Metric geometry and analysis

Date: December 9–13, 2013.

Place: Room 110, Graduate School of Science Bldg. No. 3, Department of Mathematics, Kyoto University.

Titles & Abstracts

Martin Finn-Sell (University of Goettingen)

The coarse boundary groupoid and inverse semigroup actions on spaces of graphs

Abstract: In this talk I will outline the construction of the coarse groupoid that is associated to any uniformly discrete bounded geometry metric space and consider a reduction of this groupoid that is well suited to studying the geometry of spaces at infinity. Using the notion of a partial action of a group (and associated inverse semigroups) I will prove that large girth graphs are a-T-menable at infinity in some appropriate sense. As an application of this I will explain how to get positive results for the Baum-Connes conjecture with coefficients for certain Gromov monster groups.

Yoshiyasu Fukumoto (Kyoto University)

Invariance of Finiteness of K-area under Surgery

Abstract: K-area is an invariant for Riemannian manifolds introduced by Gromov as an obstruction to the existence of positive scalar curvature. However in general it is difficult to determine whether K-area is finite or not in spite of its natural definition. In this paper, we study how the invariant changes under surgery.

Ana Khukhro (Clermont-Ferrand)

Box spaces of free groups

Abstract: Given a finitely generated group, it is often illuminating to look at the geometric properties of the group's Cayley graph. These properties are strongly linked to the structure of the group, and are thus an important tool in modern group theory. When the group is also residually finite, one can build a metric space called a box space which reflects the structure of the finite quotients of the group. Such spaces often arise as examples of metric spaces with interesting coarse-geometric properties. In this talk I will outline the connections between analytic properties of groups and coarse-geometric properties of their box spaces, and describe certain examples of box spaces of free groups with surprising behaviour.

Yoshikata Kida (Kyoto University)

Stability in orbit equivalence, central extensions of groups, and relative property (T)

Abstract: This talk is focused on an ergodic, free and measure-preserving action of a discrete countable group on a standard probability space and its orbit structure. We say that such an action is stable if the associated orbit equivalence relation is isomorphic to its direct product with the ergodic hyperfinite equivalence relation of type II_1. A question of our interests is which group admits a stable action. We discuss background, and present a sufficient condition and a necessary condition for a group with infinite center to admit a stable action. These conditions involve relative property (T) of a central subgroup.

Takefumi Kondo (Tohoku University)

Fixed point theorems for p-uniformly convex spaces

ABSTRACT: A remarkable result by Zuk gave a criterion for providing Kazhdan's property (T) by means of spectral gap of finite graphs. This result was generalized by Izeki-Nayatani and Gromov for the class of CAT(0) spaces and by Bourdon for L_p spaces. We generalize these criteria to the class of p-uniformly convex spaces.

Hirokazu Maruhashi (Kyoto University)

Vanishing of cohomology and parameter rigidity of actions of solvable Lie groups

ABSTRACT: We study smooth actions of noncompact connected Lie groups on closed manifolds. An action ρ of G on M is called parameter rigid if any action of G on M with the same orbit decomposition as that of ρ is conjugate to ρ. Namely an action is parameter rigid if it is determined only by its orbit structure. Rigidity is often related to vanishing of some first cohomology. This is the case for parameter rigidity of actions of nilpotent Lie groups as we have shown before. In this talk we discuss a generalization of this to actions of solvable Lie groups.

First we see vanishing of first cohomologies is sufficient for parameter rigidity of actions of general solvable Lie groups. However we need to assume vanishing of uncountably many cohomologies, and this strong assumption sometimes causes a trouble in application. There are two methods to weaken the assumption. First one uses volume forms on manifolds and invented by Matsumoto and Mitsumatsu. But this cannot be applied to many solvable Lie groups. Second one is a new method and this works effectively.

Based on these general theorems we obtain new examples of parameter rigid actions of solvable Lie groups. If time allows we will comment on it.

Masato Mimura (Tohoku University)

Group approximation in Cayley topology and coarse geometry: Coarse embeddings of amenable groups

ABSTRACT: Objective of this talk is to study metric geometric properties of coarse disjoint union of Cayley graphs. We employ the Cayley topology, introduced by R. I. Grigorchuk, and observe connection between large scale structure of metric spaces and group properties of Cayley limit points. In this talk, we see that a coarse disjoint union has property A of G. Yu if and only if all of the Cayley limit groups are amenable. As an application, we construct a coarse disjoint union of finite special linear groups which has property A but is of very poor compression into all uniformly convex Banach spaces. This is a joint work with Hiroki Sako.

Bogdan Nica (University of Goettingen)

Proper isometric actions of hyperbolic groups on L^p-spaces

ABSTRACT: I will discuss the following result: every non-elementary hyperbolic group G admits a proper affine isometric action on an L^p-space associated to $\mathrm{bd}(G) \times \mathrm{bd}(G)$ for p larger than the "dimension" of the boundary $\mathrm{bd}(G)$.

