

Lemma 3.1

In[1]:= **Solve**[{**y** == **t x** + **s (a x + b y)**, **z** == **a x + b y**}, {**y**, **z**}]

$$\text{Out[1]} = \left\{ \left\{ y \rightarrow -\frac{a s x + t x}{-1 + b s}, z \rightarrow -\frac{a x + b t x}{-1 + b s} \right\} \right\}$$

In[2]:= **Solve**[(**1 - b s**) **u** + (**t u + s v**) (**t + a s**) + **v (a + b t)** == **0**, {**u**}]

$$\text{Out[2]} = \left\{ \left\{ u \rightarrow \frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2} \right\} \right\}$$

In[3]:= **Collect**[**Expand**[(**x - u**)^2 + (**t x + s z - t u - s v**)^2 + (**z - v**)^2 - (**u**^2 + (**t u + s v**)^2 + **v**^2)], {**z**, **x**}]

$$\text{Out[3]} = (-2 u - 2 t^2 u - 2 s t v) x + (1 + t^2) x^2 + (-2 s t u - 2 v - 2 s^2 v + 2 s t x) z + (1 + s^2) z^2$$

In[4]:= **Solve**[(**-2 u - 2 t^2 u - 2 s t v**) **x** + (**1 + t^2**) **x**^2 + (**-2 k + 2 s t x**) **z** + (**1 + s^2**) **z**^2 == **0**, **z**]

$$\text{Out[4]} = \left\{ \left\{ z \rightarrow \frac{1}{2(1 + s^2)} \left(2 k - 2 s t x - \sqrt{(-2 k + 2 s t x)^2 - 4(1 + s^2)(-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2)} \right) \right\}, \right. \\ \left. \left\{ z \rightarrow \frac{1}{2(1 + s^2)} \left(2 k - 2 s t x + \sqrt{(-2 k + 2 s t x)^2 - 4(1 + s^2)(-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2)} \right) \right\} \right\}$$

In[5]:= **Simplify**[**s (t u + s v) + v** /. **u** -> $\frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2}$]

$$\text{Out[5]} = -\frac{(-1 + b s)(1 + s^2 + t^2) v}{1 - b s + a s t + t^2}$$

Hence, **k** is $-\frac{(-1 + b s)(1 + s^2 + t^2) v}{1 - b s + a s t + t^2}$

In[6]:= **Simplify**[**1 - (2 k)**^(-2) (**-2 k + 2 s t x**)^2 - **4 (1 + s^2)** (**-2 u x - 2 t^2 u x - 2 s t v x + x^2 + t^2 x^2**)]

$$\text{Out[6]} = \frac{1}{k^2} x (2 k s t - 2(1 + s^2)(1 + t^2) u - 2 s t v - 2 s^3 t v + x + s^2 x + t^2 x)$$

In[7]:= **Series**[- $\sqrt{1 - X}$, {**X**, **0**, **4**}]

$$\text{Out[7]} = -1 + \frac{X}{2} + \frac{X^2}{8} + \frac{X^3}{16} + \frac{5 X^4}{128} + O[X]^5$$

In[8]:= **Factor**[**2 k t s - 2 (s^2 + 1) (u + t (t u + s v))** /.

$$\left\{ u \rightarrow \frac{a v + a s^2 v + b t v + s t v}{-1 + b s - a s t - t^2}, k \rightarrow -\frac{(-1 + b s)(1 + s^2 + t^2) v}{1 - b s + a s t + t^2} \right\}]$$

$$\text{Out[8]} = -\frac{2(a + a s^2 + b t + s t)(1 + s^2 + t^2) v}{-1 + b s - a s t - t^2}$$

$$\text{In[9]:= } C0 := -\frac{2(a + as^2 + bt + st)(1 + s^2 + t^2)v}{-1 + bs - ast - t^2}$$

$$\text{In[10]:= } D0 := s^2 + t^2 + 1$$

$$\text{In[11]:= } k := -\frac{(-1 + bs)(1 + s^2 + t^2)v}{1 - bs + ast + t^2}$$

A1 is

$$\text{In[12]:= } \text{Simplify}\left[-(u + t(tu + sv)) / k /. u \rightarrow \frac{av + as^2v + btv + stv}{-1 + bs - ast - t^2}\right]$$

$$\text{Out[12]= } \frac{a + bt}{1 - bs}$$

A2 is

$$\text{In[13]:= } \text{Simplify}[D0 / (2k(s^2 + 1)) + C0^2 / (8k^3(s^2 + 1))]$$

$$\text{Out[13]= } \left(\frac{(-1 + bs - ast - t^2)(1 - 2bs + a^2(1 + s^2) + 2a(b + s)t + t^2 + b^2(s^2 + t^2))}{(2(-1 + bs)^3(1 + s^2 + t^2)v)}\right) /$$

Here,

$$\text{In[14]:= } \text{Factor}[(s^2 + t^2 + 1)(1 - bs)^2 + (as^2 + ts + bt + a)^2]$$

$$\text{Out[14]= } (1 + s^2)(1 + a^2 - 2bs + a^2s^2 + b^2s^2 + 2abt + 2ast + t^2 + b^2t^2)$$

A3 is

$$\text{In[15]:= } \text{Simplify}[2C0D0 / (8k^3(s^2 + 1)) + C0^3 / (16k^5(s^2 + 1))]$$

$$\text{Out[15]= } -\left(\frac{(a(1 + s^2) + (b + s)t)(1 - bs + ast + t^2)^2(1 - 2bs + a^2(1 + s^2) + 2a(b + s)t + t^2 + b^2(s^2 + t^2))}{(2(-1 + bs)^5(1 + s^2 + t^2)^2v^2)}\right) /$$

