# Adhesion of membranes and filaments : statics and dynamics

Olivier Pierre-Louis

ILM, Lyon, France.
olivier.pierre-louis@univ-lyon1.fr

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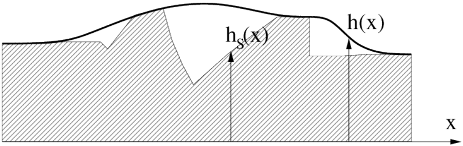
# adhesion/patterns

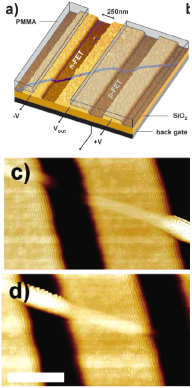
# Main question:



# Carbon nanostructures

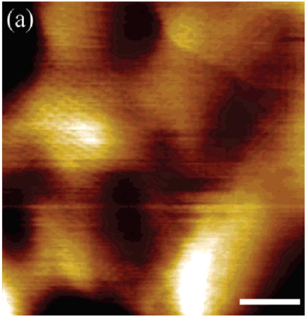
Nanotubes on patterns, or grids

Derike *et al*, NanoLetters (2001).




Graphene on rough $SiO_2$

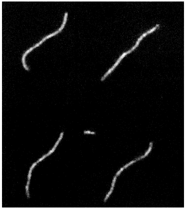
E.D. Williams *et al*, NanoLetters (2007); scale-bar 2nm.



# Biological membranes and filaments

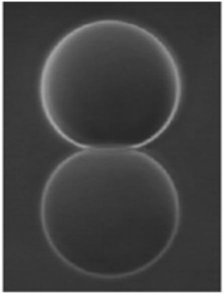
## Actin filament

A. Libchaber et al Phys Rev E 1993

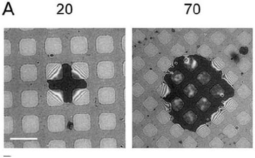


## Lipid membrane vesicle

M. Abkarian A. Viallat, Biophys J. (2005); $D = 130\mu$m

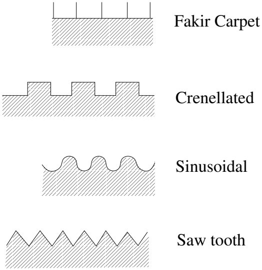


K. Sengupta et al , Soft Matter 2012



# Lignes directrices

# 1D model: Filament or membrane on patterns

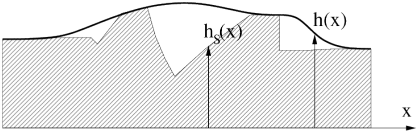



Wavelength $\lambda$
Amplitude $\epsilon$

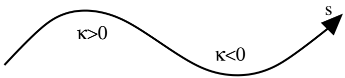
## Total energy

$$\mathcal{E} = \int ds \left[ \frac{C}{2} \kappa(s)^2 + \sigma + V(\mathbf{r}(s)) \right]$$

outside solid $V = 0$, surface $V = -\gamma$,
inside $V = +\infty$.
i.e. Deformations $\gg \ell_{eq}$
Adhesion energy $\gamma$
Bending rigidity $C$
Tension $\sigma$



# Equilibrium equations for the Euler elastica with adhesion

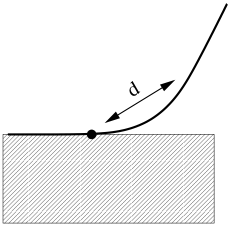


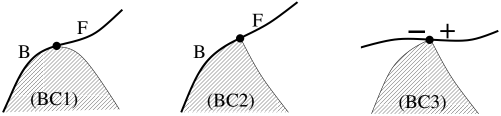

Free parts

$$\partial_{ss}\kappa + \frac{\kappa^3}{2} - \frac{\kappa}{d^2} = 0.$$

Euler-Bernoulli elastica model

Cut-off length $d = (C/\sigma)^{1/2}$




Boundary conditions

BC1

$$\kappa_F = \kappa_B - \kappa_{eq},$$

where $\kappa_{eq} = (2\gamma/C)^{1/2}$

BC2

$$\kappa_B - \kappa_{eq} \leq \kappa_F \leq \kappa_B + \kappa_{eq}.$$

Similar to the Gibbs Inequality Condition for the wetting contact angle

BC3

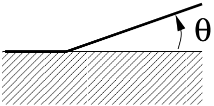
$$\kappa_+ = \kappa_-, \qquad \text{and} \qquad \partial_s\kappa_+ \leq \partial_s\kappa_-,$$

# Small slope regime

Free parts

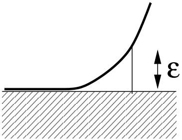
$$\partial_{xxxx} h - d^{-2} \partial_{xx} h = 0$$

$C \to 0$




BC1 $\to \sigma(1 + \cos\theta) = \gamma$
where $\theta$ contact angle
$\sigma \gg \gamma$, and $\partial_x h_s \ll 1$.

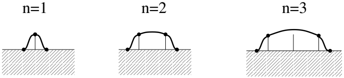
$\sigma \to 0$

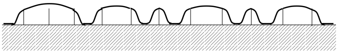


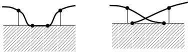

$\epsilon \kappa_{eq} \ll 1$

# Patterned substrates: a 1D model

## Constructing solutions from $n$-bridges





All possible solutions
Non-overlapping reduces the number of possible states

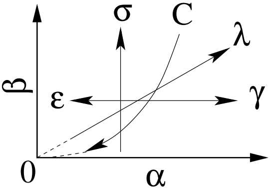
## Natural parameters

$$\beta = \frac{\lambda}{d}$$

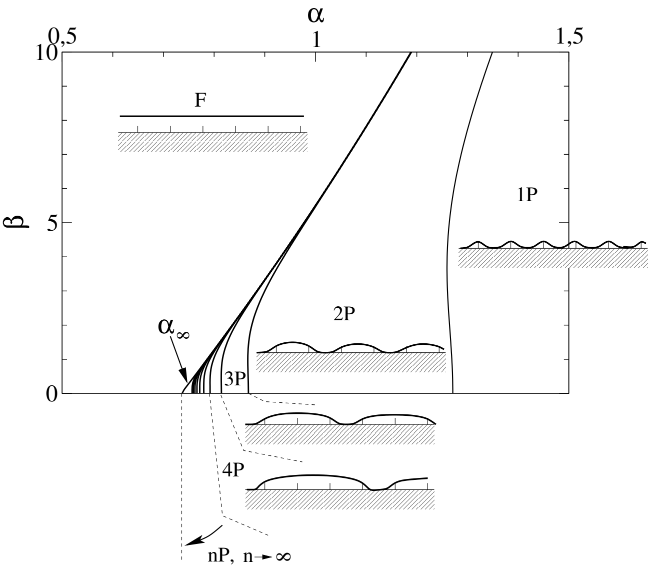
$\beta \gg 1$ tension dominates
$\beta \ll 1$ curvature dominates

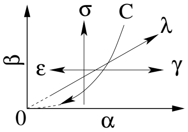
$$\alpha = \left(\frac{\kappa_{eq}}{\kappa_g}\right)^{1/2}$$

geometrical curvature $\kappa_g = 4\pi^2\epsilon/\lambda^2$
$\alpha \ll 1 \rightarrow$ Floating state
$\alpha \gg 1 \rightarrow$ Membrane follows patterns

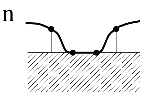


# Ground state Transitions

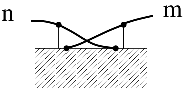


# Metastability

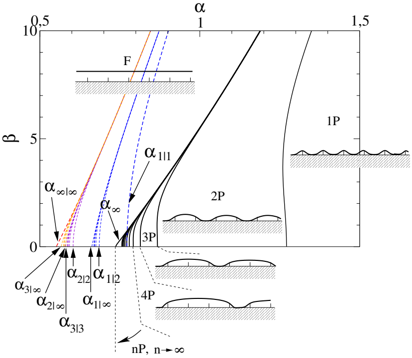


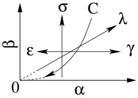

$\alpha > \alpha_{n|m}[\beta]$



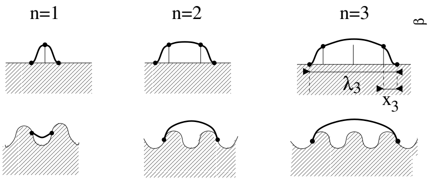

$\alpha < \alpha_{n|m}[\beta]$

$\alpha_{n|m}[\beta] > \alpha_{n|m+1}[\beta]$
$\alpha_{n|m}[\beta] = \alpha_{m|n}[\beta]$
$\rightarrow \alpha_{n|m}[\beta] > \alpha_{n|m+p}[\beta]$
But no "total order",
e.g. $\alpha_{5|5}[0] < \alpha_{4|9}[0]$, and $\alpha_{5|5}[1] > \alpha_{4|9}[1]$
$\rightarrow$ $\beta$-dependent decimation order
Non-sticky F ground state at $\alpha < \alpha_{\infty|\infty}[\beta]$

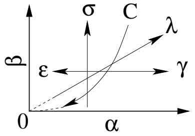


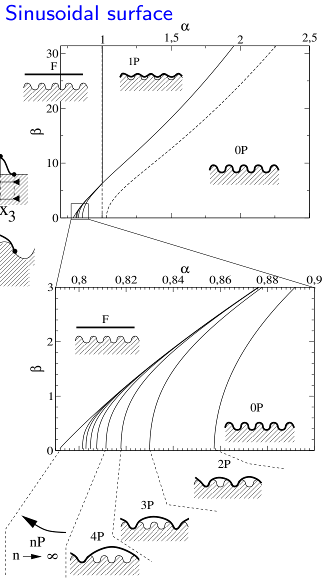



# Sinusoidal surfaces



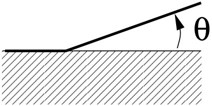

A single family of bridges for: fakir carpet and sinusoidal,
more for other surfaces



# No bending limit

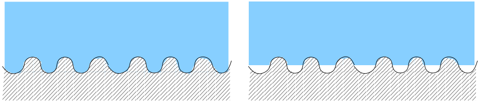
$C \to 0$




BC1 $\to \sigma(1+\cos\theta) = \gamma$
where $\theta$ contact angle
$\sigma \gg \gamma$, and $\partial_x h_s \ll 1$.