Piotr Nowak (IMPAN)

Deformations, cohomology and fixed-point properties. 1, 2

ABSTRACT: In these talks I will describe recent joint work with Uri Bader. We will be considering the stability of the cohomology of a group with coefficients in a representation on a Banach space, under metric perturbations of that representation. We will show that the vanishing of such cohomology is preserved under sufficiently small deformations, under some natural assumptions. The original motivation and one of the applications of the above deformation theorem are fixed point properties for affine actions on Banach spaces. In particular, in this setting we obtain deformation theorems for affine actions of groups with property (T). In the first lecture I will present the background on cohomology and fixed point properties on Banach spaces and in the second lecture I will discuss the deformation theorem, related examples and applications.

Shin-ichi Oguni (Ehime University)

Coronae and coarse homologies

ABSTRACT: The coarse Baum-Connes conjecture claims that two coarse homologies are naturally isomorphic for any 'good' proper metric space like a universal cover of an aspherical closed Riemannian manifold. Coronae are boundaries of unbounded proper metric spaces which are nice from the coarse geometric viewpoint. In this talk, I show that some unbounded proper metric spaces have coronae which can be used in order to compute coarse homologies. This talk is based on joint-works with Tomohiro Fukaya (Tohoku university).

Qinggang Ren (Chongqing University)

Nagata dimension and asymptotic cones

ABSTRACT: TBA

Hiroki Sako (Tokai University)

Coarse amenability and its characterizations

ABSTRACT: We study property A for coarse metric spaces introduced by Guoliang Yu. Property A is an amenability-type condition, which is less restrictive than amenability for groups. The property is often called coarse amenability. Skandalis, Tu, and Yu clarified a connection with amenability in the theory of operator algebras. They proved that a coarse metric space X has property A if and only if the uniform Roe algebra $C^*_u(X)$ is nuclear. We prove that exactness and local reflexivity of $C^*_u(X)$ also characterize property A. In the proof, the operator norm localization property plays a key role.

Yuhei Suzuki (RIMS, Kyoto)

Amenable minimal Cantor systems of free groups arising from diagonal actions

ABSTRACT: We study amenable minimal Cantor systems of free groups. We show for every free group, (explicitly given) continuum many Kirchberg algebras are realized as the crossed product of an amenable minimal Cantor system of it. In particular this shows there are continuum many Kirchberg algebras such that each of which is decomposed to the crossed products of amenable minimal Cantor systems of any virtually free group. We also give computations of K-groups for the diagonal actions of the boundary action and the odometer transformations. These computations with Matui's theorem classify their topological full groups.

Tetsu Toyoda (Suzuka National College of Technology)

Uniform estimates of nonlinear spectral gaps of finite connected graphs

ABSTRACT: As a nonlinear analogue of the linear spectral gap of a finite connected graph G, we can define the nonlinear spectral gap $\lambda_1(G, X)$ with respect to any metric space X. Recently, estimates of these invariants are required in various contexts in geometric group theory and metric geometry.

In this talk, we first briefly outline what kinds of estimates are required. Then, we present some of the recent estimates obtained in a joint work with Takefumi Kondo (Tohoku University). We focus on the behavior of the nonlinear spectral gap $\lambda_1(G, X)$ when we change a target metric space X.

Masaki Tsukamoto (Kyoto University)

Mean dimension and an embedding problem

ABSTRACT: We discuss the following question: Given a topological dynamical system, when can we embed it into $[0, 1]^Z$ (the shift on the interval)? This question has a long history. One origin is a sampling problem in Electric Engineering. Around 1999, Elon Lindenstrauss and Benjamin Weiss found that a new topological invariant "mean dimension" (defined by Gromov) is a key notion towards a good understanding of the question. I plan to explain some background materials, and then present our recent progress on the question. This talk is based on the joint works with Yonatan Gutman and Elon Lindenstrauss.

Rufus Willett (University of Hawaii)

Exactness and coarse geometry. 1, 2

ABSTRACT: Exactness is a C^*-algebraic property of locally compact groups: a group G is exact if the reduced crossed product with the group preserves short exact sequences of G-C^*-algebras. It is closely connected to coarse geometry (at least for discrete groups) via Yu's property A.

In the first lecture, I'll discuss how failures of exactness are related to Roe algebras, expanders, and so-called ghost operators. I'll sketch out why this causes problems for the Baum-Connes conjecture, and one way to get around this using exotic crossed products.

In the second lecture, I'll discuss two coarse geometric properties: fibered coarse embeddability (functionally equivalently, the Haagerup property for the boundary of the coarse groupoid), and geometric property (T). I will approach these in a slightly non-standard way, discussing how they relate to the issues in the first talk, and to each other.

Parts of this are joint work with (some of) Paul Baum, Erik Guentner, John Roe, and Guoliang Yu. I will try to keep the talks self-contained.