

OIST MINI-WORKSHOP ON GAUGE THEORY AND LOW-DIMENSIONAL TOPOLOGY

Mitsuyoshi Adachi.

Title: A gerbe-like construction in gauge theory

Abstract: For a family of closed spin manifolds, it is a natural question whether the tangent bundle along the fibers admits a spin structure, since this is related to whether gauge theory for families can be applied. In this talk, we show that for families of homotopy K3 surfaces, the obstruction $O(1)$ -gerbe to the existence of a spin structure on the vertical tangent bundle is canonically isomorphic to the obstruction $O(1)$ -gerbe to the existence of a spin structure on H^+ , the vector bundle of fiberwise harmonic self-dual 2-forms. The proof uses finite-dimensional approximations of the families Seiberg–Witten equations. Unlike in the usual applications of this method, the stabilizations and homotopies between different finite-dimensional approximations are also part of the data that must be specified explicitly. Keeping track of this data is essential for constructing the isomorphism canonically.

Paul Feehan.

Title: Towards a Bogomolov-Miyaoka-Yau inequality for symplectic 4-manifolds

Abstract: The Bogomolov-Miyaoka-Yau inequality for minimal compact complex surfaces of general type was proved in 1977 independently by Miyaoka, using methods of algebraic geometry, and by Yau, as an outgrowth of his proof of the Calabi conjectures. In this talk, we describe progress in our ongoing program to prove the conjecture that symplectic 4-manifolds obey the Bogomolov-Miyaoka-Yau inequality. Our program uses Morse theory on the gauge theoretic moduli space of non-Abelian monopoles, where the Morse function is a Hamiltonian for a natural circle action and natural two-form. We shall describe generalizations of Donaldson’s symplectic subspace criterion (1996) from finite to infinite dimensions and generalizations of Taubes’ perturbation analysis from the Seiberg-Witten to non-Abelian monopole equations. These methods have application to the problem of showing that the fundamental two-form is non-degenerate and thus an almost symplectic form on the moduli space of non-Abelian monopoles. This talk is based on joint work with Tom Leness and the monographs <https://arxiv.org/abs/2010.15789> (to appear in AMS Mathematical Surveys and Monographs), <https://arxiv.org/abs/2206.14710> and <https://arxiv.org/abs/2410.13809>.

Kristen Hendricks.

Title: Real Heegaard Floer homology and localization

Abstract: In the past few years there have been a host of remarkable topological results arising from considering “real” versions of various gauge and Floer-theoretic invariants of three- and four-dimensional manifolds equipped with involutions. Recently Guth and Manolescu defined a real version of Lagrangian Floer theory, and applied it to Ozsváth and Szabó’s three-manifold invariant Heegaard Floer homology, producing an invariant called real Heegaard Floer homology associated to a 3-manifold together with an orientation-preserving involution whose fixed set is codimension two (ie a branched double cover). In this talk we use tools from equivariant Lagrangian Floer theory, originally developed by Seidel-Smith and Large in a somewhat different context, to produce a spectral sequence from the ordinary to real Heegaard Floer homologies in their simplest “hat” version, in particular proving the existence of a rank inequality between the theories. Our results apply more generally to the real Lagrangian Floer homology of exact symplectic manifolds with antisymplectic involutions. Along the way we give a little history and context for the appearance of such spectral sequences in Heegaard Floer theory.

Hayato Imori.

Title: Unknotting number, ribbon concordance, and singular instantons

Abstract: A classical theorem of Scharlemann states that every composite knot has unknotting number greater than one. This naturally leads to the question of whether similar phenomena hold for knots related to connected sums by ribbon concordance, and to an analogous question for the Dehn surgery number of homology 3-spheres.

We show that, for a large class of slice knots obtained as band sums, any sequence of crossing changes transforming the knot into the unknot must contain crossing changes of both signs. In particular, this provides evidence for a natural extension of Scharlemann’s theorem. We also establish a 3-manifold analogue, providing an obstruction to certain homology 3-spheres being obtained by Dehn surgery on a knot in S^3 .

The main tool is equivariant singular instanton Floer theory equipped with the Chern–Simons filtration, together with its behavior under negative definite cobordisms. This talk is based on joint work with JungHwan Park and Masaki Taniguchi.

Hokuto Konno.

Title: Families 10/8 revisited

Abstract: In memory of Mikio Furuta, we revisit Furuta’s 10/8 inequality, but in the setting of families. A families version of the 10/8 theorem was considered by Baraglia in 2019, but applications of this families version have not yet been extensively explored. We consider a refinement of Baraglia’s families 10/8 theorem and investigate what it tells us about the topology of families of 4-manifolds. This is joint work in progress with Yosuke Morita.

Mike Miller Eismeier.

Title: TBA

Abstract: TBA

Jin Miyazawa.

Title: TBA

Abstract: TBA

Danny Ruberman.

Title: Equivalent exotic surfaces in small 4-manifolds

Abstract: Two S, S' in a 4-manifold are said to be exotic if they are topologically isotopic, but not smoothly isotopic. By the isotopy extension theorem, smoothly isotopic surfaces are smoothly equivalent, ie related by a self-diffeomorphism of the ambient manifold. We give a number of constructions of pairs of surfaces that are topologically isotopic, smoothly equivalent, but not smoothly isotopic. The method shows that they are not even Torelli equivalent (ie related by a diffeomorphism that induces the identity on all homology groups). The ambient manifolds are small: $CP^2 \# (S^2 \times S^2)$ and $(S^2 \times S^2) \# (S^2 \times S^2)$. We find related examples with boundary; these live in even smaller manifolds, such as a punctured $S^2 \times S^2$. This is joint work with Dave Auckly, Hokuto Konno, Anubhav Mukherjee, and Masaki Taniguchi.

Taketo Sano.

Title: TBA

Abstract: TBA

Masaki Taniguchi.

Title: The homology cobordism group is not generated by graph homology 3-spheres

Abstract: We provide a new perspective on instanton Floer theory by relating it to the Galois conjugation action on the $SL(2, \mathbb{C})$ -representation variety and to the hyperbolic geometry of 3-manifolds. As an application, we show that the homology cobordism group is not generated by graph homology 3-spheres.