Self-consistent limit for sinusoidal
but not for Fakir Carpet.

Wenzel to Cassie-Baxter transition for sinusoidal



Bico, Marzolin, Quéré (1999)

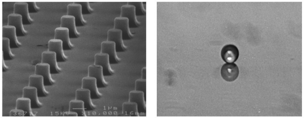
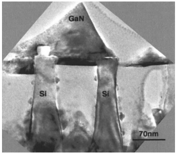


**Figure 12.** Substrate decorated with posts (the bar indicates 1 $\mu$m). If coated with a monolayer of fluorinated silanes, this substrate is found to be super-hydrophobic [37].

GaN Growth/ Si nano-pillars
Hersee *et al* J.Appl.Phys. 2005




- Liquids on micro-patterns:
  Vrancken, Kusumaatmaja, OPL, et al, Langmuir 2010
- Solids on nano-patterns:
  OPL, Saito, EPL 2009; Takano, Saito, OPL PRB 2010; Gaillard, Saito, OPL PRL 2011; Ignacio, OPL, PRB 2012

# Orders of magnitude

## Orders of magnitude

### Graphene

$C = 0.9\mathrm{eV}$, $\gamma \sim 6\mathrm{meV}\mathrm{\AA}^{-2}$, $\sigma \approx 0$
$\rightarrow \epsilon\kappa_{eq} \sim 1$.
$\epsilon = 1$nm, and $\lambda = 10$nm, $\Rightarrow \alpha \sim 2$
larger than 100nm follow

A. Incze, A. Pasturel and P. Peyla, Phys.Rev.B (2004)

Oxygen adsorption tunes bending rigidity:
12.5% oxygen $C = 40\mathrm{eV}$ Peyla et al
$\epsilon\kappa_{eq} < 1$.
$\epsilon = 1$nm, and $\lambda = 10$nm, $\Rightarrow \alpha \approx 0.6$.
Oxygen adsorption $\Rightarrow$ scan tranisiton region

## Orders of magnitude

lipid membranes(Swain and Andelman)
$C = 1.4 \times 10^{-19}$J, and $\sigma = 1.7 \times 10^{-5}\mathrm{Jm}^{-2}$
$\gamma = 5 \times 10^{-6}\mathrm{Jm}^{-2}$, $\ell_{eq} = 3$nm
Choosing
$\epsilon \approx 10\mathrm{nm} \gg \ell_{eq}$, $\lambda \approx 100\mathrm{nm} \gg \epsilon$,
we obtain
$\Rightarrow \alpha \approx 0.5$ and $\beta \approx 1$

### Nanotubes

$\sigma \approx 0$, $C = 20\mathrm{eV.nm}$, and $\gamma \approx 1\mathrm{eV.nm}^{-1}$
Choosing
$\epsilon = 5$nm, $\lambda = 50$nm
we obtain
$\alpha \approx 2$
(Nevertheless $\epsilon\kappa_{eq} \sim 1$)

# Lignes directrices

# Few-layer graphene on rough $SiO_2$

J. Nicolle, D Machon, P. Poncharal, OPL, A. San-Miguel, NanoLetters 2011

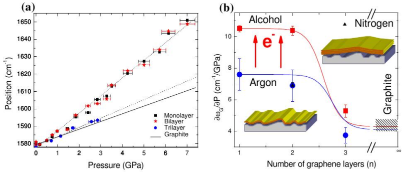



**Figure 2 (a) Evolution of the G band position under pressure for the monolayer sample (black square), bilayer sample (red star), and trilayer sample (blue circle). The solid line is the typical evolution under pressure for the graphite, and the dot lines are the linear fit of our data. (b) Summary of the G band pressure derivative position obtained in alcohol (red square), argon (blue circle) and nitrogen (green triangle) depending on the number of layers. The (e⁻) is the doping process by the alcohol which has been highlighted by our experiments. The inserts represent the samples adhesion evolution on the substrate topology depending on the number of layers.**

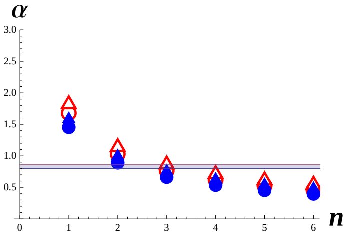
1D Model

$\rightarrow$ transition between $n = 2$ and $n = 3$

$$\alpha = \left(\frac{\kappa_{eq}}{\kappa_g}\right)^{1/2}$$

where $\kappa_{eq} = (2\gamma/C)^{1/2}$

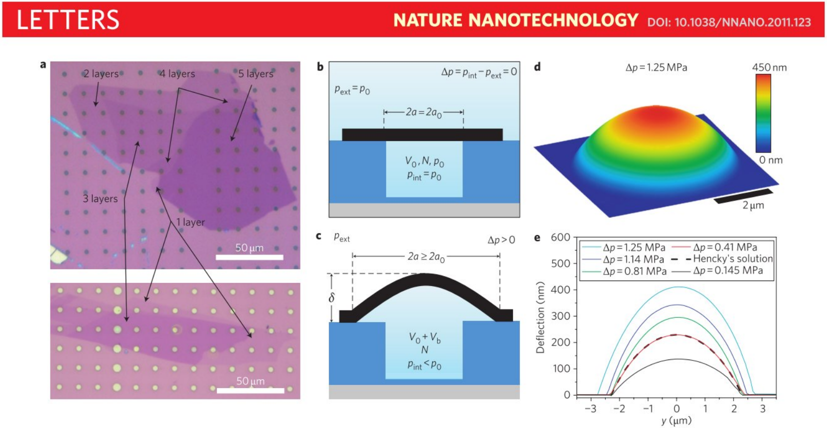
W. G. Cullen, *et al* Phys. Rev. Lett. 105, 215504 (2010)



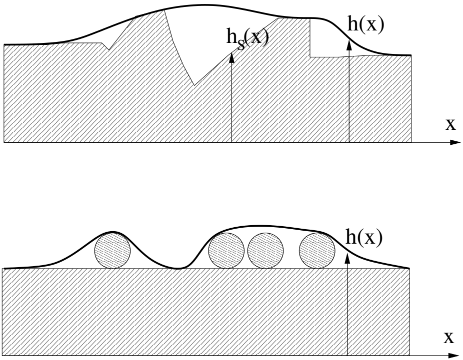
# Few-layer graphene on rough $SiO_2$

Scott Bunch et al Nat. Nanotech. (2011)

→ transition between $n = 1$ and $n = 2$

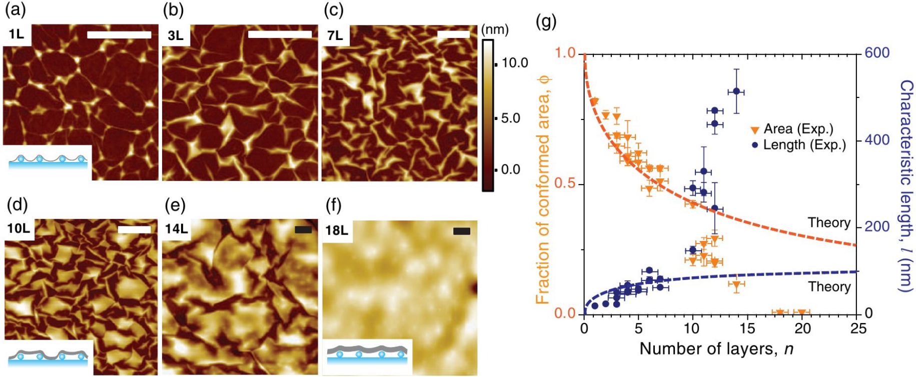


# Few-layer graphene on Nanoparticles

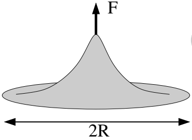


# Few-layer graphene on Nanoparticles

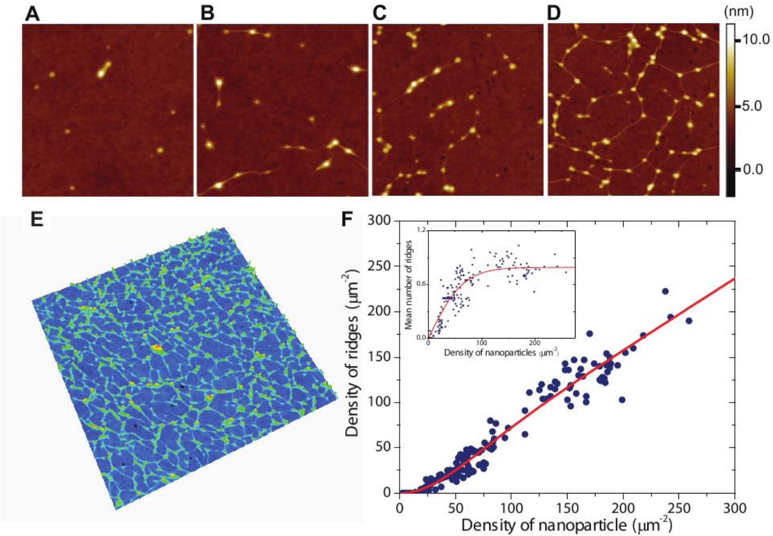
M. Yamamoto, OPL, J Huang, WG Cullen, TL Einstein, MS Fuhrer, Phys Rev X 2012




Large deviations: ~~Bending~~ $\rightarrow$ stretching
Föppl Von Karman equations $\rightarrow$ Schwerin (1929)
diameter detach. zone $2R = d(4nG/3\gamma_n)^{1/4}$
Low densities
$\phi = \exp[-\pi R^2 \rho_{np}]$



# Wrinkling of graphene



# Wrinkling of graphene

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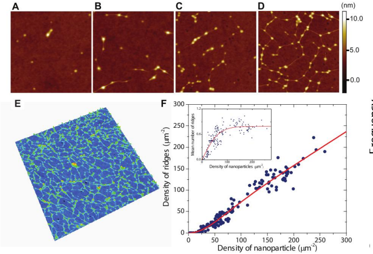
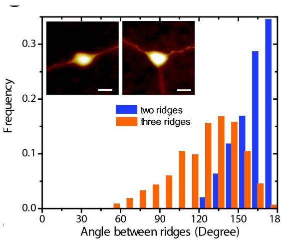



