(6)}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & -8 & 0 & 0 \\ -4 & 3 & 4 & -4 \end{pmatrix} \xrightarrow{\text{line3 += line1×(4)}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & -8 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & -8 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -8 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(8)}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(58)
$$\begin{pmatrix} 6 & 5 & -9 & -2 \\ -4 & -3 & 7 & 1 \\ 5 & 5 & -5 & -2 \end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ -4 & -3 & 7 & 1 \\ 5 & 5 & -5 & -2 \end{pmatrix} \xrightarrow{\text{line2 += line1×(4)}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & -3 & -9 & 1 \\ 5 & 5 & -5 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1×(5)}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & -3 & -9 & 1 \\ 0 & 5 & 15 & -2 \end{pmatrix} \xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & -3 & -9 & 1 \\ 0 & -1 & -3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & -1 & -3 & 0 \\ 0 & -3 & -9 & 1 \end{pmatrix} \xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -3 & -9 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(3)}} \begin{pmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(59)
$$\begin{pmatrix} -9 & 9 & -5 & 5 \\ 2 & -2 & 1 & -1 \\ -9 & 9 & -9 & 9 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} -9 & 9 & -9 & 9 \\ 2 & -2 & 1 & -1 \\ -9 & 9 & -5 & 5 \end{pmatrix} \xrightarrow{\text{line1 ×= } \left(-\frac{1}{9}\right)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 2 & -2 & 1 & -1 \\ -9 & 9 & -5 & 5 \end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1×(2)}} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ -9 & 9 & -5 & 5 \end{pmatrix} \xrightarrow{\text{line3 += line1×(9)}} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 4 & -4 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & -1 & 1 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(\frac{1}{4}\right)} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(60)

$$\begin{pmatrix} -3 & 3 & -9 & -8 \\ 2 & -2 & 6 & -7 \\ -3 & 3 & -9 & -7 \end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 2 & -2 & 6 & -7 \\ -3 & 3 & -9 & -7 \end{pmatrix} \xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 2 & -2 & 6 & -7 \\ 1 & -1 & 3 & -21 \end{pmatrix}$$
$$\xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 1 & -1 & 3 & -21 \\ 2 & -2 & 6 & -7 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line2 -= line1×(2)}} \begin{pmatrix} 1 & -1 & 3 & -21 \\ 0 & 0 & 0 & 35 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & -1 & 3 & -21 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 35 \end{pmatrix} \xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix} 1 & -1 & 3 & -21 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 35 \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2×(21)}} \begin{pmatrix} 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 35 \end{pmatrix} \xrightarrow{\text{line3 -= line2×(35)}} \begin{pmatrix} 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -2 & -6 & 7 & 0 \\ 0 & -8 & -2 & 9 \\ 1 & -3 & -5 & 7 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 1 & -3 & -5 & 7 \\ 0 & -8 & -2 & 9 \\ -2 & -6 & 7 & 0 \end{pmatrix} \xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -3 & -5 & 7 \\ 0 & -8 & -2 & 9 \\ 0 & -12 & -3 & 14 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{8}\right)} \begin{pmatrix} 1 & -3 & -5 & 7 \\ 0 & 1 & \frac{1}{4} & -\frac{9}{8} \\ 0 & -12 & -3 & 14 \end{pmatrix} \xrightarrow{\text{line1 += line2×(3)}} \begin{pmatrix} 1 & 0 & -\frac{17}{4} & \frac{29}{8} \\ 0 & 1 & \frac{1}{4} & -\frac{9}{8} \\ 0 & -12 & -3 & 14 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(12)}} \begin{pmatrix} 1 & 0 & -\frac{17}{4} & \frac{29}{8} \\ 0 & 1 & \frac{1}{4} & -\frac{9}{8} \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \xrightarrow{\text{line3 ×= (2)}} \begin{pmatrix} 1 & 0 & -\frac{17}{4} & \frac{29}{8} \\ 0 & 1 & \frac{1}{4} & -\frac{9}{8} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line3×}\left(\frac{29}{8}\right)} \begin{pmatrix} 1 & 0 & -\frac{17}{4} & 0 \\ 0 & 1 & \frac{1}{4} & -\frac{9}{8} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 += line3×}\left(\frac{9}{8}\right)} \begin{pmatrix} 1 & 0 & -\frac{17}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(62)

$$\begin{pmatrix} -1 & -5 & 1 & -1 \\ -1 & 3 & -1 & 1 \\ 7 & 7 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 7 & 7 & 0 & 0 \\ -1 & 3 & -1 & 1 \\ -1 & -5 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line1 ×= } \left(\frac{1}{7}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 3 & -1 & 1 \\ -1 & -5 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 1 \\ -1 & -5 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 1 \\ 0 & -4 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -4 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(63)

$$\begin{pmatrix} -1 & -5 & 5 & -5 \\ 1 & 4 & -5 & 4 \\ 1 & 3 & -5 & 5 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 4 & -5 & 4 \\ -1 & -5 & 5 & -5 \\ 1 & 3 & -5 & 5 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 4 & -5 & 4 \\ 0 & -1 & 0 & -1 \\ 1 & 3 & -5 & 5 \end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1}} \begin{pmatrix} 1 & 4 & -5 & 4 \\ 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix} 1 & 4 & -5 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2×(4)}} \begin{pmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line3 ×= } \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(64)