A4 is

$$\text{In[16]:= } \text{Simplify}[D0^2 / (8k^3(s^2 + 1)) + 3C0^2D0 / (16k^5(s^2 + 1)) + 5C0^4 / (128k^7(s^2 + 1))]$$

$$\text{Out[16]= } \left(\frac{(1 - bs + ast + t^2)^3(-5(a(1 + s^2) + (b + s)t)^4 - 6(-1 + bs)^2(a(1 + s^2) + (b + s)t)^2(1 + s^2 + t^2) - (-1 + bs)^4(1 + s^2 + t^2)^2)}{(8(-1 + bs)^7(1 + s^2)(1 + s^2 + t^2)^3v^3)}\right) /$$

$$\text{In[17]:= } \text{Factor}\left[\left(-5(a(1 + s^2) + (b + s)t)^4 - 6(-1 + bs)^2(a(1 + s^2) + (b + s)t)^2(1 + s^2 + t^2) - (-1 + bs)^4(1 + s^2 + t^2)^2\right)\right]$$

$$\text{Out[17]= } -(1 + s^2)(1 + a^2 - 2bs + a^2s^2 + b^2s^2 + 2abt + 2ast + t^2 + b^2t^2)(1 + 5a^2 - 2bs + s^2 + 10a^2s^2 + b^2s^2 - 2bs^3 + 5a^2s^4 + b^2s^4 + 10abt + 10ast + 10abs^2t + 10as^3t + t^2 + 5b^2t^2 + 8bst^2 + 5s^2t^2 + b^2s^2t^2)$$

Therefore A4 is

$$\frac{(1 - bs + ast + t^2)^3 (1 + a^2 - 2bs + a^2 s^2 + b^2 s^2 + 2abt + 2ast + t^2 + b^2 t^2) (1 + 5a^2 - 2bs + s^2 + 10a^2 s^2 + b^2 s^2 - 2bs^3 + 5a^2 s^4 + b^2 s^4 + 10abt + 10ast + 10abs^2 t + 10as^3 t + t^2 + 5b^2 t^2 + 8bst^2 + 5s^2 t^2 + b^2 s^2 t^2)}{(8(1 - bs)^7 (1 + s^2 + t^2)^3 v^3)}$$

In[18]:= **Expand**[(1 + 5 a² - 2 b s + s² + 10 a² s² + b² s² - 2 b s³ + 5 a² s⁴ + b² s⁴ + 10 a b t + 10 a s t + 10 a b s² t + 10 a s³ t + t² + 5 b² t² + 8 b s t² + 5 s² t² + b² s² t²) - ((s² + t² + 1) (1 - b s)² + 5 (a s² + t s + b t + a)²)]

Out[18]= 0

Proposition 3.3

In[19]:= **A2** := CT / (1 - b s)

In[20]:= **A3** := (DT + s CT1 A2) / (1 - b s)

In[21]:= **Expand**[A3]

$$\text{Out[21]= } \frac{CT CT1 s}{(1 - b s)^2} + \frac{DT}{1 - b s}$$

In[22]:= **A4** := (ET + s CT1 A3 + s DT1 A2 + s² c2 A2²) / (1 - b s)

In[23]:= **Expand**[A4]

$$\text{Out[23]= } \frac{c2 CT^2 s^2}{(1 - b s)^3} + \frac{CT CT1^2 s^2}{(1 - b s)^3} + \frac{CT1 DT s}{(1 - b s)^2} + \frac{CT DT1 s}{(1 - b s)^2} + \frac{ET}{1 - b s}$$

In[24]:= **Simplify**[a s² + t s + b t + a /. t → (1 - b s) T - a s]

Out[24]= -(-1 + b s) (a + (b + s) T)

In[25]:= **Simplify**[

$$(a^2 + b^2) s^2 + 2(a t - b) s + (b^2 + 1) t^2 + 2 a b t + a^2 + 1 /. t \rightarrow (1 - b s) T - a s]$$

Out[25]= (-1 + b s)² (1 + a² + 2 a b T + (1 + b²) T²)

$$\text{Here } R[T] := 1 + a^2 + 2 a b T + (1 + b^2) T^2$$

In[26]:= **Solve**[(s CT1 CT + (1 - b s) DT) RT == 2 (a + (b + s) T) CT², s]

$$\text{Out[26]= } \left\{ \left\{ s \rightarrow \frac{-2 a CT^2 + DT RT - 2 b CT^2 T}{-CT CT1 RT + b DT RT + 2 CT^2 T} \right\} \right\}$$

this denominator is W := b s + C K because

In[27]:= **Expand**[b (DT RT - 2 (b T + a) CT²) + CT KT /. {KT → (2 (b² + 1) T + 2 a b) CT - RT CT1}]

Out[27]= -CT CT1 RT + b DT RT + 2 CT² T

In[28]:= **Simplify**[(s² + t² + 1) (1 - b s)² + 5 (a s² + t s + b t + a)² /. t → (1 - b s) T - a s]

Out[28]= (-1 + b s)² (1 + s² + 5 (a + (b + s) T)² + (a s + (-1 + b s) T)²)

In[29]= **Collect**[**Cancel**[$RT^2 (1 - b s)^3$
 $(A4 - A2^3 (1 + s^2 + 5 (a + (b + s) T)^2 + (a s + (-1 + b s) T)^2) / RT^2)$], **s**, **Simplify**]

Out[29]= $ET RT^2 - CT^3 (1 + 5 a^2 + 10 a b T + T^2 + 5 b^2 T^2) +$
 $s (CT1 DT RT^2 + CT DT1 RT^2 - 2 b ET RT^2 - 8 CT^3 T (a + b T)) +$
 $s^2 (c2 CT^2 RT^2 + CT (CT1^2 - b DT1) RT^2 +$
 $b (-CT1 DT + b ET) RT^2 - CT^3 (1 + a^2 + 2 a b T + (5 + b^2) T^2))$

In[30]= **s := ST / WT**

In[31]= **Z1 :=** $RT^2 WT^2 (1 - b s)^3$
 $(A4 - A2^3 (1 + s^2 + 5 (a + (b + s) T)^2 + (a s + (-1 + b s) T)^2) / RT^2)$

In[32]= **Collect**[**Cancel**[**Z1**], {**ET**, **DT**, **DT1**, **CT**}, **Simplify**]

