Further development of

Atiyah-Singer index theorem and K-theory

Date: December 16--20, 2013
Place: Room 110, Graduate School of Science Bldg. No. 3,
Department of Mathematics, Kyoto University

Program:

16 (Mon)	9.30-10.30	Mathai
	11-12	Hang Wang
	14-15	Kasparov
	15.30-16.30	Kato
	17-18	Song
17 (Tues)	10-11	Piazza
	11.30-12.30	Moriyoshi
	14.30-15.30	Zuk
	16-17	Nica
18 (Wed)	10-11	Tang
	11.30-12.30	Posthuma
	14.30-15.30	Bai-Ling Wang
	16-17	Yoshikawa
19 (Thu)	9.30-10.30	Higson
	11-12	Gomi
	Afternoon	Free Discussion
20 (Fri)	9.30-10.30	Ponge
	10.40-11.40	Xie

Title and Abstract:

Kiyonori Gomi (Shinshu University)

title: Topological T-duality for Real circle bundles

abstract: Topological T-duality, initiated by Bouwkneght, Evslin and Mathai, roughtly says that the twisted K-theory of the total space of a principal circle bundle is isomorphic to that of another, generally different, principal circle bundle. The theme of my talk is a generalization of this duality motivated by orbifold superstring theory of type IIB, in which principal circle bundles are replaced by `Real' circle bundles, and K-theory is replaced by its variant $K_¥pm$.

Nigel Higson (Penn State University)

Noncommutative Geometry and Parabolic Induction

There are well-known connections between K-theory, index theory and the so-called square-integrable representations of a semisimple group. But this talk will be about the other (tempered) representations, the so-called parabolically induced representations (a basic theorem of Harish-Chandra says, roughly, that either a tempered representation is square-integrable or it is parabolically induced from a square-integrable representation). What does non-commutative geometry have to say about these? I'll discuss parabolic induction from the point of view of bimodules, including Hilbert bimodules, as well as the opposite concept parabolic restriction. My main goal will be to describe adjunction relations between the two, and their significance for representation theory and K-theory.

Gennadi Kasparov (Vanderbilt University)

Title: On the L^p Novikov and Baum-Connes conjectures.

Abstract: The right side of the Novikov and Baum-Connes conjectures is the K-theory of the reduced C^*-algebra C^*_red(G) of the group G. This algebra is the completion of the algebra L^1(G) in the norm of the algebra of operators acting on L^2(G). If we consider instead the completion of the algebra L^1(G) in the norm of the algebra of operators acting on L^p(G) we will get the Banach algebra C^p_red(G). The K-theory of this algebra serves as the right side of the L^p version of the Novikov and Baum-Connes conjectures. The construction of the assembly map in this case requires a little bit of a technique of asymptotic morphisms for Banach algebras. The current joint work in progress with Guoliang Yu aims at proving the following result: If a countable discrete group ¥Gamma acts on the space l^p(Z) properly (in the metric sense) and affine-isometrically, p¥geq 2, Z is discrete, and the linear part of the action of ¥Gamma on l^p(Z) is induced by the action of ¥Gamma on Z, then ¥Gamma satisfies the L^p Baum-Connes conjecture. I will discuss the techniques involved in this work.

Tsuyoshi Kato (Kyoto University)

A new index theory modeled on the reduced cohomology group and its application to complexity of smooth structure on open four manifolds

Abstract: In this talk we introduce a new functional analytic approach to the index theory and use it to induce Fredholm property for elliptic differential complex over non compact manifolds. We apply it to study on complexity of smooth structure on open four manifolds

Hitoshi Moriyoshi (Nagoya University)

Title: On a cyclic volume cocycle in Fractal Geometry

Abstract: Connes constructed a Fredholm module on a Cantor set and introduced an associated Dirac operator. He proved that the Dixmier trace of the operator can recapture the Minkowski dimension of Cantor set. On the other hand, the resulting Connes-Chern character gives rise to a trivial cohomology class. In this talk we introduce a secondary cyclic cocycle for the Connes-Chern character and exhibit that it can detect a phantom circle, which is a C*-subalgebra of the continuous functions on the Cantor set that is isomorphic to that of a circle. This construction can be generalized to the higher dimensional case. This is a joint work with T. Natsume.