Random bonding model with 2 parameters: $r_*$, $\Pi_b$

$$\rho_b = \rho_{np}\frac{\Pi_b}{2}\left[-\pi r_*^2\rho_{np}(2+\frac{1}{2}\pi r_*^2\rho_{np})\mathrm{e}^{-\pi\rho_{np}r_*^2}+3(1-\mathrm{e}^{-\pi\rho_{np}r_*^2})\right]$$

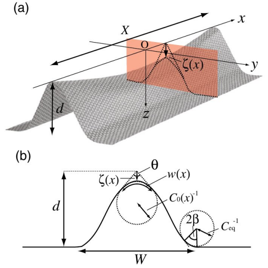
$$\rho_b \approx \frac{\Pi_b}{2}\rho_{np}^2\pi r_*^2 \qquad \rho_{np} \ll (\pi r_*^2)^{-1}$$

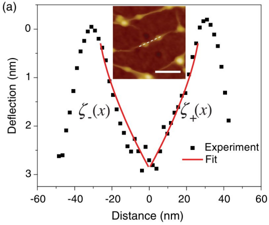
$$\rho_b \approx \frac{3\Pi_b}{2}\rho_{np} \qquad \rho_{np} \gg (\pi r_*^2)^{-1}$$

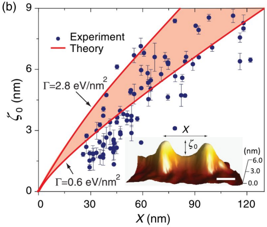
fit: $r_* \approx 120\text{nm}$, $\Pi_b \approx 0.5$

# Single wrinkle

M. Yamamoto, OPL, J Huang, WG Cullen, TL Einstein, MS Fuhrer, PRX 2012




A. Lobkovsky, S. Gentges, H. Li, D. Morse, TA. Witten, Science (1995) SV. Kusminskiy, DK. Campbell, AH. Castro Neto, F. Guinea, Phys Rev B (2011)

Longitudinal stretching vs transversal bending+adhesion

$$\mathcal{E} = (\pi-\alpha)\int dx\left[\frac{G}{8}(\csc(\alpha/2)-1)^{-1}\zeta(\partial_x\zeta)^4+\frac{\kappa}{2}(\csc(\alpha/2)-1)\frac{1}{\zeta}\right]+\gamma\mathcal{X}2d\tan(\alpha/2)$$

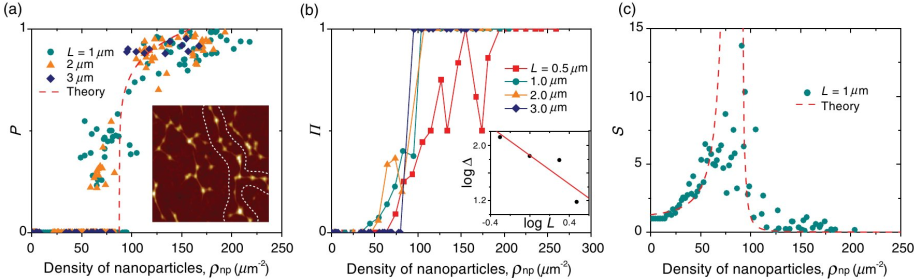
$$\zeta(x) = \frac{3^{1/2}}{2^{1/2}}\left(\frac{2C}{G}\right)^{1/6}\mathcal{X}^{2/3}\left(\frac{1}{\sin(\alpha/2)}-1\right)\left(\frac{1}{2}-\frac{|x|}{\mathcal{X}}\right)^{2/3}$$

$$\zeta(0) \sim G^{-1/5}\mathcal{X}^{4/5}\gamma^{1/5}d^{1/5} \Rightarrow \mathcal{X}_c \sim d\left(\frac{G}{\gamma}\right)^{1/4}$$

$G = 340\mathrm{J.m}^{-2}$, $\gamma = 0.1\mathrm{J.m}^{-2}$, $C = 2\times10^{-19}\mathrm{J} \rightarrow \mathcal{X}_c \approx 104\mathrm{nm} \approx r_*$

# Wrinkle percolation in graphene

M. Yamamoto, OPL, J Huang, WG Cullen, TL Einstein, MS Fuhrer, PRX 2012




Percolation threshold: $\rho_c \approx 90\mu\mathrm{m}^{-2}$, Dimensionless number $\rho_c \mathcal{X}_c^2 \approx 10$
Exponent: $\nu \approx 1.11 \pm 0.3$, models: $\nu = 4/3$
Asymmetry ratio: $Q \approx 30$, simulations: $Q = 50 \pm 26$, theory $Q \approx 200$

# Summary on graphene

Graphene on rough $SiO_2$

dimensionless parameter $\alpha$

conformal adhesion $\rightarrow$ partial unbinding? $\rightarrow$ unbinding

J. Nicolle, D Machon, P. Poncharal, OPL, A. San-Miguel, NanoLetters 2011

Graphene on nanoparticles

dimensionless parameter $\rho \mathcal{X}_c$

conformal adhesion $\rightarrow$ wrinkling $\rightarrow$ percol. of wrinkles $\rightarrow$ partial unbinding $\rightarrow$ unbinding

M. Yamamoto, OPL, J Huang, WG Cullen, TL Einstein, MS Fuhrer, PRX 2012

# Pseudo-Magnetic fields

A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, , Rev. Mod. Phys. 81, 109 (2009).
V. M. Pereira and A. H. Castro Neto, Phys. Rev. Lett. 103, 046801 (2009).
N. Levy, S. A. Burke, K. L. Meaker, M. Panlasigui, A. Zettl, F. Guinea, A. H. Castro Neto, and M. F. Crommie, Science 329, 544 (2010).

F. Guinea, M. I. Katsnelson, and A. K. Geim, Nat. Phys. 6, 30 (2010).

Strain → change bond length → change hopping amplitude → vector potential $\mathcal{A}$

$$\mathcal{A} = \frac{2\beta\hbar}{3a_0 e}(u_{xx} - u_{yy}, -2u_{xy}) \rightarrow B_{eff} = \partial_y A_x - \partial_x A_y$$

$\beta \approx 3$, and $u_{ij}$ in-plane strain
Around a single NP:

$$B_{eff} \sim \frac{\beta\hbar}{a_0 e}\left(\frac{\gamma d^2}{G}\right)^{1/3} r^{-5/3} \sin 3\phi \sim 300\mathrm{T}$$

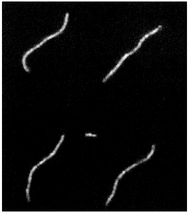
In ripples:
$B_{eff} \sim 10\mathrm{T}$

# Lignes directrices

- Unbinding transition on flat substrates:
  Rubin 1965, De Gennes 1969, Wiegel 1983, etc..
- Unbinding transition for semi-flexible filaments on flat substrates:
  Maggs, Huse, Leibler1989, Bundschuh and Lässig 2002, Benetatos and Frey 2003

Actin filament

A. Libchaber et al Phys Rev E 1993

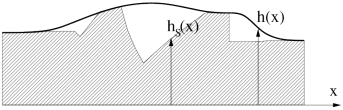


- **Filaments**
- **No tension** $\sigma = 0$
- **Sinusoidal substrates**

Free parts $x_2 - x_1 \ll L_p = 2C/k_B T$
$Z[x_1, x_2] = P[x_1, x_2] Z_{fe}[x_2 - x_1]$
Benetatos and Frey Phys Rev E 2004

$$P[x_1, x_2] = \left( \frac{3^{1/2} CB}{\pi k_B T} \right)^d \frac{\mathrm{e}^{-(C/2k_B T) \int_{x_1}^{x_2} dx (\partial_{xx} \bar{h}(x))^2}}{(x_2 - x_1)^{2d}}$$

Micro lengthscale $B = 4\delta h \delta\theta$
Macroscopic profile $\partial_{xxxx} \bar{h}(x) = 0$
Parts in adhesion
$Z_a[x_1, x_2] = P_a[x_1, x_2] Z_{fe}[x_2 - x_1]$




$$P_a[x_1, x_2] = \mathrm{e}^{-\{(C/2) \int_{x_1}^{x_2} dx (\partial_{xx} h_s(x))^2 - \gamma_T (x_2 - x_1)\}/k_B T}$$

$\gamma_T \to \gamma_0$ as $T \to 0$.
Configuration probability

$$\mathcal{P} = \prod_{i=1}^{m} P[x_i^-, x_i^+] P_a[x_i^+, x_{i+1}^-].$$

# Partition function

Blocks of length $n\lambda$
proba $P_n$

Partition function $Z_L = Z/Z_{fe}[L]$

$$Z_L = \sum_{p=1}^{L} \sum_{\{n_i\}\in\mathcal{R}(L,p)} \prod_{i=1}^{p} P_{n_i}$$

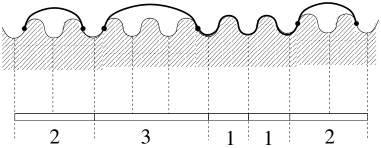
Recursion relation

$$Z_L = P_1 Z_{L-1} + P_2 Z_{L-2} + ... + P_{L-1} Z_1 + P_L$$

Generating function

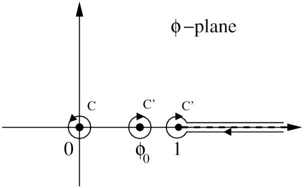
$$Z_\phi = \sum_{L=1}^{\infty} \phi^L Z_L = \frac{A_\phi}{1 - A_\phi}$$

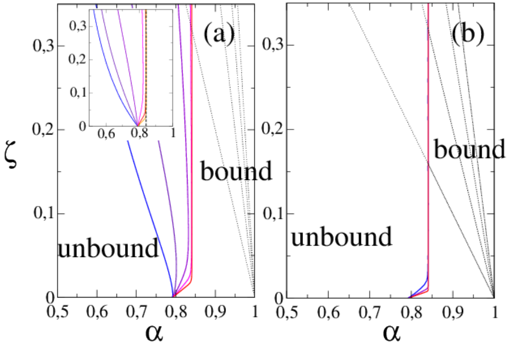
$$A_\phi = \sum_{n=1}^{\infty} P_n \phi^n$$



$$Z_L = \frac{1}{2i\pi} \oint \mathrm{d}\phi \, \phi^{-L-1} \frac{A_\phi}{1 - A_\phi}$$

Pole of $Z_\phi$ at $\phi_0$, with $A_{\phi_0} = 1$
Branch cut



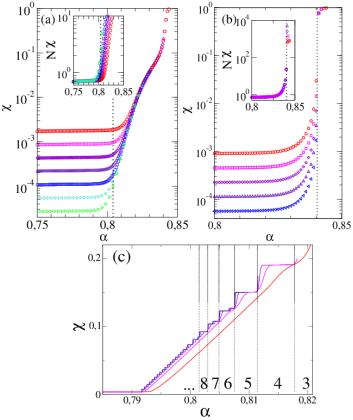

Transition $\phi_0 \to 1$
$\phi_0 < 1$ adhesion,
and Thermodynamic limit $Z_L \sim \phi_0^{-L}$
$\phi_0 \to 1$ unbound

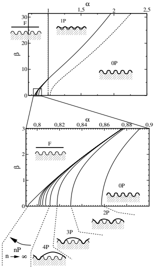
$$\begin{aligned} \zeta &= k_B T/(\lambda\gamma) \\ r &= (3^{1/2} CB/\pi k_B T)^d \lambda^{2d-2} L_m^{-2} \\ R &= r\zeta\alpha^4 = 3^{1/2} B\lambda^3/(8\pi^4 L_m^2 \epsilon^2). \end{aligned}$$

Adhesion fraction (fraction of filament in contact with substrate)

$$\chi_N = 1 - \sum_{n=1}^{N} \frac{Z_{n'}}{N Z_N} \left[ N - 2\xi_n^* - n'\chi_{n'} \right] P_n \qquad n' = N - n, \quad \text{and} \quad \chi_0 = 0$$

Order of the transition ← dim. transv. fluct.