Out[32]= $c2 CT^2 RT^2 ST^2 + CT CT1^2 RT^2 ST^2 + CT1 DT RT^2 ST (-b ST + WT) + CT DT1 RT^2 ST (-b ST + WT) +$
 $ET RT^2 (-b ST + WT)^2 + CT^3 (-ST^2 (1 + a^2 + 2 a b T + (5 + b^2) T^2) -$
 $8 ST T (a + b T) WT - (1 + 5 a^2 + 10 a b T + (1 + 5 b^2) T^2) WT^2)$

As for the coefficient of CT^3,

In[33]= **Expand**[$(-ST^2 (1 + a^2 + 2 a b T + (5 + b^2) T^2) -$
 $8 ST T (a + b T) WT - (1 + 5 a^2 + 10 a b T + (1 + 5 b^2) T^2) WT^2) +$
 $(WT^2 + ST^2 + 5 (a WT + (b WT + ST) T)^2 + (a ST - (WT - b ST) T)^2)$]

Out[33]= 0

In[34]= **WT := b ST + CT KT**

In[35]= **ST := DT RT - 2 (b T + a) CT^2**

In[36]= **ZT :=** $KT^2 (RT ET - CT^3) + RT KT DT (DT1 RT - 3 (b^2 + 1) T DT) + DT^2 RT$
 $(-a b (2 KT + T KT1) - 2 (a^2 + 1) (b^2 + 1) CT + ((a^2 + 1) c2 + (b^2 + 1) c0) RT) +$
 $2 RT CT ((b T + a) (DT KT1 CT + DT KT CT1 - DT1 KT CT) - b DT CT KT) + 4 CT^4 (b T + a)$
 $((a^2 - 1) c2 + (b^2 + 1) c0) (b T + a) - (1 / 2) a c1 RT1 + 2 a (c2 - c0) - b c1)$

In[37]= **Expand**[
 $\{Cancel[Z1] - RT CT^2 ZT /. \{KT -> RT1 CT - RT CT1, KT1 -> 2 (b^2 + 1) CT - 2 c2 RT\}\} /.$
 $\{RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,$
 $CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T\}$]

Out[37]= {0}

Lemma 3.4

In[38]= **LTD := 4 ET**

In[39]= **LTD1 := 3 ET1**

In[40]= **LTC := 3 DT**

In[41]= **LTC1 := 2 DT1**

In[42]= **LTR := 4 (b T + a) CT**

In[43]= **LTa := 2 c0 + c1 T**

In[44]= **LTb := CT1**

In[45]= **LTbTa := 2 CT**

```

In[46]:= LTR1 := 4 b CT + 2 (b T + a) CT1
In[47]:= LTK := 3 RT1 DT - 2 RT DT1 + 2 CT (2 b CT - (b T + a) CT1)
In[48]:= LTK1 := 6 (b^2 + 1) DT - 2 (d2 + 3 T d3) RT + 4 (b c1 - 2 a c2) CT
In[49]:= LTc0 := 3 d0 + d1 T
In[50]:= LTc1 := 2 d1 + 2 d2 T
In[51]:= LTc2 := d2 + 3 d3 T
In[52]:= ST1 := DT1 RT + DT RT1 - 2 b CT^2 - 4 (b T + a) CT CT1
In[53]:= WT1 := b ST1 + CT1 KT + CT KT1
In[54]:= LTS := RT LTD + DT LTR - 2 LTbTa CT^2 - 4 (b T + a) CT LTC
In[55]:= LTW := ST LTb + b LTS + KT LTC + CT LTK
In[56]:= Expand[{KT (WT LTS - ST LTW) + 2 ST (WT ST1 - ST WT1) - 4 CT ZT /.
  {KT -> RT1 CT - RT CT1, KT1 -> 2 (b^2 + 1) CT - 2 c2 RT}} /.
  {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
  CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T}]

```

Out[56]= {0}

Proposition 3.6.

(i)

```

In[57]:= Simplify[Expand[
  ZT / (c0 - c2) /. {KT -> 2 T (c0 - c2), RT -> T^2 + 1, CT -> c0 + c2 T^2, a -> 0, b -> 0}]]

```

Out[57]= $2 DT DT1 T (1 + T^2)^2 - DT^2 (1 + 6 T^2 + 5 T^4) -$
 $4 (c0 - c2) T^2 (c0^3 + 3 c0^2 c2 T^2 + 3 c0 c2^2 T^4 + c2^3 T^6 - ET (1 + T^2))$

```

In[58]:= Collect[Expand[2 DT DT1 T (1 + T^2)^2 - DT^2 (1 + 6 T^2 + 5 T^4) -
  4 (c0 - c2) T^2 (c0^3 + 3 c0^2 c2 T^2 + 3 c0 c2^2 T^4 + c2^3 T^6 - ET (1 + T^2)) /.
  {DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 d2 T + 3 d3 T^2,
  ET -> e0 + e1 T + e2 T^2 + e3 T^3 + e4 T^4}], T]

```

Out[58]= $-d0^2 + (-4 c0^4 + 4 c0^3 c2 - 6 d0^2 + d1^2 + 2 d0 d2 + 4 c0 e0 - 4 c2 e0) T^2 +$
 $(-8 d0 d1 + 4 d1 d2 + 4 d0 d3 + 4 c0 e1 - 4 c2 e1) T^3 +$
 $(-12 c0^3 c2 + 12 c0^2 c2^2 - 5 d0^2 - 2 d1^2 - 4 d0 d2 +$
 $3 d2^2 + 6 d1 d3 + 4 c0 e0 - 4 c2 e0 + 4 c0 e2 - 4 c2 e2) T^4 +$
 $(-8 d0 d1 + 8 d2 d3 + 4 c0 e1 - 4 c2 e1 + 4 c0 e3 - 4 c2 e3) T^5 +$
 $(-12 c0^2 c2^2 + 12 c0 c2^3 - 3 d1^2 - 6 d0 d2 + 2 d2^2 + 4 d1 d3 + 5 d3^2 + 4 c0 e2 - 4 c2 e2 +$
 $4 c0 e4 - 4 c2 e4) T^6 + (-4 d1 d2 - 4 d0 d3 + 8 d2 d3 + 4 c0 e3 - 4 c2 e3) T^7 +$
 $(-4 c0 c2^3 + 4 c2^4 - d2^2 - 2 d1 d3 + 6 d3^2 + 4 c0 e4 - 4 c2 e4) T^8 + d3^2 T^{10}$