Bogdan Nica (University of Goettingen)

K-homological finiteness and hyperbolic groups

This talk is concerned with the following finiteness property for theK-homology of a C*-algebra: all the K-homology classes can berepresented by cycles which are finitely summable over the same dense sub-algebra, and with the same degree of summability. I will discussresults which say that the following C*-algebras enjoy this finitenessproperty:- the C*-crossed product defined by the action of a torsion-freehyperbolic group on its boundary;- the reduced group C*-algebra of a torsion-free, a-T-menable hyperbolic group. This is joint work with Heath Emerson (Victoria).

Paolo Piazza (Universita' di Roma "La Sapienza")

Title: Stratified spaces and the Novikov conjecture

Abstract. Witt spaces are examples of stratified pseudomanifolds admitting a homology L-class and, therefore, admitting higher signatures. Albin, Leichtnam Mazzeo and myself have proved that if the Strong Novikov conjecture holds for the fundamental group of a Witt space then these higher signatures are stratified homotopy invariants (Ann. Sci. ENS, vol 45, 2012). Banagl has enlarged the

class of stratified pseudomanifolds admitting a homology L-class by requiring the existence of a Lagrangian structure along the strata. On these psudomanifolds is therefore possible to formulate a Novikov conjecture. In this talk I will explain recent results, obtained once again in collaboration with Albin, Leichtnam and Mazzeo, arXiv 1308.2844, showing that despite a rather sophisticated elliptic theory it is still possible to prove that the Strong Novikov Conjecture implies the Novikov conjecture. In particular, we obtain the stratified homotopy invariance of the higher signatures of a large class of stratified pseudomanifolds.

Raphael Ponge (Seoul National University)

Noncommutative geometry and conformal geometry

Abstract: In this talk we shall report on a applications to conformal geometry of the framework of noncommutative geometry, and more specifically twisted spectral triples, conformal geometry and noncommutative. One result is a local index formula in conformal geometry taking into account the action of the group of conformal diffeomorphisms. One byproduct is a cohomological interpretation of conformal invariants constructed by Branson-Orsted. Another result is an index formula for twisted spectral triples on noncommutative tori associated to conformal weights. A further result is a version of Vafa-Witten inequality for twisted spectral triples. This uses a notion of Poincare duality for twisted spectral triples. In particular, natural examples of twisted spectral triples occurring as Poincare duals of ordinary spectral triples. One geometric application is a version of Vafa-Witten in conformal geometry. There are also versions of the Vafa-Witten inequality for spectral triples over noncommutative tori associated to conformal weights and for duals of discrete cocompact subgroups of semisimple Lie groups. (This is joint work with Hang Wang.)

Hessel Posthuma (University of Amsterdam)

Title: Lie groupoids, cyclic homology and index theory.

Abstract: In this talk I will describe a general index theorem for Lie groupoids computing certain pairings between element in the K-theory of the convolution algebra and cyclic cohomology classes. The classes in K-theory are associated to elliptic operators invariant under a proper action of the Lie groupoid, and the cyclic cohomology classes come from its smooth groupoid cohomology. This index theorem generalizes various well-known index theorems (in cohomological form) such as the foliation index theorem, and the index theorem for covering spaces.

Yanli Song (University of Tronto)

Title : equivariant index theory of non-compact manifold and bordism

Abstract : For any compact, spin^c manifold, we can construct a Dirac operator whose index is finite dimensional. It is well-know that the index is invariant under bordism. In this talk, I will discuss how to associate a well-define index for a non-compact G-manifold with mild assumptions. Instead of finite dimensional virtual representation of G, its index will take values in the completion of character ring R(G). In addition, I will introduce a non-compact bordism such that the index of non-compact G-manifold is invariant under it. I will explain how this work is related to the quantization commutes with reduction theorem.