$$\begin{pmatrix} 0 & 6 & 3 & -2 \\ -1 & 9 & 5 & 1 \\ 0 & -4 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} -1 & 9 & 5 & 1 \\ 0 & 6 & 3 & -2 \\ 0 & -4 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line1 } \times= (-1)} \begin{pmatrix} 1 & -9 & -5 & -1 \\ 0 & 6 & 3 & -2 \\ 0 & -4 & -2 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{6}\right)} \begin{pmatrix} 1 & -9 & -5 & -1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{3} \\ 0 & -4 & -2 & 1 \end{pmatrix} \xrightarrow{\text{line1 } += \text{ line2}\times(9)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{3} \\ 0 & -4 & -2 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } += \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 & 0 & -\frac{1}{3} \end{pmatrix} \xrightarrow{\text{line3 } \times= (-3)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } += \text{ line3}\times(4)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & -3 & 5 & 7 \\ 2 & -4 & 8 & 7 \\ -1 & 0 & -2 & 3 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(2)} \begin{pmatrix} 1 & -3 & 5 & 7 \\ 0 & 2 & -2 & -7 \\ -1 & 0 & -2 & 3 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}} \begin{pmatrix} 1 & -3 & 5 & 7 \\ 0 & 2 & -2 & -7 \\ 0 & -3 & 3 & 10 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } += \text{ line2}\times(2)} \begin{pmatrix} 1 & -3 & 5 & 7 \\ 0 & 2 & -2 & -7 \\ 0 & 1 & -1 & -4 \end{pmatrix} \xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -3 & 5 & 7 \\ 0 & 1 & -1 & -4 \\ 0 & 2 & -2 & -7 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } += \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & -4 \\ 0 & 2 & -2 & -7 \end{pmatrix} \xrightarrow{\text{line3 } -= \text{ line2}\times(2)} \begin{pmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } += \text{ line3}\times(5)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}\times(4)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 & 3 & -3 \\ 5 & -1 & -4 & 0 \\ -3 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}\times(5)} \begin{pmatrix} 1 & 1 & 3 & -3 \\ 0 & -6 & -19 & 15 \\ -3 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}\times(3)} \begin{pmatrix} 1 & 1 & 3 & -3 \\ 0 & -6 & -19 & 15 \\ 0 & 3 & 10 & -8 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \times= \left(-\frac{1}{6}\right)} \begin{pmatrix} 1 & 1 & 3 & -3 \\ 0 & 1 & \frac{19}{6} & -\frac{5}{2} \\ 0 & 3 & 10 & -8 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & -\frac{1}{2} \\ 0 & 1 & \frac{19}{6} & -\frac{5}{2} \\ 0 & 3 & 10 & -8 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } -= \text{ line2}\times(3)} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & -\frac{1}{2} \\ 0 & 1 & \frac{19}{6} & -\frac{5}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \xrightarrow{\text{line3 } \times= (2)} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & -\frac{1}{2} \\ 0 & 1 & \frac{19}{6} & -\frac{5}{2} \\ 0 & 0 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } += \text{ line3}\times\left(\frac{1}{6}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{19}{6} & -\frac{5}{2} \\ 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line3}\times\left(\frac{19}{6}\right)} \begin{pmatrix} 1 & 0 & 0 & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 4 & -2 & -2 \\ 1 & 3 & -1 & -3 \\ -1 & 0 & -3 & 8 \end{pmatrix} \xrightarrow{\text{line2 } -= \text{ line1}} \begin{pmatrix} 1 & 4 & -2 & -2 \\ 0 & -1 & 1 & -1 \\ -1 & 0 & -3 & 8 \end{pmatrix} \xrightarrow{\text{line3 } += \text{ line1}} \begin{pmatrix} 1 & 4 & -2 & -2 \\ 0 & -1 & 1 & -1 \\ 0 & 4 & -5 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & 4 & -2 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 4 & -5 & 6 \end{pmatrix} \xrightarrow{\text{line1 } -= \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & 2 & -6 \\ 0 & 1 & -1 & 1 \\ 0 & 4 & -5 & 6 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } -= \text{ line2}\times(4)} \begin{pmatrix} 1 & 0 & 2 & -6 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{\text{line3 } \times= (-1)} \begin{pmatrix} 1 & 0 & 2 & -6 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } -= \text{ line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line2 } += \text{ line3}} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

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$$\begin{pmatrix}2&1&-2&2\\5&5&-3&7\\2&2&-1&3\end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix}0&-1&-1&-1\\5&5&-3&7\\2&2&-1&3\end{pmatrix} \xrightarrow{\text{line2 -= line3×(2)}} \begin{pmatrix}0&-1&-1&-1\\1&1&-1&1\\2&2&-1&3\end{pmatrix}$$
$$\xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix}1&1&-1&1\\0&-1&-1&-1\\2&2&-1&3\end{pmatrix} \xrightarrow{\text{line3 -= line1×(2)}} \begin{pmatrix}1&1&-1&1\\0&-1&-1&-1\\0&0&1&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix}1&1&-1&1\\0&1&1&1\\0&0&1&1\end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix}1&0&-2&0\\0&1&1&1\\0&0&1&1\end{pmatrix}$$
$$\xrightarrow{\text{line1 += line3×(2)}} \begin{pmatrix}1&0&0&2\\0&1&1&1\\0&0&1&1\end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix}1&0&0&2\\0&1&0&0\\0&0&1&1\end{pmatrix}$$

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$$\begin{pmatrix}2&4&-2&8\\3&7&-5&-1\\-2&-5&4&4\end{pmatrix} \xrightarrow{\text{line1 ×= }\left(\frac{1}{2}\right)} \begin{pmatrix}1&2&-1&4\\3&7&-5&-1\\-2&-5&4&4\end{pmatrix} \xrightarrow{\text{line2 -= line1×(3)}} \begin{pmatrix}1&2&-1&4\\0&1&-2&-13\\-2&-5&4&4\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix}1&2&-1&4\\0&1&-2&-13\\0&-1&2&12\end{pmatrix} \xrightarrow{\text{line1 -= line2×(2)}} \begin{pmatrix}1&0&3&30\\0&1&-2&-13\\0&-1&2&12\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2}} \begin{pmatrix}1&0&3&30\\0&1&-2&-13\\0&0&0&-1\end{pmatrix} \xrightarrow{\text{line3 ×= (−1)}} \begin{pmatrix}1&0&3&30\\0&1&-2&-13\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line3×(30)}} \begin{pmatrix}1&0&3&0\\0&1&-2&-13\\0&0&0&1\end{pmatrix} \xrightarrow{\text{line2 += line3×(13)}} \begin{pmatrix}1&0&3&0\\0&1&-2&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}2&5&-8&-8\\-2&3&-8&-8\\-5&-1&-3&-3\end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix}0&8&-16&-16\\-2&3&-8&-8\\-5&-1&-3&-3\end{pmatrix} \xrightarrow{\text{line3 -= line2×(3)}} \begin{pmatrix}0&8&-16&-16\\-2&3&-8&-8\\1&-10&21&21\end{pmatrix}$$
$$\xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix}1&-10&21&21\\-2&3&-8&-8\\0&8&-16&-16\end{pmatrix} \xrightarrow{\text{line2 += line1×(2)}} \begin{pmatrix}1&-10&21&21\\0&-17&34&34\\0&8&-16&-16\end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix}1&-10&21&21\\0&8&-16&-16\\0&-17&34&34\end{pmatrix} \xrightarrow{\text{line2 ×= }\left(\frac{1}{8}\right)} \begin{pmatrix}1&-10&21&21\\0&1&-2&-2\\0&-17&34&34\end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2×(10)}} \begin{pmatrix}1&0&1&1\\0&1&-2&-2\\0&-17&34&34\end{pmatrix} \xrightarrow{\text{line3 += line2×(17)}} \begin{pmatrix}1&0&1&1\\0&1&-2&-2\\0&0&0&0\end{pmatrix}$$