(iv)

```

In[59]:= a0 := s3 g7

```

```

In[60]:= a1 := -(s2 g7 + s3 g6)

```

```

In[61]:= a2 := s1 g7 + s2 g6 + s3 g5

```

```

In[62]:= a3 := -(g7 + s1 g6 + s2 g5 + s3 g4)

```

In[63]:= **a4 := g6 + s1 g5 + s2 g4 + s3 g3**

In[64]:= **a5 := - (g5 + s1 g4 + s2 g3 + s3 g2)**

In[65]:= **a6 := g4 + s1 g3 + s2 g2 + s3 g1**

In[66]:= **a7 := - (g3 + s1 g2 + s2 g1 + s3)**

In[67]:= **a8 := g2 + s1 g1 + s2**

In[68]:= **a9 := - (g1 + s1)**

In[69]:= **Solve[{a1 == 0, a9 == 0, a5 - a3 - a7 == 0}, {s1, s2, s3}]**

Out[69]= $\{ \{s1 \rightarrow -g1, s2 \rightarrow - (g6 (g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7)) / (-g1 g6 + g3 g6 - g5 g6 + g7 - g2 g7 + g4 g7), s3 \rightarrow (g7 (-g1 g2 + g3 + g1 g4 - g5 - g1 g6 + g7)) / (g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7)\} \}$

In[70]:= **G1 := g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7**

In[71]:= **G2 := g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7**

In[72]:= **s1 := -g1**

In[73]:= **s2 := $\frac{g6 G2}{G1}$**

In[74]:= **s3 := $\frac{-g7 G2}{G1}$**

In[75]:= **a0**

Out[75]= $- \left((g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7) g7^2 \right) / (g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7)$

In[76]:= **Factor[a0 - a2 + a4 - a6 + a8 - 1]**

Out[76]= $- \left((g1 g6 - g7) (1 + g1^2 - 2 g2 + g2^2 - 2 g1 g3 + g3^2 + 2 g4 - 2 g2 g4 + g4^2 + 2 g1 g5 - 2 g3 g5 + g5^2 - 2 g6 + 2 g2 g6 - 2 g4 g6 + g6^2 - 2 g1 g7 + 2 g3 g7 - 2 g5 g7 + g7^2) \right) / (g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7)$

In[77]:= **Simplify** $\left[(1 + g1^2 - 2 g2 + g2^2 - 2 g1 g3 + g3^2 + 2 g4 - 2 g2 g4 + g4^2 + 2 g1 g5 - 2 g3 g5 + g5^2 - 2 g6 + 2 g2 g6 - 2 g4 g6 + g6^2 - 2 g1 g7 + 2 g3 g7 - 2 g5 g7 + g7^2) - \left((g1 - g3 + g5 - g7)^2 + (1 - g2 + g4 - g6)^2 \right) \right]$

Out[77]= 0

In[78]:= **G3 := - (g1 g6 - g7)**

In[79]:= **Collect** $\left[\left(\frac{s1}{3} + Y \right)^3 - s1 \left(\frac{s1}{3} + Y \right)^2 + s2 \left(\frac{s1}{3} + Y \right) - s3, Y \right]$

Out[79]= $-\frac{2 S1^3}{27} + \frac{S1 S2}{3} - S3 + \left(-\frac{S1^2}{3} + S2 \right) Y + Y^3$

$$\text{In[80]:= Factor}\left[4\left(-\left(-\frac{s1^2}{3} + s2\right)/3\right)^3 - \left(-\frac{2s1^3}{27} + \frac{s1s2}{3} - s3\right)^2\right]$$