## Actin on nanogrooves

Benetatos and Frey

$$(J/k_BT_f)^2(e^{J/k_BT_f} - 1) \approx 2(L_m^2 J^2/CB)^2,$$

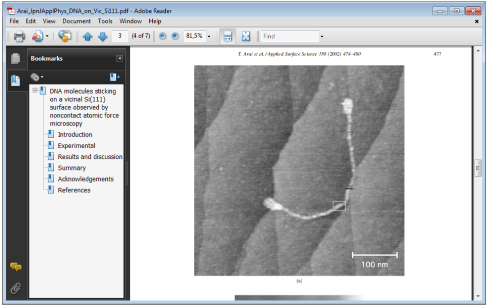
$C \sim 6 \times 10^{-26}$J.m,
$J \sim k_BT$
$L_m \sim 5$ nm
$\rightarrow T \approx 2000$K

## Nano-patterns

Line of binders $\sim k_BT$ perp to nanogrooves
$\epsilon \sim 50$ nm
$\lambda \sim 700$ nm at room temperature.

## DNA along atomic steps

T. Arai et al Appl Surf Sci (2001): Si(111)
Yoshida et al BioPhys Journ. sapphire steps
$\epsilon \sim 1$ nm, $\lambda \sim 10$nm, $L_p = 2C/k_BT \sim$ 50nm,
$\rightarrow \gamma \sim k_BT$ per nm
$\sim$ Van der Waals interaction




C. Misbah, OPL, Y. Saito, Rev. Mod. Phys. 2010

# Lignes directrices

# Conclusion

- 1D model: infinite staircase of transitions at $T = 0$
  OPL, Phys. Rev. E 78, 021603 (2008)

Exp. Collab.:

# Conclusion

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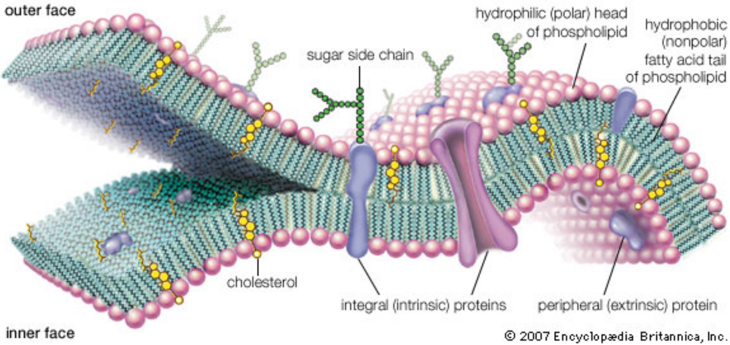
- J.P. Rieu, ILM-Lyon, AM Sfarghiu INSA-Lyon Lipid Membranes

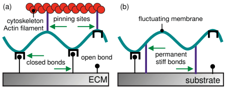
# adhesion/patterns

# adhesion/patterns

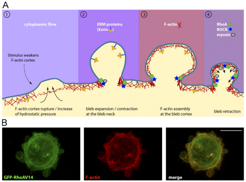
# Lipid membranes



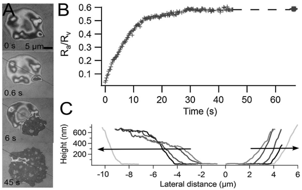
# Geometric confinement, double well potential



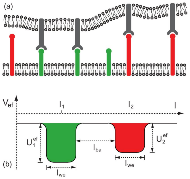

Speck, Vink Phys Rev E 2012




Oliver T. Fackler, Robert Grosse JCB 2008

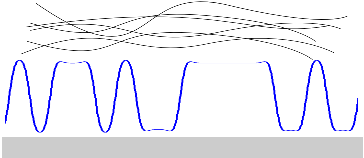



Sengupta Limozin PRL 2010




Rozycki , Lipowsky and T.R. Weikl NJP 2012

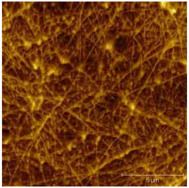
## Generic question: membrane in a two well potential



# Permeability of membrane environment

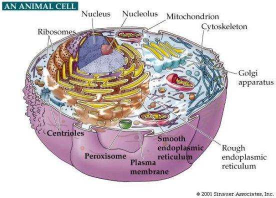
Biological environment is permeable

- Extracellular environment: collagen network



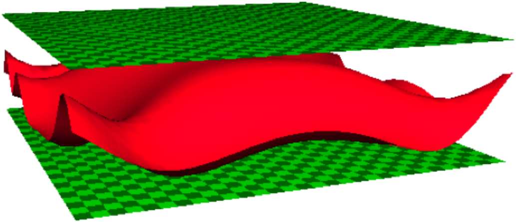

M. Loparik, Basel

- Cytoskeleton: Actin cortex, molecular crowding



# adhesion/patterns

# Model

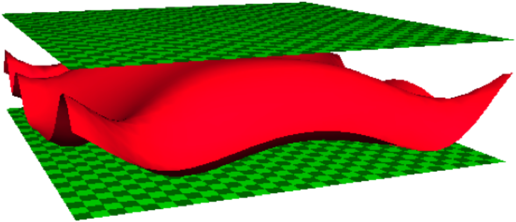


Ingredients

- Interaction with walls

Assumptions

# Model

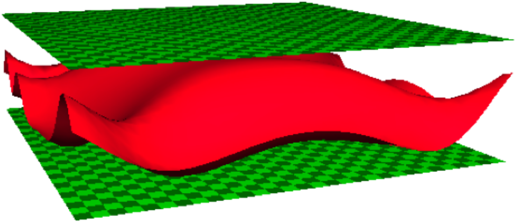


Ingredients

- Interaction with walls
- Membrane mechanics

Assumptions

# Model

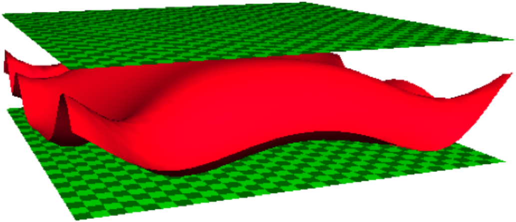


Ingredients

- Interaction with walls
- Membrane mechanics
- Hydrodynamics

Assumptions

# Model



Ingredients

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- Membrane mechanics
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Assumptions

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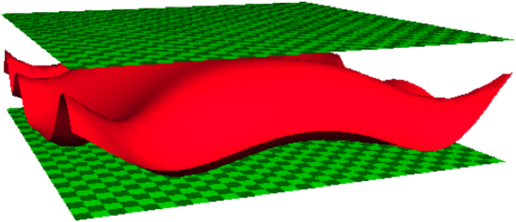

Ingredients

- Interaction with walls
- Membrane mechanics
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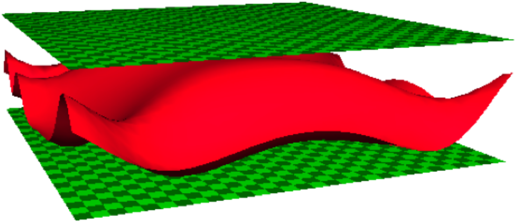
Assumptions

- Zero-tickness membrane

# Model



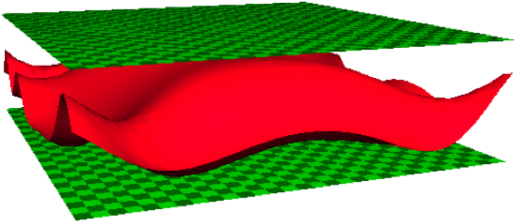
Ingredients

- Interaction with walls
- Membrane mechanics
- Hydrodynamics
- Wall permeability

Assumptions

- Zero-tickness membrane
- Up-down symmetric

# Model



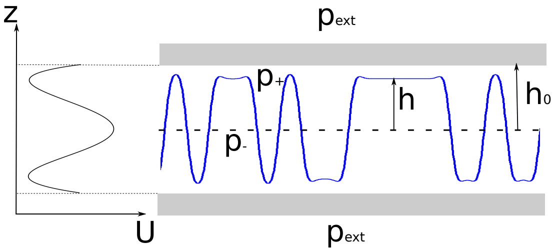
Ingredients

- Interaction with walls
- Membrane mechanics
- Hydrodynamics
- Wall permeability

Assumptions

- Zero-tickness membrane
- Up-down symmetric
- Flat walls

# Model




- Incompressible Stokes fluid: $0 = -\nabla p_\pm + \mu \Delta \mathbf{v}_\pm$ and $\nabla \cdot \mathbf{v} = 0$

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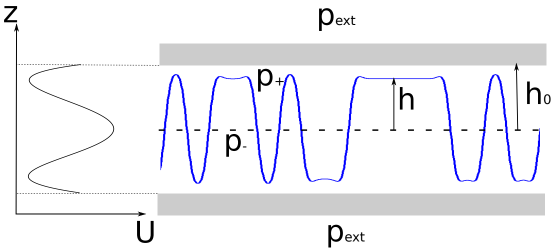