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$$\begin{pmatrix}4&-4&-3&-3\\3&-3&-2&6\\-8&8&6&2\end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix}1&-1&-1&-9\\3&-3&-2&6\\-8&8&6&2\end{pmatrix} \xrightarrow{\text{line2 -= line1×(3)}} \begin{pmatrix}1&-1&-1&-9\\0&0&1&33\\-8&8&6&2\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(8)}} \begin{pmatrix}1&-1&-1&-9\\0&0&1&33\\0&0&-2&-70\end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix}1&-1&0&24\\0&0&1&33\\0&0&-2&-70\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix}1&-1&0&24\\0&0&1&33\\0&0&0&-4\end{pmatrix} \xrightarrow{\text{line3 ×= }\left(-\frac{1}{4}\right)} \begin{pmatrix}1&-1&0&24\\0&0&1&33\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line3×(24)}} \begin{pmatrix}1&-1&0&0\\0&0&1&33\\0&0&0&1\end{pmatrix} \xrightarrow{\text{line2 -= line3×(33)}} \begin{pmatrix}1&-1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix} 7 & 0 & 8 & 9 \\ 1 & 3 & -5 & -5 \\ -1 & -2 & 3 & 3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 3 & -5 & -5 \\ 7 & 0 & 8 & 9 \\ -1 & -2 & 3 & 3 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{-}= \text{line1}\times(7)} \begin{pmatrix} 1 & 3 & -5 & -5 \\ 0 & -21 & 43 & 44 \\ -1 & -2 & 3 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}} \begin{pmatrix} 1 & 3 & -5 & -5 \\ 0 & -21 & 43 & 44 \\ 0 & 1 & -2 & -2 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 3 & -5 & -5 \\ 0 & 1 & -2 & -2 \\ 0 & -21 & 43 & 44 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & -21 & 43 & 44 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(21)} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line3}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times(2)} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

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$$\begin{pmatrix} 8 & 3 & 2 & -3 \\ -2 & -5 & 8 & 5 \\ -1 & 3 & -7 & -3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} -1 & 3 & -7 & -3 \\ -2 & -5 & 8 & 5 \\ 8 & 3 & 2 & -3 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{\times}= (-1)} \begin{pmatrix} 1 & -3 & 7 & 3 \\ -2 & -5 & 8 & 5 \\ 8 & 3 & 2 & -3 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)} \begin{pmatrix} 1 & -3 & 7 & 3 \\ 0 & -11 & 22 & 11 \\ 8 & 3 & 2 & -3 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(8)} \begin{pmatrix} 1 & -3 & 7 & 3 \\ 0 & -11 & 22 & 11 \\ 0 & 27 & -54 & -27 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -3 & 7 & 3 \\ 0 & 27 & -54 & -27 \\ 0 & -11 & 22 & 11 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(\frac{1}{27}\right)} \begin{pmatrix} 1 & -3 & 7 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & -11 & 22 & 11 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & -11 & 22 & 11 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(11)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -9 & -1 & 6 & 1 \\ -3 & 8 & -4 & -8 \\ 1 & -5 & 3 & 5 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -5 & 3 & 5 \\ -3 & 8 & -4 & -8 \\ -9 & -1 & 6 & 1 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(3)} \begin{pmatrix} 1 & -5 & 3 & 5 \\ 0 & -7 & 5 & 7 \\ -9 & -1 & 6 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(9)} \begin{pmatrix} 1 & -5 & 3 & 5 \\ 0 & -7 & 5 & 7 \\ 0 & -46 & 33 & 46 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(-\frac{1}{7}\right)} \begin{pmatrix} 1 & -5 & 3 & 5 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & -46 & 33 & 46 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(5)} \begin{pmatrix} 1 & 0 & -\frac{4}{7} & 0 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & -46 & 33 & 46 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(46)} \begin{pmatrix} 1 & 0 & -\frac{4}{7} & 0 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & 0 & \frac{1}{7} & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{\times}= (7)} \begin{pmatrix} 1 & 0 & -\frac{4}{7} & 0 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{+}= \text{line3}\times\left(\frac{4}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{5}{7} & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{5}{7}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} -9 & 3 & 9 & -9 \\ -7 & -1 & 7 & -6 \\ -5 & -8 & 5 & -2 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times(2)} \begin{pmatrix} 1 & 19 & -1 & -5 \\ -7 & -1 & 7 & -6 \\ -5 & -8 & 5 & -2 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(7)} \begin{pmatrix} 1 & 19 & -1 & -5 \\ 0 & 132 & 0 & -41 \\ -5 & -8 & 5 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(5)} \begin{pmatrix} 1 & 19 & -1 & -5 \\ 0 & 132 & 0 & -41 \\ 0 & 87 & 0 & -27 \end{pmatrix} \xrightarrow{\text{line2} \mathrel{\times}= \left(\frac{1}{132}\right)} \begin{pmatrix} 1 & 19 & -1 & -5 \\ 0 & 1 & 0 & -\frac{41}{132} \\ 0 & 87 & 0 & -27 \end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(19)} \begin{pmatrix} 1 & 0 & -1 & \frac{119}{132} \\ 0 & 1 & 0 & -\frac{41}{132} \\ 0 & 87 & 0 & -27 \end{pmatrix} \xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(87)} \begin{pmatrix} 1 & 0 & -1 & \frac{119}{132} \\ 0 & 1 & 0 & -\frac{41}{132} \\ 0 & 0 & 0 & \frac{1}{44} \end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{\times}= (44)} \begin{pmatrix} 1 & 0 & -1 & \frac{119}{132} \\ 0 & 1 & 0 & -\frac{41}{132} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times\left(\frac{119}{132}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{41}{132} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{41}{132}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix}-8&6&-4&1\\5&-1&0&-1\\3&3&-3&-1\end{pmatrix}\xrightarrow{\text{line1 += line3×(3)}}\begin{pmatrix}1&15&-13&-2\\5&-1&0&-1\\3&3&-3&-1\end{pmatrix}\xrightarrow{\text{line2 -= line1×(5)}}\begin{pmatrix}1&15&-13&-2\\0&-76&65&9\\3&3&-3&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1×(3)}}\begin{pmatrix}1&15&-13&-2\\0&-76&65&9\\0&-42&36&5\end{pmatrix}\xrightarrow{\text{line2 ×= }\left(-\frac{1}{76}\right)}\begin{pmatrix}1&15&-13&-2\\0&1&-\frac{65}{76}&-\frac{9}{76}\\0&-42&36&5\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2×(15)}}\begin{pmatrix}1&0&-\frac{13}{76}&-\frac{17}{76}\\0&1&-\frac{65}{76}&-\frac{9}{76}\\0&-42&36&5\end{pmatrix}\xrightarrow{\text{line3 += line2×(42)}}\begin{pmatrix}1&0&-\frac{13}{76}&-\frac{17}{76}\\0&1&-\frac{65}{76}&-\frac{9}{76}\\0&0&\frac{3}{38}&\frac{1}{38}\end{pmatrix}$$
$$\xrightarrow{\text{line3 ×= }\left(\frac{38}{3}\right)}\begin{pmatrix}1&0&-\frac{13}{76}&-\frac{17}{76}\\0&1&-\frac{65}{76}&-\frac{9}{76}\\0&0&1&\frac{1}{3}\end{pmatrix}\xrightarrow{\text{line1 += line3×}\left(\frac{13}{76}\right)}\begin{pmatrix}1&0&0&-\frac{1}{6}\\0&1&-\frac{65}{76}&-\frac{9}{76}\\0&0&1&\frac{1}{3}\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3×}\left(\frac{65}{76}\right)}\begin{pmatrix}1&0&0&-\frac{1}{6}\\0&1&0&\frac{1}{6}\\0&0&1&\frac{1}{3}\end{pmatrix}$$