$$\begin{aligned} \text{Out[80]= } & - \left((g1 g2 - g3 - g1 g4 + g5 + g1 g6 - g7) \right. \\ & \left(-g1^4 g2 g6^3 + 4 g1^2 g2^2 g6^3 + g1^3 g3 g6^3 - 8 g1 g2 g3 g6^3 + g1^3 g2 g3 g6^3 + 4 g3^2 g6^3 - \right. \\ & g1^2 g3^2 g6^3 + g1^4 g4 g6^3 - 8 g1^2 g2 g4 g6^3 + 8 g1 g3 g4 g6^3 - g1^3 g3 g4 g6^3 + \\ & 4 g1^2 g4^2 g6^3 - g1^3 g5 g6^3 + 8 g1 g2 g5 g6^3 - g1^3 g2 g5 g6^3 - 8 g3 g5 g6^3 + \\ & 2 g1^2 g3 g5 g6^3 - 8 g1 g4 g5 g6^3 + g1^3 g4 g5 g6^3 + 4 g5^2 g6^3 - g1^2 g5^2 g6^3 - g1^4 g6^4 + \\ & 8 g1^2 g2 g6^4 - 8 g1 g3 g6^4 + g1^3 g3 g6^4 - 8 g1^2 g4 g6^4 + 8 g1 g5 g6^4 - g1^3 g5 g6^4 + \\ & 4 g1^2 g6^5 + 4 g1^5 g6^2 g7 - 17 g1^3 g2 g6^2 g7 - g1^3 g2^2 g6^2 g7 + 17 g1^2 g3 g6^2 g7 - \\ & 8 g1^4 g3 g6^2 g7 + 19 g1^2 g2 g3 g6^2 g7 - 18 g1 g3^2 g6^2 g7 + 4 g1^3 g3^2 g6^2 g7 + \\ & 17 g1^3 g4 g6^2 g7 + 2 g1^3 g2 g4 g6^2 g7 - 19 g1^2 g3 g4 g6^2 g7 - g1^3 g4^2 g6^2 g7 - \\ & 17 g1^2 g5 g6^2 g7 + 8 g1^4 g5 g6^2 g7 - 19 g1^2 g2 g5 g6^2 g7 + 36 g1 g3 g5 g6^2 g7 - \\ & 8 g1^3 g3 g5 g6^2 g7 + 19 g1^2 g4 g5 g6^2 g7 - 18 g1 g5^2 g6^2 g7 + 4 g1^3 g5^2 g6^2 g7 - \\ & 16 g1^3 g6^3 g7 - 8 g1 g2 g6^3 g7 - g1^3 g2 g6^3 g7 + 8 g3 g6^3 g7 + 17 g1^2 g3 g6^3 g7 + \\ & 8 g1 g4 g6^3 g7 + g1^3 g4 g6^3 g7 - 8 g5 g6^3 g7 - 17 g1^2 g5 g6^3 g7 - 8 g1 g6^4 g7 - \\ & 8 g1^4 g6 g7^2 + 45 g1^2 g2 g6 g7^2 + 8 g1^4 g2 g6 g7^2 - 18 g1^2 g2^2 g6 g7^2 - \\ & 45 g1 g3 g6 g7^2 + 8 g1^3 g3 g6 g7^2 - 9 g1 g2 g3 g6 g7^2 - 8 g1^3 g2 g3 g6 g7^2 + \\ & 27 g3^2 g6 g7^2 - 45 g1^2 g4 g6 g7^2 - 8 g1^4 g4 g6 g7^2 + 36 g1^2 g2 g4 g6 g7^2 + \\ & 9 g1 g3 g4 g6 g7^2 + 8 g1^3 g3 g4 g6 g7^2 - 18 g1^2 g4^2 g6 g7^2 + 45 g1 g5 g6 g7^2 - \\ & 8 g1^3 g5 g6 g7^2 + 9 g1 g2 g5 g6 g7^2 + 8 g1^3 g2 g5 g6 g7^2 - 54 g3 g5 g6 g7^2 - \\ & 9 g1 g4 g5 g6 g7^2 - 8 g1^3 g4 g5 g6 g7^2 + 27 g5^2 g6 g7^2 + 62 g1^2 g6^2 g7^2 - \\ & 17 g1^2 g2 g6^2 g7^2 - 45 g1 g3 g6^2 g7^2 + 17 g1^2 g4 g6^2 g7^2 + 45 g1 g5 g6^2 g7^2 + \\ & 4 g6^3 g7^2 + 4 g1^3 g7^3 - 27 g1 g2 g7^3 - 8 g1^3 g2 g7^3 + 27 g1 g2^2 g7^3 + 4 g1^3 g2^2 g7^3 + \\ & 27 g3 g7^3 - 27 g2 g3 g7^3 + 27 g1 g4 g7^3 + 8 g1^3 g4 g7^3 - 54 g1 g2 g4 g7^3 - \\ & 8 g1^3 g2 g4 g7^3 + 27 g3 g4 g7^3 + 27 g1 g4^2 g7^3 + 4 g1^3 g4^2 g7^3 - 27 g5 g7^3 + \\ & 27 g2 g5 g7^3 - 27 g4 g5 g7^3 - 72 g1 g6 g7^3 + 45 g1 g2 g6 g7^3 + 27 g3 g6 g7^3 - \\ & 45 g1 g4 g6 g7^3 - 27 g5 g6 g7^3 + 27 g7^4 - 27 g2 g7^4 + 27 g4 g7^4) \left. \right) / \\ & (27 (g1 g6 - g3 g6 + g5 g6 - g7 + g2 g7 - g4 g7)^3) \end{aligned}$$

```
In[81]:= G4 := -g14 g2 g63 + 4 g12 g22 g63 + g13 g3 g63 - 8 g1 g2 g3 g63 + g13 g2 g3 g63 + 4 g32 g63 -
g12 g32 g63 + g14 g4 g63 - 8 g12 g2 g4 g63 + 8 g1 g3 g4 g63 - g13 g3 g4 g63 + 4 g12 g42 g63 -
g13 g5 g63 + 8 g1 g2 g5 g63 - g13 g2 g5 g63 - 8 g3 g5 g63 + 2 g12 g3 g5 g63 - 8 g1 g4 g5 g63 +
g13 g4 g5 g63 + 4 g52 g63 - g12 g52 g63 - g14 g64 + 8 g12 g2 g64 - 8 g1 g3 g64 + g13 g3 g64 -
8 g12 g4 g64 + 8 g1 g5 g64 - g13 g5 g64 + 4 g12 g65 + 4 g15 g62 g7 - 17 g13 g2 g62 g7 -
g13 g22 g62 g7 + 17 g12 g3 g62 g7 - 8 g14 g3 g62 g7 + 19 g12 g2 g3 g62 g7 - 18 g1 g32 g62 g7 +
4 g13 g32 g62 g7 + 17 g13 g4 g62 g7 + 2 g13 g2 g4 g62 g7 - 19 g12 g3 g4 g62 g7 -
g13 g42 g62 g7 - 17 g12 g5 g62 g7 + 8 g14 g5 g62 g7 - 19 g12 g2 g5 g62 g7 + 36 g1 g3 g5 g62 g7 -
8 g13 g3 g5 g62 g7 + 19 g12 g4 g5 g62 g7 - 18 g1 g52 g62 g7 + 4 g13 g52 g62 g7 -
16 g13 g63 g7 - 8 g1 g2 g63 g7 - g13 g2 g63 g7 + 8 g3 g63 g7 + 17 g12 g3 g63 g7 +
8 g1 g4 g63 g7 + g13 g4 g63 g7 - 8 g5 g63 g7 - 17 g12 g5 g63 g7 - 8 g1 g64 g7 - 8 g14 g6 g72 +
45 g12 g2 g6 g72 + 8 g14 g2 g6 g72 - 18 g12 g22 g6 g72 - 45 g1 g3 g6 g72 + 8 g13 g3 g6 g72 -
9 g1 g2 g3 g6 g72 - 8 g13 g2 g3 g6 g72 + 27 g32 g6 g72 - 45 g12 g4 g6 g72 - 8 g14 g4 g6 g72 +
36 g12 g2 g4 g6 g72 + 9 g1 g3 g4 g6 g72 + 8 g13 g3 g4 g6 g72 - 18 g12 g42 g6 g72 +
45 g1 g5 g6 g72 - 8 g13 g5 g6 g72 + 9 g1 g2 g5 g6 g72 + 8 g13 g2 g5 g6 g72 - 54 g3 g5 g6 g72 -
9 g1 g4 g5 g6 g72 - 8 g13 g4 g5 g6 g72 + 27 g52 g6 g72 + 62 g12 g62 g72 - 17 g12 g2 g62 g72 -
45 g1 g3 g62 g72 + 17 g12 g4 g62 g72 + 45 g1 g5 g62 g72 + 4 g63 g72 + 4 g13 g73 -
27 g1 g2 g73 - 8 g13 g2 g73 + 27 g1 g22 g73 + 4 g13 g22 g73 + 27 g3 g73 - 27 g2 g3 g73 +
27 g1 g4 g73 + 8 g13 g4 g73 - 54 g1 g2 g4 g73 - 8 g13 g2 g4 g73 + 27 g3 g4 g73 +
27 g1 g42 g73 + 4 g13 g42 g73 - 27 g5 g73 + 27 g2 g5 g73 - 27 g4 g5 g73 - 72 g1 g6 g73 +
45 g1 g2 g6 g73 + 27 g3 g6 g73 - 45 g1 g4 g6 g73 - 27 g5 g6 g73 + 27 g74 - 27 g2 g74 + 27 g4 g74
```

```
In[82]:= g1 := SymmetricPolynomial[1, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[83]:= g2 := SymmetricPolynomial[2, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[84]:= g3 := SymmetricPolynomial[3, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[85]:= g4 := SymmetricPolynomial[4, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[86]:= g5 := SymmetricPolynomial[5, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[87]:= g6 := SymmetricPolynomial[6, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[88]:= g7 := SymmetricPolynomial[7, {x1, x2, x3, x4, x5, x6, x7}]
```

```
In[89]:= G1 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