- Incompressible Stokes fluid: $0 = -\nabla p_{\pm} + \mu \Delta \mathbf{v}_{\pm}$ and $\nabla \cdot \mathbf{v} = 0$
- No slip at the membrane and at the wall: $\mathbf{v}_{+}|_{z=h(x,t)} = \mathbf{v}_{-}|_{z=h(x,t)}$ and $v_{x\pm}|_{z=\pm h_0} = 0$

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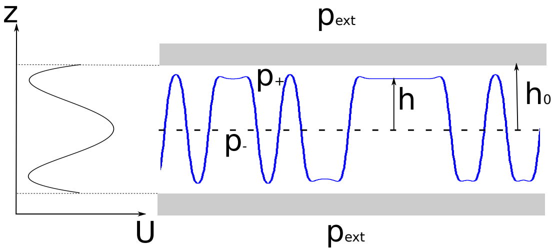


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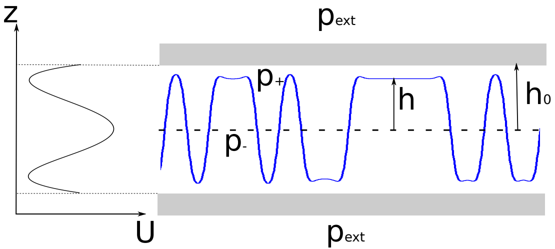
- Permeable walls: $v_{z\pm}|_{z=\pm h_0} = \pm\nu(p_{\pm} - p_{ext})$

# Model

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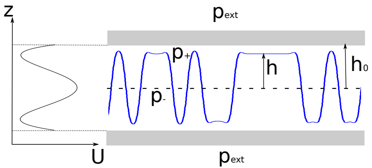
- Permeable walls: $v_{z\pm}|_{z=\pm h_0} = \pm\nu(p_\pm - p_{ext})$
- Membrane equilibrium: $[\sigma]_-^+ \cdot \mathbf{n} + \mathbf{f} = 0$
  $\sigma_{ij} = \mu(\partial_i v_j + \partial_j v_i) - p\delta_{ij}$

# Model




- Double-well adhesion potential $\mathcal{E}_a = \int\int d\mathcal{A}\,\mathcal{U}(h)$.
- Membrane bending rigidity $\mathcal{E}_b = \int\int d\mathcal{A}\,\frac{\kappa}{2} C^2$.
- Membrane inextensibility $\mathcal{E}_\Sigma = \int\int d\mathcal{A}\,\Sigma$.

Variation

$$\delta\left(\mathcal{E}_a + \mathcal{E}_b + \mathcal{E}_\Sigma\right) = \int\int d\mathcal{A}\,(\mathbf{f}\cdot\delta r)$$

Membrane forces

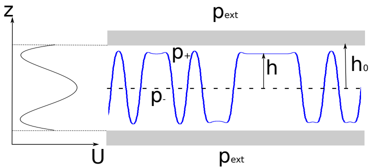
$$f_{t_j} = g^{ij}\partial_{s_i}(\Sigma + \mathcal{U}(\mathbf{r})) - \nabla\mathcal{U}(\mathbf{r})\cdot\mathbf{t}_j,$$

$$f_n = -\kappa\left(\Delta_b\mathcal{C} + \frac{\mathcal{C}^3}{2} - 2\mathcal{C}c_G\right) + (\Sigma + \mathcal{U}(\mathbf{r}))\mathcal{C} - \nabla\mathcal{U}(\mathbf{r})\cdot\mathbf{n}.$$

Area conservation $\rho_{2D} = [1 + (\nabla_{xy}h)^2]^{1/2}$

$$\partial_t\rho_{2D} + \nabla_{xy}.(\rho_{2D}\mathbf{v}_{2D}) = 0$$

# Model




## Lubrication expansion

$$x \sim \mathcal{O}(1), \quad y \sim \mathcal{O}(1), \quad h \sim h_0 << 1, \quad |\nabla h| << 1$$

Poiseuille flow

$$\mathbf{v}_{xy} = \frac{z^2}{2\mu} \nabla_{xy} p + \mathbf{a} z + \mathbf{b}$$

Force ( constant tension)

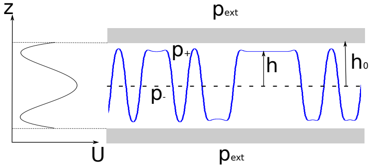
$$\mathbf{f} = \left(-\kappa \Delta_{xy}^2 h + \mathcal{S}_0 \Delta_{xy} h - \mathcal{U}'(h)\right) \hat{\mathbf{z}}$$

Membrane Area

$$\mathcal{A} = \int\int dxdy [1 + (\nabla_{yx} h)^2]^{1/2} \approx \mathcal{L}_x \mathcal{L}_y + \Delta\mathcal{A}$$

$$\Delta\mathcal{A} = \int\int dxdy \frac{1}{2} (\nabla_{yx} h)^2$$

# Model




$$f_z = -\kappa\Delta_{xy}^2 h + \mathcal{S}_0 \Delta_{xy} h - \mathcal{U}'(h)$$

Evolution equation for $h$

$$\partial_t h = -\nabla_{xy}\left\{\frac{h_0^3}{24\mu}\left[1-\left(\frac{h}{h_0}\right)^2\right]^3 \nabla_{xy} f_z - \frac{3}{4}\left[\frac{1}{3}\left(\frac{h}{h_0}\right)^3 - \frac{h}{h_0}\right]\mathbf{j}\right\} + \frac{\nu}{2} f_z$$

**j** satisfies

$$\nabla_{xy}^2 \mathbf{j} - \frac{3\mu\nu}{h_0^3}\mathbf{j} = \frac{\nu}{2}\left[\left(\frac{h}{h_0}\right)^3 - 3\frac{h}{h_0}\right]\nabla_{xy} f_z$$

$\mathcal{S}_0$ determined from

$$\partial_t \Delta\mathcal{A} = 0$$

# adhesion/patterns

# Lignes directrices

# Permeable environment

Normalized permeability: $\bar{\nu} = 12\mu\nu\kappa^{1/2}/(h_0\mathcal{U}_0^{1/2}) \gg 1$

$$\partial_t h = \frac{\nu}{2}(-\kappa\Delta^2 h + \mathcal{S}_0\Delta h - \mathcal{U}'(h)),$$

$$\mathcal{S}_0 = \frac{\int\int dxdy\Big(\kappa\Delta^2 h + \mathcal{U}'(h)\Big)\Delta h}{\int\int dxdy(\Delta h)^2}.$$

similar to Swift-Hohenberg equation (constant tension)

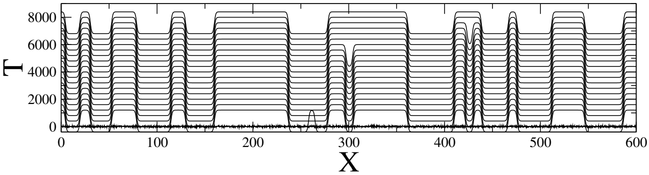
similar to TDGL $\qquad \partial_T H = \Delta H - U'(H).$

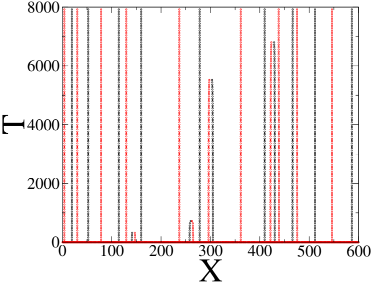
Difference from Time-Dependent-Ginzburg-Landau equation (TDGL):

- 4th order $\Leftarrow$ bending rigidity
- Time-dependent tension $\mathcal{S}_0$

# 1D Phase separation with non-conserved order parameter

TDGL $$\partial_T H = \partial_X^2 H - U'(H).$$

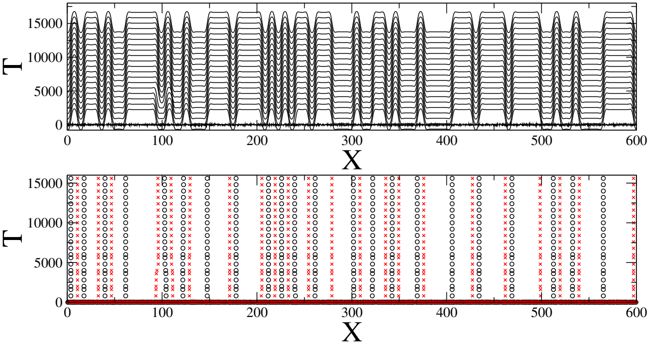



- Coarsening, $\lambda \sim \ln t$

# 1D Frozen dynamics: permeable case

TDGL4 $\partial_T H = -\partial_X^4 H - U'(H).$



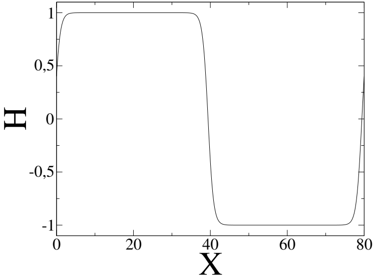

- Frozen dynamics ⇐ Oscillatory interaction between Domain Walls (Kinks)
- Disorder ⇐ Linear spectrum of spinodal instability

# Kinks

TDGL and CH

Steady-state equation $$-\partial_X^2 H(X) + U'(H) = 0$$

One single branch of steady-states parametrized by $\lambda$
Kink solution with $U(H) = -H_m^2 H^2/2 + H^4/4$




Exponentially decreasing kink tail $H = H_m + R(X - X_k)$

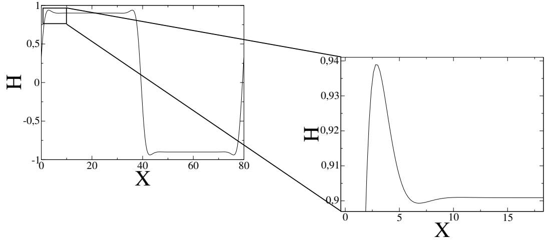
$$R(\ell) = A \exp[-U_m''^{1/2}\ell].$$

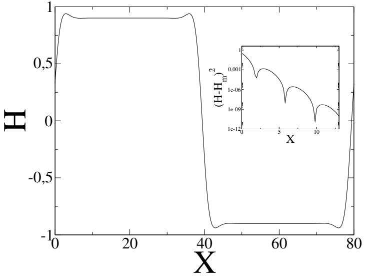
# Oscillatory kink tails

TDGL4 and CH4, or general equation

Steady-state equation $\partial_X^4 H(X) + U'(H) = 0$

many steady-states, periodic, chaotic, etc.
Kink solution with $U(H) = -H_m^2 H^2/2 + H^4/4$