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$$\begin{pmatrix}-3&-2&-7&-5\\0&-1&-2&-6\\-1&-2&-5&-9\end{pmatrix}\xrightarrow{\text{line1 ↔ line3}}\begin{pmatrix}-1&-2&-5&-9\\0&-1&-2&-6\\-3&-2&-7&-5\end{pmatrix}\xrightarrow{\text{line1 ×= (−1)}}\begin{pmatrix}1&2&5&9\\0&-1&-2&-6\\-3&-2&-7&-5\end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(3)}}\begin{pmatrix}1&2&5&9\\0&-1&-2&-6\\0&4&8&22\end{pmatrix}\xrightarrow{\text{line2 ×= (−1)}}\begin{pmatrix}1&2&5&9\\0&1&2&6\\0&4&8&22\end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2×(2)}}\begin{pmatrix}1&0&1&-3\\0&1&2&6\\0&4&8&22\end{pmatrix}\xrightarrow{\text{line3 -= line2×(4)}}\begin{pmatrix}1&0&1&-3\\0&1&2&6\\0&0&0&-2\end{pmatrix}$$
$$\xrightarrow{\text{line3 ×= }\left(-\frac{1}{2}\right)}\begin{pmatrix}1&0&1&-3\\0&1&2&6\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 += line3×(3)}}\begin{pmatrix}1&0&1&0\\0&1&2&6\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line3×(6)}}\begin{pmatrix}1&0&1&0\\0&1&2&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}-3&2&4&-8\\3&3&-9&-5\\2&3&-7&-6\end{pmatrix}\xrightarrow{\text{line1 += line2}}\begin{pmatrix}0&5&-5&-13\\3&3&-9&-5\\2&3&-7&-6\end{pmatrix}\xrightarrow{\text{line2 -= line3}}\begin{pmatrix}0&5&-5&-13\\1&0&-2&1\\2&3&-7&-6\end{pmatrix}$$
$$\xrightarrow{\text{line1 ↔ line2}}\begin{pmatrix}1&0&-2&1\\0&5&-5&-13\\2&3&-7&-6\end{pmatrix}\xrightarrow{\text{line3 -= line1×(2)}}\begin{pmatrix}1&0&-2&1\\0&5&-5&-13\\0&3&-3&-8\end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line3×(2)}}\begin{pmatrix}1&0&-2&1\\0&-1&1&3\\0&3&-3&-8\end{pmatrix}\xrightarrow{\text{line2 ×= (−1)}}\begin{pmatrix}1&0&-2&1\\0&1&-1&-3\\0&3&-3&-8\end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line2×(3)}}\begin{pmatrix}1&0&-2&1\\0&1&-1&-3\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 -= line3}}\begin{pmatrix}1&0&-2&0\\0&1&-1&-3\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3×(3)}}\begin{pmatrix}1&0&-2&0\\0&1&-1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}-3&2&9&-5\\-1&-1&3&-1\\-2&-6&6&0\end{pmatrix}\xrightarrow{\text{line1} \leftrightarrow \text{line3}}\begin{pmatrix}-2&-6&6&0\\-1&-1&3&-1\\-3&2&9&-5\end{pmatrix}\xrightarrow{\text{line1} \times= \left(-\frac{1}{2}\right)}\begin{pmatrix}1&3&-3&0\\-1&-1&3&-1\\-3&2&9&-5\end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line1}}\begin{pmatrix}1&3&-3&0\\0&2&0&-1\\-3&2&9&-5\end{pmatrix}\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(3)}\begin{pmatrix}1&3&-3&0\\0&2&0&-1\\0&11&0&-5\end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(5)}\begin{pmatrix}1&3&-3&0\\0&2&0&-1\\0&1&0&0\end{pmatrix}\xrightarrow{\text{line2} \leftrightarrow \text{line3}}\begin{pmatrix}1&3&-3&0\\0&1&0&0\\0&2&0&-1\end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(3)}\begin{pmatrix}1&0&-3&0\\0&1&0&0\\0&2&0&-1\end{pmatrix}\xrightarrow{\text{line3} \mathrel{-}= \text{line2}\times(2)}\begin{pmatrix}1&0&-3&0\\0&1&0&0\\0&0&0&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3} \times= (-1)}\begin{pmatrix}1&0&-3&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}-2&-8&2&7\\1&-8&7&9\\-6&-3&-8&-1\end{pmatrix}\xrightarrow{\text{line1} \leftrightarrow \text{line2}}\begin{pmatrix}1&-8&7&9\\-2&-8&2&7\\-6&-3&-8&-1\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&-8&7&9\\0&-24&16&25\\-6&-3&-8&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(6)}\begin{pmatrix}1&-8&7&9\\0&-24&16&25\\0&-51&34&53\end{pmatrix}\xrightarrow{\text{line2} \times= \left(-\frac{1}{24}\right)}\begin{pmatrix}1&-8&7&9\\0&1&-\frac{2}{3}&-\frac{25}{24}\\0&-51&34&53\end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{+}= \text{line2}\times(8)}\begin{pmatrix}1&0&\frac{5}{3}&\frac{2}{3}\\0&1&-\frac{2}{3}&-\frac{25}{24}\\0&-51&34&53\end{pmatrix}\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(51)}\begin{pmatrix}1&0&\frac{5}{3}&\frac{2}{3}\\0&1&-\frac{2}{3}&-\frac{25}{24}\\0&0&0&-\frac{1}{8}\end{pmatrix}$$
$$\xrightarrow{\text{line3} \times= (-8)}\begin{pmatrix}1&0&\frac{5}{3}&\frac{2}{3}\\0&1&-\frac{2}{3}&-\frac{25}{24}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times\left(\frac{2}{3}\right)}\begin{pmatrix}1&0&\frac{5}{3}&0\\0&1&-\frac{2}{3}&-\frac{25}{24}\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{25}{24}\right)}\begin{pmatrix}1&0&\frac{5}{3}&0\\0&1&-\frac{2}{3}&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}-2&-4&-5&2\\1&1&3&-1\\-2&-6&-1&1\end{pmatrix}\xrightarrow{\text{line1} \leftrightarrow \text{line2}}\begin{pmatrix}1&1&3&-1\\-2&-4&-5&2\\-2&-6&-1&1\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&1&3&-1\\0&-2&1&0\\-2&-6&-1&1\end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&1&3&-1\\0&-2&1&0\\0&-4&5&-1\end{pmatrix}\xrightarrow{\text{line2} \times= \left(-\frac{1}{2}\right)}\begin{pmatrix}1&1&3&-1\\0&1&-\frac{1}{2}&0\\0&-4&5&-1\end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}}\begin{pmatrix}1&0&\frac{7}{2}&-1\\0&1&-\frac{1}{2}&0\\0&-4&5&-1\end{pmatrix}\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(4)}\begin{pmatrix}1&0&\frac{7}{2}&-1\\0&1&-\frac{1}{2}&0\\0&0&3&-1\end{pmatrix}\xrightarrow{\text{line3} \times= \left(\frac{1}{3}\right)}\begin{pmatrix}1&0&\frac{7}{2}&-1\\0&1&-\frac{1}{2}&0\\0&0&1&-\frac{1}{3}\end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times\left(\frac{7}{2}\right)}\begin{pmatrix}1&0&0&\frac{1}{6}\\0&1&-\frac{1}{2}&0\\0&0&1&-\frac{1}{3}\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{1}{2}\right)}\begin{pmatrix}1&0&0&\frac{1}{6}\\0&1&0&-\frac{1}{6}\\0&0&1&-\frac{1}{3}\end{pmatrix}$$