```
Out[89]=  $\frac{2436076}{5}$ 
```

```
In[90]:= G2 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

```
Out[90]=  $\frac{664}{5}$ 
```

```
In[91]:= G3 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

```
Out[91]=  $\frac{27382}{25}$ 
```

```
In[92]:= G4 /. {x1 → 1, x2 → -2, x3 → 2, x4 → 3, x5 → 4, x6 → -10, x7 → 1 / 10}
```

```
Out[92]=  $-\frac{4831113262058724096}{15625}$ 
```


In[93]:= $a_0 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[93]= $-\frac{382464}{609019}$

In[94]:= $a_1 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[94]= 0

In[95]:= $a_2 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[95]= $\frac{2505121816}{15225475}$

In[96]:= $a_3 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[96]= $-\frac{3662378874}{3045095}$

In[97]:= $a_4 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[97]= $\frac{29627935751}{15225475}$

In[98]:= $a_5 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[98]= $-\frac{5493568311}{6090190}$

In[99]:= $a_6 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[99]= $-\frac{14502009729}{60901900}$

In[100]:= $a_7 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[100]= $\frac{1831189437}{6090190}$

In[101]:= $a_8 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[101]= $-\frac{4181509819}{60901900}$

In[102]:= $a_9 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[102]= 0

In[103]:= $a_5 - a_3 - a_7 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[103]= 0

In[104]:= $a_0 - a_2 + a_4 - a_6 + a_8 - 1 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}$

Out[104]= $\frac{2374252147}{1218038}$

In[105]:= **NSolve**[$(a_0 + a_1 X + a_2 X^2 + a_3 X^3 + a_4 X^4 + a_5 X^5 + a_6 X^6 + a_7 X^7 + a_8 X^8 + a_9 X^9 + X^{10}) == 0 / \{x_1 \rightarrow 1, x_2 \rightarrow -2, x_3 \rightarrow 2, x_4 \rightarrow 3, x_5 \rightarrow 4, x_6 \rightarrow -10, x_7 \rightarrow 1/10\}, \{X\}$]

Out[105]= $\{\{X \rightarrow -10.\}, \{X \rightarrow -2.\}, \{X \rightarrow -0.0519793\}, \{X \rightarrow 0.1\}, \{X \rightarrow 0.13882\}, \{X \rightarrow 1.\}, \{X \rightarrow 1.81316\}, \{X \rightarrow 2.\}, \{X \rightarrow 3.\}, \{X \rightarrow 4.\}\}$

Lemma 3.8

In[106]:= **Solve**[$y == a x + b (x^2 + y^2)$, y]

Out[106]= $\left\{ \left\{ y \rightarrow \frac{1 - \sqrt{1 - 4 a b x - 4 b^2 x^2}}{2 b} \right\}, \left\{ y \rightarrow \frac{1 + \sqrt{1 - 4 a b x - 4 b^2 x^2}}{2 b} \right\} \right\}$

In[107]:= **Series** $\left[\frac{1 - \sqrt{1 - 4 a b x - 4 b^2 x^2}}{2 b}, \{x, 0, 4\} \right]$

Out[107]= $a x + (b + a^2 b) x^2 + (2 a b^2 + 2 a^3 b^2) x^3 + (b^3 + 6 a^2 b^3 + 5 a^4 b^3) x^4 + O[x]^5$

In[108]:= **P**[x] := $c_0 x^2 + c_2 y^2 + d_0 x^3 + d_1 x^2 y + d_2 x y^2 +$
 $d_3 y^3 + e_0 x^4 + e_1 x^3 y + e_2 x^2 y^2 + e_3 x y^3 + e_4 y^4 /.$
 $\{y \rightarrow a x + (1 + a^2) b x^2 + 2 a b^2 (1 + a^2) x^3 + (1 + a^2) (1 + 5 a^2) b^3 x^4\}$

In[109]:= **Coefficient**[**P**[x], x , 2]

Out[109]= $c_0 + a^2 c_2$

In[110]:= **Coefficient**[**P**[x], x , 3]

Out[110]= $2 a b c_2 + 2 a^3 b c_2 + d_0 + a d_1 + a^2 d_2 + a^3 d_3$

In[111]:= **Coefficient**[**P**[x], x , 4]

Out[111]= $b^2 c_2 + 6 a^2 b^2 c_2 + 5 a^4 b^2 c_2 + b d_1 + a^2 b d_1 + 2 a b d_2 +$
 $2 a^3 b d_2 + 3 a^2 b d_3 + 3 a^4 b d_3 + e_0 + a e_1 + a^2 e_2 + a^3 e_3 + a^4 e_4$

The proof of Theorem 2.5.

To derive the explicit form of NT, we identify ZT with a function ZT[u], or ZT[T] where u and T are variables for differentiation in L_T and in T, respectively. (u is a fictional variable for L_T).

Therefore, LTZ : L_T[ZT] is given by

In[112]:= **ZTu**[u] := ZT /. {KT → KT[u], RT → RT[u], ET → ET[u],
 CT → CT[u], DT → DT[u], DT1 → DT1[u], b → b[u], a → a[u], KT1 → KT1[u],
 c2 → c2[u], c0 → c0[u], CT1 → CT1[u], c1 → c1[u], RT1 → RT1[u]}

In[113]:= **LTZ** := D[ZTu[u], u] /.
 {KT'[u] → LTK, KT[u] → KT, RT'[u] → LTR, RT[u] → RT, ET'[u] → LTE, ET[u] → ET,