Oscillatory kink tail $H = H_m + R(X - X_k)$

$$R(\ell) = A\cos(2^{-1/2} U_m''^{1/4}\ell + \alpha)\exp[-2^{-1/2} U_m''^{1/4}\ell].$$

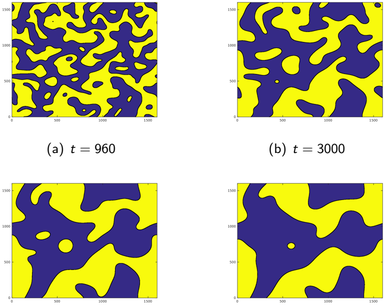
⇒ Oscillatory interactions between Domain Walls (kinks)

# Time-Dependent Ginzburg-Landau equation

normalization $H = h/h_0$

$$\partial_T H = \Delta H - U'(H)$$
$$U(h) = \frac{1}{4}\left(H_m^2 - H^2\right)^2, \quad H_m = 0.7$$

TDGL



(a) $t = 960$ (b) $t = 3000$

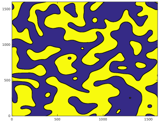
(c) $t = 6000$ (d) $t = 9000$

coarsening $\lambda \sim t^{1/2}$

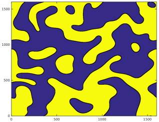
# TDGL4

$$\partial_T H = -\Delta^2 H - U'(H)$$

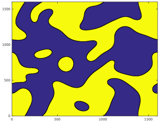
## TDGL4



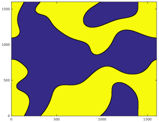
(a) $t = 3000$



(b) $t = 6000$



(c) $t = 12000$



(d) $t = 24000$

coarsening $\lambda \sim t^{1/2}$

# Permeable environment

Full dynamics with area conservation

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H))$$

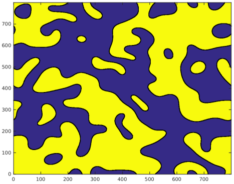
$$\Sigma = \frac{\int\int dXdY \Big(\Delta^2 H + U'(H)\Big)\Delta H}{\int\int dXdY (\Delta H)^2}$$

3 regimes $\Delta A^* = \Delta A/\mathcal{L}_x\mathcal{L}_y$

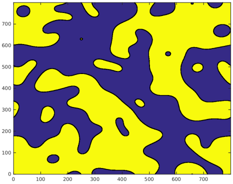
- $\Delta A^* < (0.93 \pm 0.03) \times 10^{-2}$ Frozen domains
- $(0.930.03) \times 10^2 < \Delta A^* < (5.53 \pm 0.15) \times 10^{-2}$ coarsening with domains + wrinkles
- $(5.53 \pm 0.15) \times 10^{-2} < \Delta A^*$ frozen wrinkles

# Permeable environment

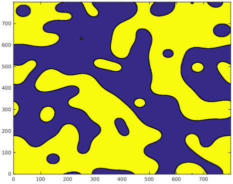
## Nonconserved equation, W=1.5, $\Delta A/A = 0.0465$



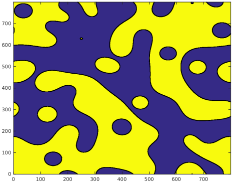
(a) $t = 960$



(b) $t = 12000$



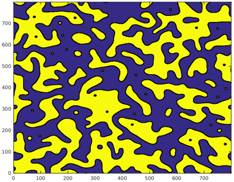
(c) $t = 102000$



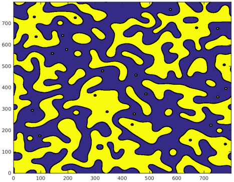
(d) $t = 801000$

# Permeable environment

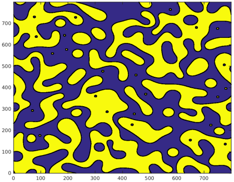
## Nonconserved equation, W=1, $\Delta A/A = 0.1072$



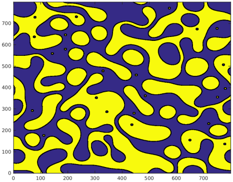

(a) $t = 960$




(b) $t = 12000$



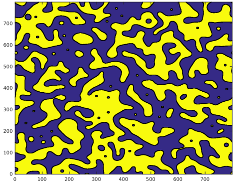

(c) $t = 102000$



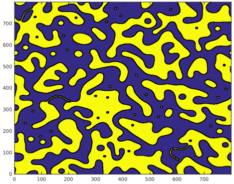

(d) $t = 801000$

# Permeable environment

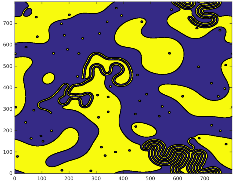
## Nonconserved equation, W=0.95, $\Delta A/A = 0.1195$



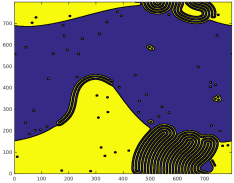

(a) $t = 960$




(b) $t = 12000$



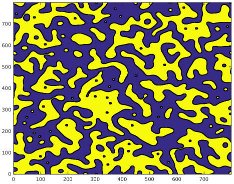

(c) $t = 102000$



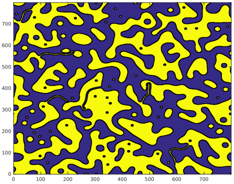

(d) $t = 801000$

# Permeable environment

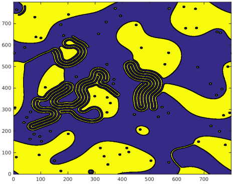
## Nonconserved equation, W=0.9, $\Delta A/A = 0.1338$



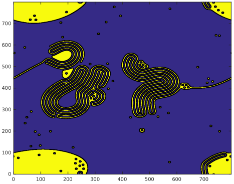

(a) $t = 960$




(b) $t = 12000$



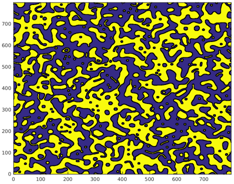

(c) $t = 102000$



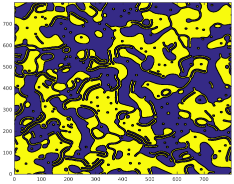

(d) $t = 801000$

# Permeable environment

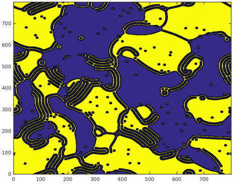
## Nonconserved equation, W=0.7, $\Delta A/A = 0.2244$



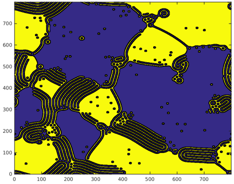

(a) $t = 960$




(b) $t = 12000$



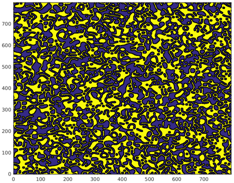

(c) $t = 102000$



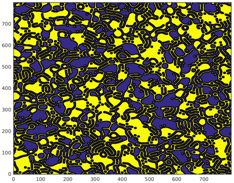

(d) $t = 801000$

# Permeable environment

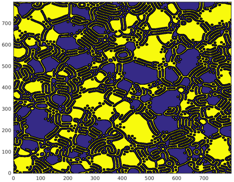
## Nonconserved equation, W=0.5, $\Delta A/A = 0.4515$



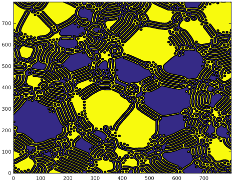

(a) $t = 960$




(b) $t = 12000$




(c) $t = 102000$

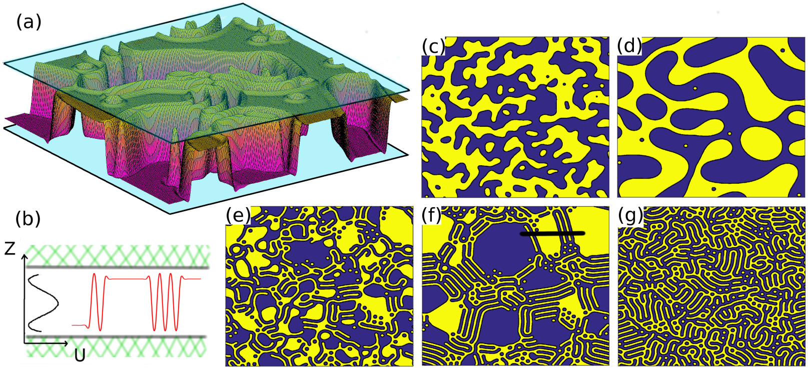



(d) $t = 2001000$

# Permeable environment

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H))$$

$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right) \Delta H}{\int\int dXdY (\Delta H)^2}$$




Normalized excess area $\Delta A^* = \Delta A/(\mathcal{L}_x \mathcal{L}_y)$

(c) and (d): $\Delta A^* = 0.88 \cdot 10^{-2}$ at $T = 15$ and $T = 8 \cdot 10^5$

(e) and (f): $\Delta A^* = 3.61 \cdot 10^{-2}$ at $T = 10^4$ and $T = 8 \cdot 10^5$

(g): $\Delta A^* = 5.68 \cdot 10^{-2}$ at $T = 8 \cdot 10^5$

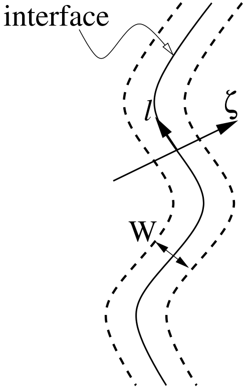
# Lignes directrices

# Domain Wall dynamics

$$\partial_T H = -\Delta^2 H + \Sigma\Delta H - U'(H)$$

direction $\zeta$ perpendicular to domain wall: $\Delta \approx \partial_{\zeta\zeta} + K\partial_\zeta$

$$-V_{DW}\partial_\zeta H \approx -\partial_\zeta^4 H - 2K\partial_\zeta^3 H + \Sigma\partial_\zeta^2 H + \Sigma K\partial_\zeta H - U'(H)$$