Xiang Tang (Washington University)

Title: An index theorem for strongly pseudoconvex domains with isolated singularities

Abstract: In the 80s, Baum, Douglas, and Taylor developed a K-homology theory for strongly pseudoconvex domains. In this talk, we will discuss an extension of this theory to study an index problem for strongly pseudoconvex domains with isolated singularities. This is work in progress with R. Douglas and G. Yu.

Mathai Varghese (University of Adelaide)

Title: Quantizing group actions

The "quantization commutes with reduction" problem was solved for compact groups acting on compact symplectic manifolds by Meinrenken, with Tian-Zhang giving a direct analytic proof in the late 1990s. Since then, generalizations have been obtained to cases where only the group, or the orbit space of the action, is compact. The main result which I will discuss is a generalization to settings where the group, the manifold and the orbit space may all be noncompact, as long as the symplectic reduction at zero is compact. The method used builds on the Tian-Zhang approach in the compact case.
This is joint work with Peter Hochs.

Bai-Ling Wang (Australian National University)

Title: Twisted geometric cycles and twisted K-homology

Abstract: In string theory, D-branes were proposed as a mechanism for providing boundary conditions for the dynamics of open strings moving in space-time. Motivated by the Freed–Witten anomaly cancellation condition for type II D-branes, the author defined twisted Spin^c manifolds as a mathematical notion of D-branes in string theory. In recent joint work with Paul Baum and Alan Carey, we investigate the role of these D-branes in relating twisted K-homology and twisted Spin^c bordism group. In particular, we establish an equivalence between geometric and analytic (twisted) K-homology for any finite CW complex. The new ingredient in the proof is to introduce a periodic Spin^c (co)bordism group.

Hang Wang (University of Adelaide)

Localised indices and L^2-Lefschetz fixed point formulae for orbifolds

We focus on a class of localized indices for the Dirac type operators on a complete Riemannian orbifold, where a discrete group acts properly, co-compactly and isometrically. These localized indices, generalising the L^2 index of Atiyah, are obtained by taking Hattori-Stallings type traces of the higher index for the Dirac type operators along conjugacy classes of the discrete group. Applying the local index technique, we also obtain L^2 version of the Lefschetz fixed point formula for orbifolds. These cohomological formulae for the localised indices give rise to a class of refined topological invariants for the quotient orbifold. We shall explain some further development of the localised indices. This is joint work with Bai-Ling Wang.

Zhizhang Xie (Texas A&M University)

Title: Higher rho invariants and the moduli space of positive scalar curvature metrics

Abstract: Given a closed smooth manifold M which carries a positive scalar curvature metric, one can associate an abelian group P(M) to the space of positive scalar curvature metrics on this manifold. The group of all diffeomorphisms of the manifold naturally acts on P(M). The moduli group of positive scalar curvature metrics is defined to be the quotient abelian group of this action, i.e. the coinvariant of the action. In this talk, I will talk about how to use the higher rho invariant and the finite part of the K-theory of the group C*-algebra of the fundamental group of M to give a lower estimate of the rank of the moduli group. This is joint work with Guoliang Yu.

Ken-Ichi Yoshikawa (Kyoto University)

Title: An analytic torsion invariant for $K3$ surfaces with involution

Abstract: In 2004, we introduced an invariant of $K3$ surfaces with anti-symplectic holomorphic involution, which we constructed using equivariant analytic torsion. It is known that the moduli space of those $K3$ surfaces with

involution consists of 75 distinct families. In our recent work with S. Ma, we get an expression for the analytic torsion invariant as an explicit function on the moduli space of K3 surfaces with involution, at least for 72 families. In the talk, I would like to report our recent progress in the analytic torsion invariant.

Andrzej Zuk (Universite Paris 7)

Title: TBA