 CT'[u] → LTC, CT[u] → CT, DT'[u] → LTD, DT[u] → DT, DT1'[u] → LTD1, DT1[u] → DT1,
 b'[u] → LTb, b[u] → b, a'[u] → LTa, a[u] → a, KT1'[u] → LTK1, KT1[u] → KT1,
 c2'[u] → Ltc2, c2[u] → c2, c0'[u] → Ltc0, c0[u] → c0, CT1'[u] → LTC1,
 CT1[u] → CT1, c1'[u] → Ltc1, c1[u] → c1, RT1'[u] → LTR1, RT1[u] → RT1}

Z'[T] is given by the following ZT1 :

In[114]:= **ZTT**[T] := ZT /. {KT → KT[T], RT → RT[T], ET → ET[T], CT → CT[T],
 DT → DT[T], DT1 → DT1[T], KT1 → KT1[T], CT1 → CT1[T], RT1 → RT1[T]}

In[115]:= **ZT1** := D[ZTT[T], T] /. {KT'[T] → KT1, KT[T] → KT, RT'[T] → RT1,
 RT[T] → RT, ET'[T] → ET1, ET[T] → ET, CT'[T] → CT1, CT[T] → CT,
 DT'[T] → DT1, DT[T] → DT, DT1'[T] → DT2, DT1[T] → DT1, KT1'[T] → KT2,
 KT1[T] → KT1, CT1'[T] → 2 c2, CT1[T] → CT1, RT1'[T] → RT2, RT1[T] → RT1}

The explicit form of N[T] is the following :

ln[116]:= **NT** := -5 RT KT^2 ET1 (RT DT - 2 (b T + a) CT^2) + DT^3 RT B1 + 2 DT^2 DT1 RT^2 B2 -
DT^2 RT^2 KT ((3 d3 T + d2) (5 RT - (b^2 + 1) T^2) + (d1 T + 3 d0) (b^2 + 1)) +
DT^2 B3 + 2 (b T + a) DT DT1 RT CT B4 + 10 (b T + a) DT DT2 RT^2 KT CT^2 +
DT B5 - 4 (b T + a) DT1 RT KT CT^3 (5 (b T + a) CT1 + 2 b CT) +
4 (b T + a) CT^4 KT (3 d0 B6 + d1 B7 + d2 B8 - 3 d3 T B9) + 4 CT^4 B10

ln[117]:= **B1** := KT (42 (b^2 + 1) RT - 20 (a^2 + b^2 + 1)) -
4 c2 RT (3 a b RT - 2 (a^2 + b^2 + 1 - a^2 b^2) T + 2 a b (a^2 + 1)) +
2 (b^2 + 1) c1 RT (3 RT + 4 a b T + 4 a^2 + 4) - 4 (b^2 + 1) RT RT1 c0

ln[118]:= **B2** := - (4 (b^2 + 1) T + 5 a b) KT - ((b^2 + 1) T^2 + a^2 + 1) KT1 +
4 (a^2 + 1) (b^2 + 1) CT - 2 ((a^2 + 1) c2 + (b^2 + 1) c0) RT

ln[119]:= **B3** := KT^2 (-4 a RT CT1 + 12 (b^2 + 1) (b T + a) T CT +
8 a b (b T + a) CT + 12 b RT CT + 2 b RT (2 c0 + c1 T)) +
KT KT1 (4 a b T (b T + a) CT - 12 (b T + a) RT CT + b T RT (2 c0 + c1 T) + a T RT CT1) +
KT (-4 a (b^2 + 6) RT CT^2 - (18 (b^2 + 1) T (b T + a) + 16 a b (b T + a) - 4 b (a^2 + 1))
RT CT CT1 + 8 (a^2 + 1) (b^2 + 1) (b T + a) CT^2 -
8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT CT + 4 a b T (b c1 - 2 a c2) RT CT +
4 a (b^2 + 1) (2 c0 + c1 T) RT CT - 2 a c2 (2 c0 + c1 T) RT^2 - 2 b c0 CT1 RT^2 -
12 (b^2 + 1) (b T + a) T RT1 CT^2 - 8 a b (b T + a) RT1 CT^2 - 4 (b T + a) RT RT1 CT CT1 +
4 b RT RT1 CT^2 - 4 (b T + a) RT^2 CT1^2 - 8 c2 (b T + a) RT^2 CT + 4 b RT^2 CT CT1) -
2 (b T + a) KT1 CT (2 a b T RT1 CT + 5 RT1 RT CT + 6 RT^2 CT1) -
4 (b T + a) KT2 RT CT^2 (RT + a b T) -
8 (a^2 + 1) (b^2 + 1) (b T + a) RT1 CT^3 +
8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT RT1 CT^2 -
8 (a^2 + 1) (b^2 + 1) (b T + a) RT CT^2 CT1

ln[120]:= **B4** := 2 (3 (b^2 + 1) T + 5 a b) KT CT + 5 RT KT CT1 + 2 ((b^2 + 1) T^2 + a^2 + 1) KT1 CT -
8 (a^2 + 1) (b^2 + 1) CT^2 + 4 RT CT ((a^2 + 1) c2 + (b^2 + 1) c0)

ln[121]:= **B5** := 9 CT^2 KT^3 + KT^2 CT^2 (6 RT1 CT + 4 RT CT1 + 8 b (b T + a) CT - 8 (b T + a)^2 CT1) -
8 (b T + a)^2 KT KT1 CT^3 +
KT CT^2 (-8 (b T + a) (b c1 - 2 a c2) RT CT - 8 b (b T + a) RT CT1 CT + 8 b^2 RT CT^2 -
4 b (b T + a) RT CT CT1 - 48 (b T + a)^2 CT ((a^2 - 1) c2 + (b^2 + 1) c0) +
24 a c1 (b T + a) RT1 CT - 48 (b T + a) CT (2 a (c2 - c0) - b c1) +
12 (b T + a)^2 RT CT1^2 - 8 b (b T + a) RT1 CT^2 + 8 (b T + a)^2 RT1 CT CT1 +
16 c2 (b T + a)^2 RT CT - 8 b (b T + a) RT CT CT1) +
8 (b T + a)^2 KT1 CT^3 (RT1 CT + 3 RT CT1) + 8 (b T + a)^2 RT CT^4 KT2 -
16 b (b T + a) RT CT^4 ((a^2 - 1) c2 + (b^2 + 1) c0) -
32 (b T + a)^2 RT CT^3 CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) +
16 a (b T + a) RT RT1 c1 CT^3 CT1 + 4 a b RT RT1 c1 CT^4 +
8 a (b^2 + 1) (b T + a) RT c1 CT^4 - 8 b RT CT^4 (2 a (c2 - c0) - b c1) -
32 (b T + a) RT CT^3 CT1 (2 a (c2 - c0) - b c1)

ln[122]:= **B6** := - (b^2 + 1) (b T + a) + 2 a

ln[123]:= **B7** := - (b^2 + 1) T (b T + a) + a RT1 + 2 (a T + b)

ln[124]:= **B8** := - (a^2 - 3) (b T + a) + a T RT1 - 4 a - 4 (b T + a) RT

ln[125]:= **B9** := (a^2 - 1) (b T + a) + 2 a + 4 (b T + a) RT