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$$\begin{pmatrix}-1&-3&5&0\\-2&-1&5&-2\\2&-2&-2&3\end{pmatrix}\xrightarrow{\text{line1} \times= (-1)}\begin{pmatrix}1&3&-5&0\\-2&-1&5&-2\\2&-2&-2&3\end{pmatrix}\xrightarrow{\text{line2} \mathrel{+}= \text{line1}\times(2)}\begin{pmatrix}1&3&-5&0\\0&5&-5&-2\\2&-2&-2&3\end{pmatrix}$$
$$\xrightarrow{\text{line3} \mathrel{-}= \text{line1}\times(2)}\begin{pmatrix}1&3&-5&0\\0&5&-5&-2\\0&-8&8&3\end{pmatrix}\xrightarrow{\text{line2} \times= \left(\frac{1}{5}\right)}\begin{pmatrix}1&3&-5&0\\0&1&-1&-\frac{2}{5}\\0&-8&8&3\end{pmatrix}$$
$$\xrightarrow{\text{line1} \mathrel{-}= \text{line2}\times(3)}\begin{pmatrix}1&0&-2&\frac{6}{5}\\0&1&-1&-\frac{2}{5}\\0&-8&8&3\end{pmatrix}\xrightarrow{\text{line3} \mathrel{+}= \text{line2}\times(8)}\begin{pmatrix}1&0&-2&\frac{6}{5}\\0&1&-1&-\frac{2}{5}\\0&0&0&-\frac{1}{5}\end{pmatrix}$$
$$\xrightarrow{\text{line3} \times= (-5)}\begin{pmatrix}1&0&-2&\frac{6}{5}\\0&1&-1&-\frac{2}{5}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1} \mathrel{-}= \text{line3}\times\left(\frac{6}{5}\right)}\begin{pmatrix}1&0&-2&0\\0&1&-1&-\frac{2}{5}\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2} \mathrel{+}= \text{line3}\times\left(\frac{2}{5}\right)}\begin{pmatrix}1&0&-2&0\\0&1&-1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}-1&1&5&-2\\8&-8&0&-3\\-6&6&9&-2\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&-1&-5&2\\8&-8&0&-3\\-6&6&9&-2\end{pmatrix}\xrightarrow{\text{line2 }-=\text{ line1}\times(8)}\begin{pmatrix}1&-1&-5&2\\0&0&40&-19\\-6&6&9&-2\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line1}\times(6)}\begin{pmatrix}1&-1&-5&2\\0&0&40&-19\\0&0&-21&10\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(\frac{1}{40}\right)}\begin{pmatrix}1&-1&-5&2\\0&0&1&-\frac{19}{40}\\0&0&-21&10\end{pmatrix}$$
$$\xrightarrow{\text{line1 }+=\text{ line2}\times(5)}\begin{pmatrix}1&-1&0&-\frac{3}{8}\\0&0&1&-\frac{19}{40}\\0&0&-21&10\end{pmatrix}\xrightarrow{\text{line3 }+=\text{ line2}\times(21)}\begin{pmatrix}1&-1&0&-\frac{3}{8}\\0&0&1&-\frac{19}{40}\\0&0&0&\frac{1}{40}\end{pmatrix}$$
$$\xrightarrow{\text{line3 }\times=(40)}\begin{pmatrix}1&-1&0&-\frac{3}{8}\\0&0&1&-\frac{19}{40}\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line3}\times\left(\frac{3}{8}\right)}\begin{pmatrix}1&-1&0&0\\0&0&1&-\frac{19}{40}\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 }+=\text{ line3}\times\left(\frac{19}{40}\right)}\begin{pmatrix}1&-1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}0&-2&2&6\\-1&1&-5&2\\-1&3&-7&-5\end{pmatrix}\xrightarrow{\text{line1 }\leftrightarrow\text{ line2}}\begin{pmatrix}-1&1&-5&2\\0&-2&2&6\\-1&3&-7&-5\end{pmatrix}\xrightarrow{\text{line1 }\times=(-1)}\begin{pmatrix}1&-1&5&-2\\0&-2&2&6\\-1&3&-7&-5\end{pmatrix}$$
$$\xrightarrow{\text{line3 }+=\text{ line1}}\begin{pmatrix}1&-1&5&-2\\0&-2&2&6\\0&2&-2&-7\end{pmatrix}\xrightarrow{\text{line2 }\times=\left(-\frac{1}{2}\right)}\begin{pmatrix}1&-1&5&-2\\0&1&-1&-3\\0&2&-2&-7\end{pmatrix}$$
$$\xrightarrow{\text{line1 }+=\text{ line2}}\begin{pmatrix}1&0&4&-5\\0&1&-1&-3\\0&2&-2&-7\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}\times(2)}\begin{pmatrix}1&0&4&-5\\0&1&-1&-3\\0&0&0&-1\end{pmatrix}$$
$$\xrightarrow{\text{line3 }\times=(-1)}\begin{pmatrix}1&0&4&-5\\0&1&-1&-3\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 }+=\text{ line3}\times(5)}\begin{pmatrix}1&0&4&0\\0&1&-1&-3\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 }+=\text{ line3}\times(3)}\begin{pmatrix}1&0&4&0\\0&1&-1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix}1&-3&-4&6\\-2&6&1&-7\\2&-6&-2&8\end{pmatrix}\xrightarrow{\text{line2 }+=\text{ line1}\times(2)}\begin{pmatrix}1&-3&-4&6\\0&0&-7&5\\2&-6&-2&8\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line1}\times(2)}\begin{pmatrix}1&-3&-4&6\\0&0&-7&5\\0&0&6&-4\end{pmatrix}$$
$$\xrightarrow{\text{line2 }+=\text{ line3}}\begin{pmatrix}1&-3&-4&6\\0&0&-1&1\\0&0&6&-4\end{pmatrix}\xrightarrow{\text{line2 }\times=(-1)}\begin{pmatrix}1&-3&-4&6\\0&0&1&-1\\0&0&6&-4\end{pmatrix}$$
$$\xrightarrow{\text{line1 }+=\text{ line2}\times(4)}\begin{pmatrix}1&-3&0&2\\0&0&1&-1\\0&0&6&-4\end{pmatrix}\xrightarrow{\text{line3 }-=\text{ line2}\times(6)}\begin{pmatrix}1&-3&0&2\\0&0&1&-1\\0&0&0&2\end{pmatrix}$$
$$\xrightarrow{\text{line3 }\times=\left(\frac{1}{2}\right)}\begin{pmatrix}1&-3&0&2\\0&0&1&-1\\0&0&0&1\end{pmatrix}\xrightarrow{\text{line1 }-=\text{ line3}\times(2)}\begin{pmatrix}1&-3&0&0\\0&0&1&-1\\0&0&0&1\end{pmatrix}$$
$$\xrightarrow{\text{line2 }+=\text{ line3}}\begin{pmatrix}1&-3&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