Domain Wall motion by curvature and interactions

$$V_{DW} = -\frac{1}{\alpha}\Big([W]_-^+ + K\xi_{DW}\Big)$$

$\alpha = \int d\zeta(\partial_\zeta H)^2$

- Interaction term:

$$W(H) = \frac{1}{2}\partial_{\zeta\zeta}(\partial_\zeta H)^2 - \frac{3}{2}(\partial_{\zeta\zeta} H)^2 - \frac{\Sigma}{2}(\partial_\zeta H)^2 + U(H)$$

  Isolated domain wall $[W]_-^+ = 0$

- Effective Domain wall energy:

$$\xi_{DW} = \int d\zeta\Big(\frac{1}{2}(\partial_\zeta^2 H)^2 + \frac{1}{2}\Sigma(\partial_\zeta H)^2 + U(H)\Big)$$

# Domain Wall dynamics

- Fixed tension
- Isolated domain walls $[W]_-^+ = 0$

$$V_{DW} = -\frac{\xi_{DW}}{\alpha} K$$

One single lengthscale domain size $R$

$$V_{DW} \sim \partial_t R$$

$$K \sim \frac{1}{R}$$

leading to

$$\partial_t R \sim \frac{\xi_{DW}}{\alpha} \frac{1}{R} \Rightarrow R \sim \left( \frac{\xi_{DW}}{\alpha} t \right)^{1/2}$$

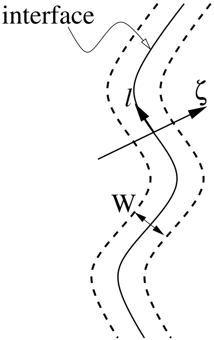
TDGL and TDGL4 in 2D

# Tension dynamics

Domain wall length

$$L_{DW} = \int_{DW} d\ell \qquad \Rightarrow \partial_T L_{DW} = \int_{DW} d\ell V_{DW} K$$

$$\Delta \mathcal{A} = \int\int dX\, dY \frac{1}{2}(\nabla H)^2 \approx L_{DW}\alpha/2$$




$\alpha = \int d\zeta (\partial_\zeta H)^2$

Area conservation

$$0 = \partial_T(2\Delta\mathcal{A}) = \partial_T\Big(L_{DW}\alpha\Big)$$

$$= -\int d\ell\Big([W]_-^+ + \xi_{DW}K\Big)K + L_{DW}\partial_T\Sigma\partial_\Sigma\alpha$$

Tension dynamics

$$\partial_T\Sigma = \frac{\xi_{DW}\int d\ell K^2 + \int d\ell [W]_-^+ K}{L_{DW}\partial_\Sigma\alpha}$$

# Tension dynamics

## Tension dynamics

$$\partial_T \Sigma = \frac{\xi_{DW} \int d\ell K^2 + \int d\ell [W]_-^+ K}{L_{DW} \partial_\Sigma \alpha}$$

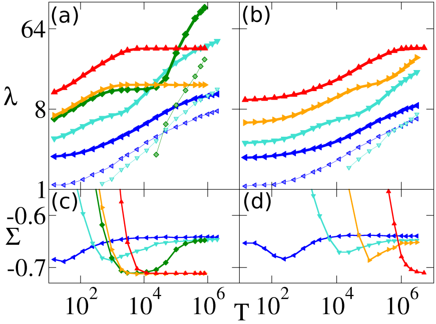
- Isolated wall $[W]_-^+ \approx 0$
- $\alpha \searrow$ for increasing $\Sigma \Rightarrow \partial_\Sigma \alpha < 0$
- Domain wall energy $\xi_{DW} \nearrow$ for increasing $\Sigma$
  $\xi_{DW} = 0$ for $\Sigma = \Sigma_c = -1.0226 H_m$

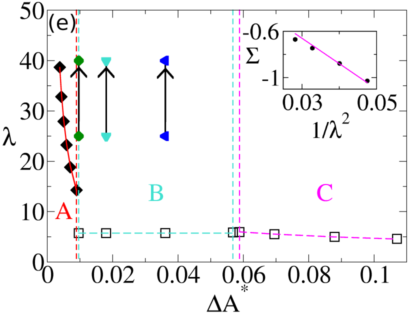
$\Rightarrow \Sigma \searrow \Sigma_c$ and $V_{DW} \to 0$

# Tension dynamics

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H)),$$

$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right)\Delta H}{\int\int dXdY (\Delta H)^2}.$$



# Frozen wavelength at small $\Delta\mathcal{A}$

Effective Domain wall energy

$$\xi_{DW}(\Sigma) = \int d\zeta \Big(\frac{1}{2}(\partial_\zeta^2 H)^2 + U(H) + \frac{1}{2}\Sigma(\partial_\zeta H)^2\Big)$$

$$E_{DW}(\Sigma) = \int d\zeta \Big(\frac{1}{2}(\partial_{\zeta\zeta} H)^2 + U(H)\Big)$$

$$\Delta\mathcal{A} \approx \frac{L_{DW}}{2} \int d\zeta (\partial_\zeta H)^2$$

$$\xi_{DW}(\Sigma) = E_{DW}(\Sigma) + \Sigma\Delta\mathcal{A}$$

Wavelength of frozen structures

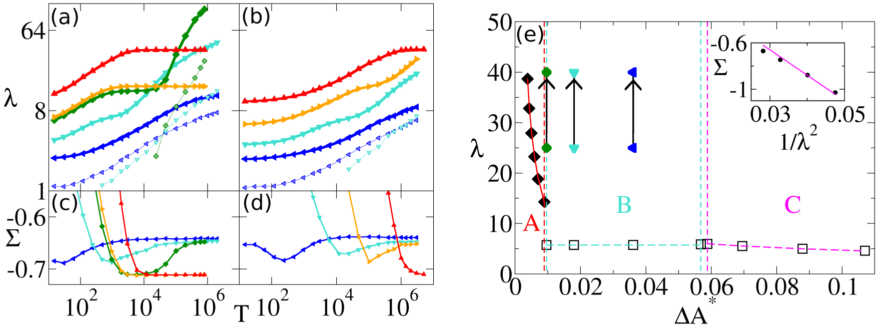
$$\xi_{DW}(\Sigma_c) = 0 \Rightarrow \bar{\lambda} = \frac{\mathcal{L}_x\mathcal{L}_y}{L_{DW}} = \frac{E_{DW}(\Sigma_c)}{-\Sigma_c \Delta A^*}$$

$\Delta A^* = \Delta\mathcal{A}/(\mathcal{L}_x\mathcal{L}_y)$

# Permeable environment

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H)),$$

$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right)\Delta H}{\int\int dXdY (\Delta H)^2}.$$

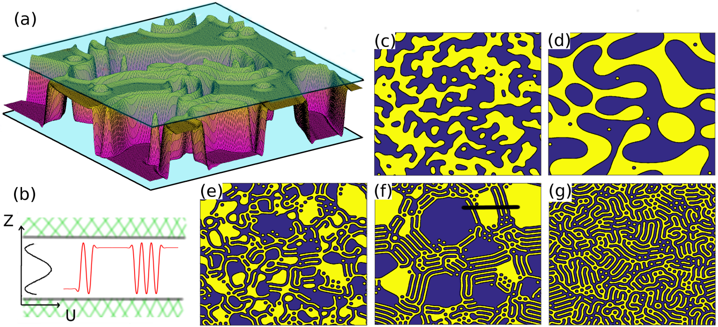


# Lignes directrices

# Permeable environment

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H)),$$

$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right)\Delta H}{\int\int dXdY (\Delta H)^2}.$$




Normalized excess area $\Delta A^* = \Delta A/(\mathcal{L}_x \mathcal{L}_y)$
(c) and (d): $\Delta A^* = 0.88 \cdot 10^{-2}$ at $T = 15$ and $T = 8 \cdot 10^5$
(e) and (f): $\Delta A^* = 3.61 \cdot 10^{-2}$ at $T = 10^4$ and $T = 8 \cdot 10^5$
(g): $\Delta A^* = 5.68 \cdot 10^{-2}$ at $T = 8 \cdot 10^5$

# Coarsening at coexistence

Effective energy and true energy

$$\xi(\Sigma) = \int\int dXdY\Big(\frac{1}{2}(\Delta H)^2 + U(H) + \frac{1}{2}\Sigma(\nabla H)^2\Big)$$
$$E(\Sigma) = \int\int dXdY\Big(\frac{1}{2}(\Delta H)^2 + U(H)\Big)$$

Coexistence at $\Sigma_{nl} \approx -0.9225 H_m$

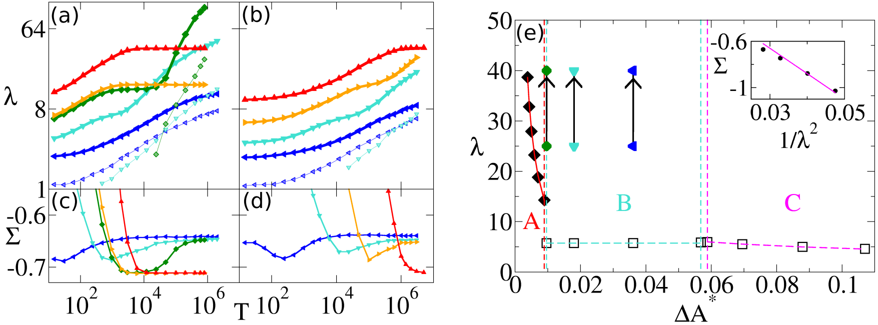
$$\xi_{flat}(\Sigma_{nl}) = \xi_{wrinkles}(\Sigma_{nl})$$

oscillatory interaction between kinks $\Rightarrow E_{2DW} > E_{1Wrinkle}$

# Permeable environment

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H)),$$

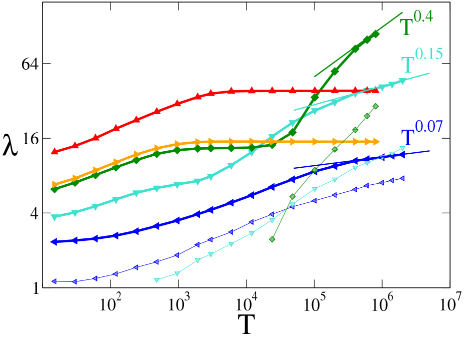
$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right)\Delta H}{\int\int dXdY (\Delta H)^2}.$$



# Permeable environment

$$\partial_t H = -\Delta^2 H + \Sigma \Delta H - U'(H)),$$

$$\Sigma = \frac{\int\int dXdY \left(\Delta^2 H + U'(H)\right) \Delta H}{\int\int dXdY (\Delta H)^2}.$$




No universal exponent?