```
In[126]:= B10 := KT^2 (2 b CT - 4 (b T + a) CT1) - 2 (b T + a) CT KT KT1 +
KT (a RT1 c1 CT - 4 (b T + a) CT ((a^2 - 1) c2 + (b^2 + 1) c0) - 4 a (c2 - c0) CT +
2 b c1 CT - 2 a (b T + a)^2 c2 (2 c0 + c1 T) - 2 b (b T + a)^2 c0 CT1 +
2 a b (b T + a) c1 CT + a (b T + a)^2 c1 CT1 + (1 / 2) (b T + a) RT1 c1 (2 c0 + c1 T) -
2 (b T + a) (c2 - c0) (2 c0 + c1 T) + (b T + a) c1 CT1) +
8 b (b T + a)^2 ((a^2 - 1) c2 + (b^2 + 1) c0) CT^2 +
16 (b T + a)^3 CT CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) -
8 a (b T + a)^2 RT1 c1 CT CT1 - 2 a b (b T + a) RT1 c1 CT^2 -
4 a (b^2 + 1) (b T + a)^2 c1 CT^2 + 4 b (b T + a) CT^2 (2 a (c2 - c0) - b c1) +
16 (b T + a)^2 CT CT1 (2 a (c2 - c0) - b c1)
```

On the other hand, the definition of $N[T]$ is given by the following :

```
In[127]:= NTdef := -KT (LTZ - KT^2 RT LTE) - 2 ST ZT1
```

```
In[128]:= Expand[{NT - NTdef /. {KT -> RT1 CT - RT CT1,
KT1 -> 2 (b^2 + 1) CT - 2 c2 RT, KT2 -> 2 (b^2 + 1) CT1 - 2 c2 RT1}} /.
{RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
RT2 -> 2 (b^2 + 1), CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T, CT2 -> 2 c2,
DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 T d2 + 3 d3 T^2, DT2 -> 2 d2 + 6 d3 T}]
```

```
Out[128]= {0}
```

```
In[129]:= Simplify[Coefficient[{NT /. {KT -> RT1 CT - RT CT1,
KT1 -> 2 (b^2 + 1) CT - 2 c2 RT, KT2 -> 2 (b^2 + 1) CT1 - 2 c2 RT1}} /.
{RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
RT2 -> 2 (b^2 + 1), CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T,
CT2 -> 2 c2, DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 T d2 + 3 d3 T^2,
DT2 -> 2 d2 + 6 d3 T, ET1 -> e1 + 2 e2 T + 3 e3 T^2 + 4 e4 T^3}, T, 14]]
```

```
Out[129]= {-20 (-2 b c2^2 + d3 + b^2 d3) (2 b^5 c2 (-2 c0 c2 + a d2) d3 +
4 b^3 c2 (-2 c0 c2 + c2^2 + a d2) d3 + 2 b c2 (-2 c0 c2 + 2 c2^2 + a d2) d3 +
b^6 c0 d3^2 + (c0 - (1 + a^2) c2) d3^2 + (1 + b^2)^2 c1^2 (-c2^3 - 2 b c2 d3 + e4 + b^2 e4) -
(1 + b^2) c1 (b^4 d2 d3 + 2 b^2 (-3 a c2^2 + d2) d3 + (-2 a c2^2 + d2) d3 -
2 b (c2^2 d2 + a d3^2 - 2 a c2 e4) - 2 b^3 (c2^2 d2 + a d3^2 - 2 a c2 e4)) +
b^2 (c0 (4 c2^4 + 3 d3^2) - 2 c2 (2 c2^4 + 2 a c2^2 d2 + d3^2 + 3 a^2 d3^2 - 2 a^2 c2 e4)) +
b^4 (c0 (4 c2^4 + 3 d3^2) - c2 (4 a c2^2 d2 + d3^2 + a^2 (5 d3^2 - 4 c2 e4)))}
```