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$$\begin{pmatrix} 2 & 0 & -9 & 0 \\ -1 & 0 & 3 & 0 \\ 5 & -1 & -5 & -5 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} -1 & 0 & 3 & 0 \\ 2 & 0 & -9 & 0 \\ 5 & -1 & -5 & -5 \end{pmatrix} \xrightarrow{\text{line1} \times= (-1)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 2 & 0 & -9 & 0 \\ 5 & -1 & -5 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -3 & 0 \\ 5 & -1 & -5 & -5 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line1} \times (5)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -1 & 10 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & -1 & 10 & -5 \\ 0 & 0 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= (-1)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -10 & 5 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3} \times= \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -10 & 5 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line1} += \text{line3} \times (3)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & 5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} += \text{line3} \times (10)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 1 & 0 & -3 \\ 3 & 0 & 1 & 2 \\ 2 & 2 & -1 & -8 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1}} \begin{pmatrix} 2 & 1 & 0 & -3 \\ 1 & -1 & 1 & 5 \\ 2 & 2 & -1 & -8 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & -1 & 1 & 5 \\ 2 & 1 & 0 & -3 \\ 2 & 2 & -1 & -8 \end{pmatrix}$$
$$\xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & 1 & 5 \\ 0 & 3 & -2 & -13 \\ 2 & 2 & -1 & -8 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & -1 & 1 & 5 \\ 0 & 3 & -2 & -13 \\ 0 & 4 & -3 & -18 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line2}} \begin{pmatrix} 1 & -1 & 1 & 5 \\ 0 & 3 & -2 & -13 \\ 0 & 1 & -1 & -5 \end{pmatrix} \xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & -1 & 1 & 5 \\ 0 & 1 & -1 & -5 \\ 0 & 3 & -2 & -13 \end{pmatrix}$$
$$\xrightarrow{\text{line1} += \text{line2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ 0 & 3 & -2 & -13 \end{pmatrix} \xrightarrow{\text{line3} -= \text{line2} \times (3)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line2} += \text{line3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

(88)

$$\begin{pmatrix} 2 & 6 & 8 & -5 \\ 2 & 6 & 9 & -5 \\ 1 & 5 & 2 & -3 \end{pmatrix} \xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 2 & 6 & 9 & -5 \\ 2 & 6 & 8 & -5 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 0 & -4 & 5 & 1 \\ 2 & 6 & 8 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line3} -= \text{line1} \times (2)} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 0 & -4 & 5 & 1 \\ 0 & -4 & 4 & 1 \end{pmatrix} \xrightarrow{\text{line2} -= \text{line3}} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & -4 & 4 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 0 & -4 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line2} \times= \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 5 & 2 & -3 \\ 0 & 1 & -1 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1} -= \text{line2} \times (5)} \begin{pmatrix} 1 & 0 & 7 & -\frac{7}{4} \\ 0 & 1 & -1 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line1} -= \text{line3} \times (7)} \begin{pmatrix} 1 & 0 & 0 & -\frac{7}{4} \\ 0 & 1 & -1 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line2} += \text{line3}} \begin{pmatrix} 1 & 0 & 0 & -\frac{7}{4} \\ 0 & 1 & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 3 & -9 & 2 & 3 \\ 3 & -9 & -1 & 7 \\ -2 & 6 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 0 & 0 & 3 & -4 \\ 3 & -9 & -1 & 7 \\ -2 & 6 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line2 += line3}} \begin{pmatrix} 0 & 0 & 3 & -4 \\ 1 & -3 & 0 & 2 \\ -2 & 6 & 1 & -5 \end{pmatrix}$$
$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 3 & -4 \\ -2 & 6 & 1 & -5 \end{pmatrix} \xrightarrow{\text{line3 += line1}\times(2)} \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & -4 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(3)} \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } \times= (-1)} \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line3}\times(2)} \begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 5 & -5 & 9 & 3 \\ 2 & -2 & 3 & 3 \\ 1 & -1 & 2 & -1 \end{pmatrix} \xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & 3 \\ 5 & -5 & 9 & 3 \end{pmatrix} \xrightarrow{\text{line2 -= line1}\times(2)} \begin{pmatrix} 1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 5 \\ 5 & -5 & 9 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1}\times(5)} \begin{pmatrix} 1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & -1 & 8 \end{pmatrix} \xrightarrow{\text{line2 } \times= (-1)} \begin{pmatrix} 1 & -1 & 2 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & -1 & 8 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}\times(2)} \begin{pmatrix} 1 & -1 & 0 & 9 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & -1 & 8 \end{pmatrix} \xrightarrow{\text{line3 += line2}} \begin{pmatrix} 1 & -1 & 0 & 9 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line3 } \times= \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & -1 & 0 & 9 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line3}\times(9)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}\times(5)} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(91)

