In our paper [MNN24] the assertion of Proposition 3.2 is correct. However, **the proof includes some confusing typos.** We show a correct proof.

Proposition 3.2. *Assume that* $p = 3$. *Let* L *be given by (1.8). If* $\lambda \in \mathbb{C} \setminus \sigma_L$, *then* $(L-\lambda)^{-1}[1]$ *and* $(L-\lambda)^{-1}[(u_n^{\pm})^2]$ *can be explicitly written as follows:*

$$(L-\lambda)^{-1}[1] = \frac{(3-\lambda) - 3(u_n^{\pm})^2}{\lambda^2 - 2\lambda - 3(\alpha^2-1)^2} \text{ and } (L-\lambda)^{-1}[(u_n^{\pm})^2] = \frac{(2\alpha^2 - \alpha^4) - (1+\lambda)(u_n^{\pm})^2}{\lambda^2 - 2\lambda - 3(\alpha^2-1)^2}, \quad (3.3)$$

where $\alpha := u_n^{\pm}(0)$ *given by (2.6).*

Proof. Let $u := u_n^{\pm}$ for simplicity. Multiplying (1.5) by u' and integrating it over $[0, x]$ we have

$$\frac{\varepsilon^2 u'^2}{2} - \frac{u^2}{2} + \frac{u^4}{4} = -\frac{\alpha^2}{2} + \frac{\alpha^4}{4},$$

and hence

$$L[u^2] = 2\varepsilon^2 u u'' + 2\varepsilon^2 u'^2 - u^2 + u^4 = 3u^2 - 2\alpha^2 + \alpha^4. \quad (3.4)$$

Using $L[1] = -1 + 3u^2$ and (3.4), by direct calculation we can check that

$$(L-\lambda)\left[\frac{(3-\lambda) - 3u^2}{\lambda^2 - 2\lambda - 3(\alpha^2-1)^2}\right] = 1 \quad \text{and} \quad (L-\lambda)\left[\frac{(2\alpha^2-\alpha^4) - (1+\lambda)u^2}{\lambda^2 - 2\lambda - 3(\alpha^2-1)^2}\right] = u^2.$$

□

In the paper [MNN24, page 47],

$$\frac{\varepsilon^2 u'^2}{2} + \frac{u^2}{2} - \frac{u^4}{4} = \frac{\alpha^2}{2} - \frac{\alpha^4}{4}$$

is not correct and it should be

$$\frac{\varepsilon^2 u'^2}{2} - \frac{u^2}{2} + \frac{u^4}{4} = -\frac{\alpha^2}{2} + \frac{\alpha^4}{4},$$

and

$$L[1] = 1 - 3u^2$$

is not correct and it should be

$$L[1] = -1 + 3u^2.$$

However, (3,3) is correct. This note is written on Nov. 19th, 2024.

References

[MNN24] Y. Miyamoto, H. Nakamura and K. Nishigaki, *Exact periods and exact critical values for Hopf bifurcations from multi-peak solutions of the shadow Gierer-Meinhardt model in one spatial dimension*, J. Differential Equations **413** (2024), 34–69.