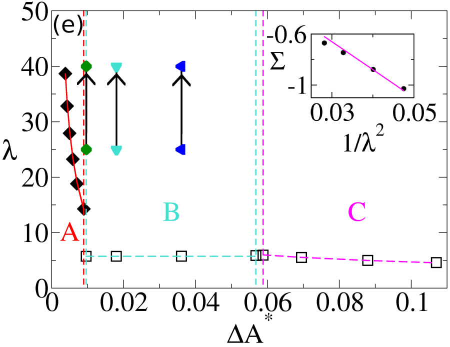
# Lignes directrices

# Frozen wrinkled state

- Excess area

$$\Delta A^* = \frac{\Delta \mathcal{A}}{\mathcal{L}_x \mathcal{L}_y} = \frac{L \int d\zeta (\partial_\zeta H)^2}{L\lambda} = \frac{(\langle H^2 \rangle / \lambda) \int d\bar{\zeta} (\partial_{\bar{\zeta}} \bar{H})^2}{\lambda}$$

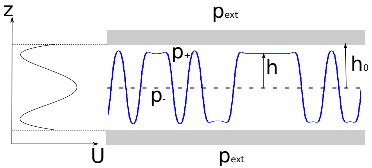
  approximately constant wrinkle shape $\Rightarrow \Delta A^* \lambda^2 / \langle H^2 \rangle \approx 3.3$
- Wavelength-Amplitude relation in stead-states
  Balance of tension and bending: $H/\lambda^4 \sim \Sigma H/\lambda^2 \;\Rightarrow\; \Sigma\lambda^2 = \mathrm{cst} \approx -22$



# adhesion/patterns

# Lignes directrices

# Model




Normalized permeability: $\bar{\nu} = 12\mu\nu\kappa^{1/2}/(h_0\mathcal{U}_0^{1/2}) \ll 1$

$$f_z = -\kappa\Delta_{xy}^2 h + \mathcal{S}_0\Delta_{xy} h - \mathcal{U}'(h)$$

Evolution equation for $h$ neglect $\mathbf{j}$

$$\partial_t h = -\nabla_{xy}\left\{\frac{h_0^3}{24\mu}\left[1-\left(\frac{h}{h_0}\right)^2\right]^3 \nabla_{xy} f_z\right\}$$

$\mathcal{S}_0$ determined from

$$\partial_t \Delta\mathcal{A} = 0$$

# Model

Evolution equations

$$\partial_t h = \nabla\Big(\mathcal{M}(h)\nabla(\kappa\Delta^2 h - \mathcal{S}_0\Delta h + \mathcal{U}'(h))\Big)$$

$$\mathcal{S}_0 = \frac{\int\int dxdy \nabla\Big(\mathcal{M}(h)\nabla(\kappa\Delta^2 h + \mathcal{U}'(h))\Big)\Delta h}{\int\int dxdy\nabla(\mathcal{M}(h)\nabla\Delta h)\Delta h}.$$

$\mathcal{M}(h) = (h_0^3/(24\mu))(1-(h/h_0)^2)^3$

Similar results as permeable (non-conserved) case

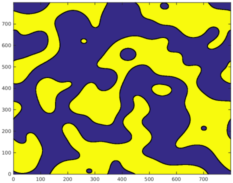
Differences:

- Slower dynamics $\Rightarrow$ further simplification $\mathcal{M}(h) = 1$
- smaller $A_c = (0.4 \pm 0.03) \times 10^{-2}$

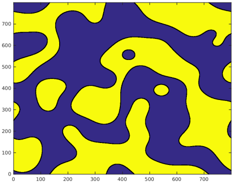
# Lignes directrices

# Impermeable environment

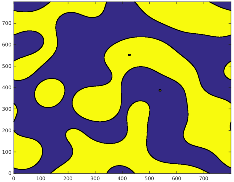
## Conserved tilde equation, tdgl4 W=0.9, $\Delta A/A = 0.0352$



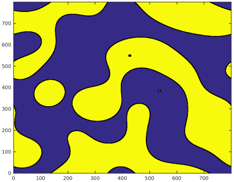

(a) $t = 960$




(b) $t = 102000$



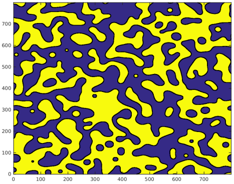

(c) $t = 801000$



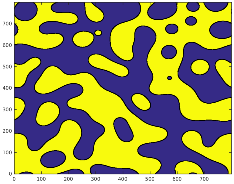

(d) $t = 4302000$

# Impermeable environment

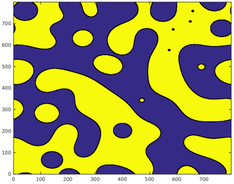
## Conserved tilde equation, W=1.5, $\Delta A/A = 0.0465$



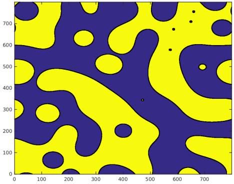

(a) $t = 960$




(b) $t = 102000$



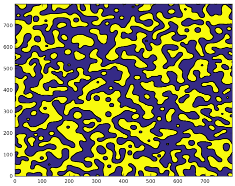

(c) $t = 801000$



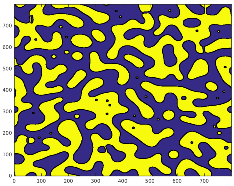

(d) $t = 5001000$

# Impermeable environment

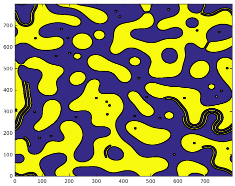
## Conserved tilde equation, W=1, $\Delta A/A = 0.1072$



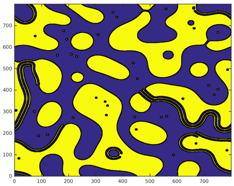

(a) $t = 960$




(b) $t = 102000$



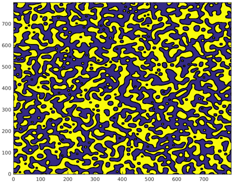

(c) $t = 801000$



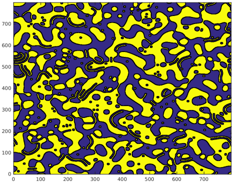

(d) $t = 2001000$

# Impermeable environment

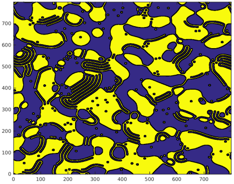
## Conserved tilde equation, W=0.7, $\Delta A/A = 0.2244$



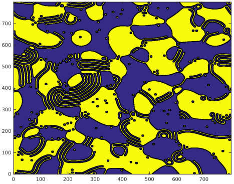

(a) $t = 960$




(b) $t = 102000$



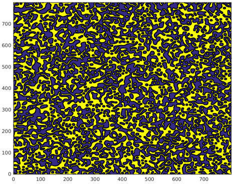

(c) $t = 801000$



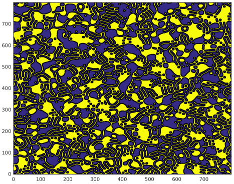

(d) $t = 2001000$

# Impermeable environment

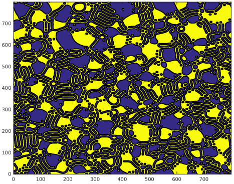
## Conserved tilde equation, W=0.5, $\Delta A/A = 0.4515$



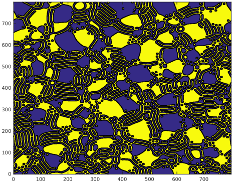

(a) $t = 960$




(b) $t = 102000$



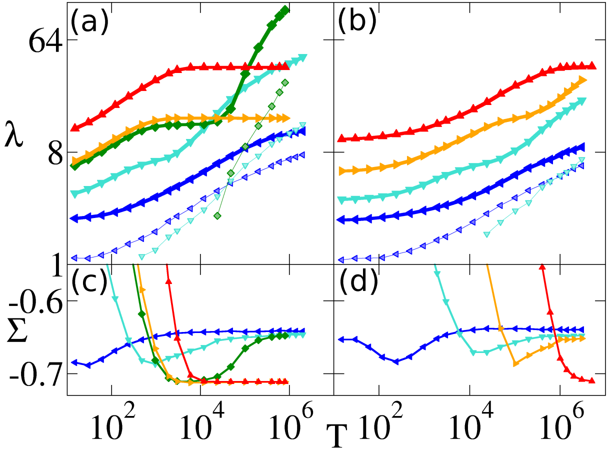

(c) $t = 801000$




(d) $t = 2001000$

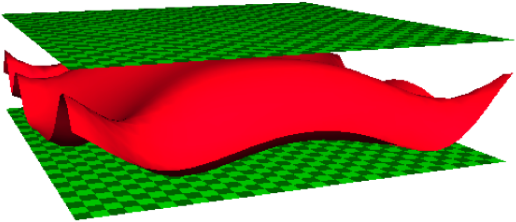
# Impermeable environment

Conserved dynamics similar to non-conserved but slower



# adhesion/patterns

# Conclusion



Confined membranes, lubrication limit

- Membrane confinement ⇒ frozen dynamics or coarsening
- Permeable / impermeable: similar dynamics
- Experiments?

T.B.T. To, T. Le Goff, O. Pierre-Louis, preprint (2017)
T. Le Goff, T.B.T. To, O. Pierre-Louis, EPJE 40, 1 (2017)
T. Le Goff, P. Politi, O. Pierre-Louis, Phys Rev E 92 022918 (2015)
T. LeGoff, O. Pierre-Louis, P. Politi, J. of Stat. Mech.: Theor. and Exp. 2015 (8), P08004 (2015)
T. Le Goff, P. Politi, and O. Pierre-Louis Phys Rev E 90 032114 (2014)

# Conclusion

Tung B.T. To [1,3]
Thomas Le Goff [1,4]
Paolo Politi [2]

[1] CNRS, ILM, Lyon, France
[2] CNR, ISC, Florence, Italy
[3] Univ. Fed. Fluminense, Nietroi, Rio, Brasil
[4] Marseille, France

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