$$\begin{pmatrix} 7 & 4 & -1 & -1 \\ -2 & 2 & -6 & -1 \\ -5 & -1 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line1 += line2}\times(3)} \begin{pmatrix} 1 & 10 & -19 & -4 \\ -2 & 2 & -6 & -1 \\ -5 & -1 & -3 & 0 \end{pmatrix} \xrightarrow{\text{line2 += line1}\times(2)} \begin{pmatrix} 1 & 10 & -19 & -4 \\ 0 & 22 & -44 & -9 \\ -5 & -1 & -3 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1}\times(5)} \begin{pmatrix} 1 & 10 & -19 & -4 \\ 0 & 22 & -44 & -9 \\ 0 & 49 & -98 & -20 \end{pmatrix} \xrightarrow{\text{line2 } \times= \left(\frac{1}{22}\right)} \begin{pmatrix} 1 & 10 & -19 & -4 \\ 0 & 1 & -2 & -\frac{9}{22} \\ 0 & 49 & -98 & -20 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line2}\times(10)} \begin{pmatrix} 1 & 0 & 1 & \frac{1}{11} \\ 0 & 1 & -2 & -\frac{9}{22} \\ 0 & 49 & -98 & -20 \end{pmatrix} \xrightarrow{\text{line3 -= line2}\times(49)} \begin{pmatrix} 1 & 0 & 1 & \frac{1}{11} \\ 0 & 1 & -2 & -\frac{9}{22} \\ 0 & 0 & 0 & \frac{1}{22} \end{pmatrix}$$
$$\xrightarrow{\text{line3 } \times= (22)} \begin{pmatrix} 1 & 0 & 1 & \frac{1}{11} \\ 0 & 1 & -2 & -\frac{9}{22} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 -= line3}\times\left(\frac{1}{11}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & -\frac{9}{22} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}\times\left(\frac{9}{22}\right)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 9 & -2 & -6 & -6 \\ -4 & 0 & 1 & 1 \\ 6 & 3 & 4 & 4 \end{pmatrix} \xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ -4 & 0 & 1 & 1 \\ 6 & 3 & 4 & 4 \end{pmatrix} \xrightarrow{\text{line2 += line1×(4)}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ 0 & -8 & -15 & -15 \\ 6 & 3 & 4 & 4 \end{pmatrix}$$
$$\xrightarrow{\text{line3 -= line1×(6)}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ 0 & -8 & -15 & -15 \\ 0 & 15 & 28 & 28 \end{pmatrix} \xrightarrow{\text{line3 += line2×(2)}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ 0 & -8 & -15 & -15 \\ 0 & -1 & -2 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ 0 & -1 & -2 & -2 \\ 0 & -8 & -15 & -15 \end{pmatrix} \xrightarrow{\text{line2 ×= (−1)}} \begin{pmatrix} 1 & -2 & -4 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -8 & -15 & -15 \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -8 & -15 & -15 \end{pmatrix} \xrightarrow{\text{line3 += line2×(8)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line3×(2)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} -7 & 7 & -7 & -4 \\ 3 & -4 & 5 & 2 \\ 9 & -2 & -5 & 3 \end{pmatrix} \xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} -1 & -1 & 3 & 0 \\ 3 & -4 & 5 & 2 \\ 9 & -2 & -5 & 3 \end{pmatrix} \xrightarrow{\text{line1 ×= (−1)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 3 & -4 & 5 & 2 \\ 9 & -2 & -5 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line2 -= line1×(3)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & -7 & 14 & 2 \\ 9 & -2 & -5 & 3 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(9)}} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & -7 & 14 & 2 \\ 0 & -11 & 22 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{7}\right)} \begin{pmatrix} 1 & 1 & -3 & 0 \\ 0 & 1 & -2 & -\frac{2}{7} \\ 0 & -11 & 22 & 3 \end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & -1 & \frac{2}{7} \\ 0 & 1 & -2 & -\frac{2}{7} \\ 0 & -11 & 22 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(11)}} \begin{pmatrix} 1 & 0 & -1 & \frac{2}{7} \\ 0 & 1 & -2 & -\frac{2}{7} \\ 0 & 0 & 0 & -\frac{1}{7} \end{pmatrix} \xrightarrow{\text{line3 ×= (−7)}} \begin{pmatrix} 1 & 0 & -1 & \frac{2}{7} \\ 0 & 1 & -2 & -\frac{2}{7} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line1 -= line3×}\left(\frac{2}{7}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -\frac{2}{7} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 += line3×}\left(\frac{2}{7}\right)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(94)
$$\begin{pmatrix} -4 & -5 & -1 & -1 \\ -2 & -7 & -8 & 2 \\ -1 & 2 & 5 & -2 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} -1 & 2 & 5 & -2 \\ -2 & -7 & -8 & 2 \\ -4 & -5 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line1 ×= (−1)}} \begin{pmatrix} 1 & -2 & -5 & 2 \\ -2 & -7 & -8 & 2 \\ -4 & -5 & -1 & -1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1×(2)}} \begin{pmatrix} 1 & -2 & -5 & 2 \\ 0 & -11 & -18 & 6 \\ -4 & -5 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line3 += line1×(4)}} \begin{pmatrix} 1 & -2 & -5 & 2 \\ 0 & -11 & -18 & 6 \\ 0 & -13 & -21 & 7 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{11}\right)} \begin{pmatrix} 1 & -2 & -5 & 2 \\ 0 & 1 & \frac{18}{11} & -\frac{6}{11} \\ 0 & -13 & -21 & 7 \end{pmatrix} \xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & -\frac{19}{11} & \frac{10}{11} \\ 0 & 1 & \frac{18}{11} & -\frac{6}{11} \\ 0 & -13 & -21 & 7 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(13)}} \begin{pmatrix} 1 & 0 & -\frac{19}{11} & \frac{10}{11} \\ 0 & 1 & \frac{18}{11} & -\frac{6}{11} \\ 0 & 0 & \frac{3}{11} & -\frac{1}{11} \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{11}{3}\right)} \begin{pmatrix} 1 & 0 & -\frac{19}{11} & \frac{10}{11} \\ 0 & 1 & \frac{18}{11} & -\frac{6}{11} \\ 0 & 0 & 1 & -\frac{1}{3} \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line3×}\left(\frac{19}{11}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{18}{11} & -\frac{6}{11} \\ 0 & 0 & 1 & -\frac{1}{3} \end{pmatrix} \xrightarrow{\text{line2 -= line3×}\left(\frac{18}{11}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} \end{pmatrix}$$

(95)
$$\begin{pmatrix} -2 & -3 & 7 & 3 \\ -6 & -1 & 6 & 1 \\ 9 & -7 & 7 & 7 \end{pmatrix} \xrightarrow{\text{line3 += line1×(4)}} \begin{pmatrix} -2 & -3 & 7 & 3 \\ -6 & -1 & 6 & 1 \\ 1 & -19 & 35 & 19 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 1 & -19 & 35 & 19 \\ -6 & -1 & 6 & 1 \\ -2 & -3 & 7 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line1×(6)}} \begin{pmatrix} 1 & -19 & 35 & 19 \\ 0 & -115 & 216 & 115 \\ -2 & -3 & 7 & 3 \end{pmatrix} \xrightarrow{\text{line3 += line1×(2)}} \begin{pmatrix} 1 & -19 & 35 & 19 \\ 0 & -115 & 216 & 115 \\ 0 & -41 & 77 & 41 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ×= } \left(-\frac{1}{115}\right)} \begin{pmatrix} 1 & -19 & 35 & 19 \\ 0 & 1 & -\frac{216}{115} & -1 \\ 0 & -41 & 77 & 41 \end{pmatrix} \xrightarrow{\text{line1 += line2×(19)}} \begin{pmatrix} 1 & 0 & -\frac{79}{115} & 0 \\ 0 & 1 & -\frac{216}{115} & -1 \\ 0 & -41 & 77 & 41 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line2×(41)}} \begin{pmatrix} 1 & 0 & -\frac{79}{115} & 0 \\ 0 & 1 & -\frac{216}{115} & -1 \\ 0 & 0 & -\frac{1}{115} & 0 \end{pmatrix} \xrightarrow{\text{line3 ×= (−115)}} \begin{pmatrix} 1 & 0 & -\frac{79}{115} & 0 \\ 0 & 1 & -\frac{216}{115} & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line3×}\left(\frac{79}{115}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{216}{115} & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\text{line2 += line3×}\left(\frac{216}{115}\right)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(96)
$$\begin{pmatrix} -1 & -1 & 0 & -2 \\ -1 & -5 & 3 & 5 \\ -1 & -5 & 4 & 7 \end{pmatrix} \xrightarrow{\text{line1 ×= (−1)}} \begin{pmatrix} 1 & 1 & 0 & 2 \\ -1 & -5 & 3 & 5 \\ -1 & -5 & 4 & 7 \end{pmatrix} \xrightarrow{\text{line2 += line1}} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & -4 & 3 & 7 \\ -1 & -5 & 4 & 7 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1}} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & -4 & 3 & 7 \\ 0 & -4 & 4 & 9 \end{pmatrix} \xrightarrow{\text{line2 −= line3}} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & -4 & 4 & 9 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & -4 & 4 & 9 \\ 0 & 0 & -1 & -2 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & -1 & -\frac{9}{4} \\ 0 & 0 & -1 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line1 −= line2}} \begin{pmatrix} 1 & 0 & 1 & \frac{17}{4} \\ 0 & 1 & -1 & -\frac{9}{4} \\ 0 & 0 & -1 & -2 \end{pmatrix} \xrightarrow{\text{line3 ×= (−1)}} \begin{pmatrix} 1 & 0 & 1 & \frac{17}{4} \\ 0 & 1 & -1 & -\frac{9}{4} \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{\text{line1 −= line3}} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{4} \\ 0 & 1 & -1 & -\frac{9}{4} \\ 0 & 0 & 1 & 2 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 0 & 0 & \frac{9}{4} \\ 0 & 1 & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

(97)
$$\begin{pmatrix} 0 & -3 & 1 & 1 \\ -2 & 1 & -1 & 1 \\ -3 & 3 & -2 & 0 \end{pmatrix} \xrightarrow{\text{line2 −= line3}} \begin{pmatrix} 0 & -3 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ -3 & 3 & -2 & 0 \end{pmatrix} \xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -3 & 1 & 1 \\ -3 & 3 & -2 & 0 \end{pmatrix}$$
$$\xrightarrow{\text{line3 += line1×(3)}} \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -3 & 1 & 1 \\ 0 & -3 & 1 & 3 \end{pmatrix} \xrightarrow{\text{line2 −= line3}} \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & -3 & 1 & 3 \end{pmatrix}$$
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -3 & 1 & 3 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(-\frac{1}{3}\right)} \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$
$$\xrightarrow{\text{line1 += line2×(2)}} \begin{pmatrix} 1 & 0 & \frac{1}{3} & -1 \\ 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(-\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & \frac{1}{3} & -1 \\ 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line1 += line3}} \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{1}{3} & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\xrightarrow{\text{line2 += line3}} \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -2 & 6 & -1 \\ -4 & -4 & -4 & 7 \\ 3 & -1 & 7 & -3 \end{pmatrix} \xrightarrow{\text{line3 -= line1}} \begin{pmatrix} 2 & -2 & 6 & -1 \\ -4 & -4 & -4 & 7 \\ 1 & 1 & 1 & -2 \end{pmatrix} \xrightarrow{\text{line1 ↔ line3}} \begin{pmatrix} 1 & 1 & 1 & -2 \\ -4 & -4 & -4 & 7 \\ 2 & -2 & 6 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 += line1×(4)}} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & -1 \\ 2 & -2 & 6 & -1 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(2)}} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & -1 \\ 0 & -4 & 4 & 3 \end{pmatrix}$$

(98)
$$\xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & -4 & 4 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line2 ×= } \left(-\frac{1}{4}\right)} \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & -1 & -\frac{3}{4} \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & 2 & -\frac{5}{4} \\ 0 & 1 & -1 & -\frac{3}{4} \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow{\text{line3 ×= } (-1)} \begin{pmatrix} 1 & 0 & 2 & -\frac{5}{4} \\ 0 & 1 & -1 & -\frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×}\left(\frac{5}{4}\right)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & -\frac{3}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{line2 += line3×}\left(\frac{3}{4}\right)} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & -7 & 1 \\ -7 & -4 & 5 & -9 \\ 4 & 3 & -2 & 6 \end{pmatrix} \xrightarrow{\text{line1 -= line3}} \begin{pmatrix} 0 & -4 & -5 & -5 \\ -7 & -4 & 5 & -9 \\ 4 & 3 & -2 & 6 \end{pmatrix} \xrightarrow{\text{line2 += line3×(2)}} \begin{pmatrix} 0 & -4 & -5 & -5 \\ 1 & 2 & 1 & 3 \\ 4 & 3 & -2 & 6 \end{pmatrix}$$

$$\xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -4 & -5 & -5 \\ 4 & 3 & -2 & 6 \end{pmatrix} \xrightarrow{\text{line3 -= line1×(4)}} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & -4 & -5 & -5 \\ 0 & -5 & -6 & -6 \end{pmatrix}$$

(99)
$$\xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -6 & -6 \end{pmatrix} \xrightarrow{\text{line1 -= line2×(2)}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -6 & -6 \end{pmatrix}$$

$$\xrightarrow{\text{line3 += line2×(5)}} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \xrightarrow{\text{line3 ×= } (-1)} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line3}} \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -7 & -5 & -3 & -3 \\ 7 & 5 & 5 & 5 \\ 4 & 3 & 4 & 4 \end{pmatrix} \xrightarrow{\text{line1 += line2}} \begin{pmatrix} 0 & 0 & 2 & 2 \\ 7 & 5 & 5 & 5 \\ 4 & 3 & 4 & 4 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(2)}} \begin{pmatrix} 0 & 0 & 2 & 2 \\ -1 & -1 & -3 & -3 \\ 4 & 3 & 4 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{line1 ↔ line2}} \begin{pmatrix} -1 & -1 & -3 & -3 \\ 0 & 0 & 2 & 2 \\ 4 & 3 & 4 & 4 \end{pmatrix} \xrightarrow{\text{line1 ×= } (-1)} \begin{pmatrix} 1 & 1 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 4 & 3 & 4 & 4 \end{pmatrix}$$

(100)
$$\xrightarrow{\text{line3 -= line1×(4)}} \begin{pmatrix} 1 & 1 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & -1 & -8 & -8 \end{pmatrix} \xrightarrow{\text{line2 ↔ line3}} \begin{pmatrix} 1 & 1 & 3 & 3 \\ 0 & -1 & -8 & -8 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{line2 ×= } (-1)} \begin{pmatrix} 1 & 1 & 3 & 3 \\ 0 & 1 & 8 & 8 \\ 0 & 0 & 2 & 2 \end{pmatrix} \xrightarrow{\text{line1 -= line2}} \begin{pmatrix} 1 & 0 & -5 & -5 \\ 0 & 1 & 8 & 8 \\ 0 & 0 & 2 & 2 \end{pmatrix} \xrightarrow{\text{line3 ×= } \left(\frac{1}{2}\right)} \begin{pmatrix} 1 & 0 & -5 & -5 \\ 0 & 1 & 8 & 8 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 += line3×(5)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 8 & 8 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{\text{line2 -= line3×(8)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$