CLASSIFICATION OF CERTAIN GENERALIZED BERNOULLI ACTIONS OF MAPPING CLASS GROUPS

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Abstract. We present a classification result up to conjugacy of generalized Bernoulli actions of mapping class groups arising geometrically. As a consequence of a rigidity result due to the author, this gives a classification of such actions up to orbit equivalence.

1. Introduction

By a discrete group we mean a discrete and countable group. We refer as *standard finite measure space* (X, μ) to a standard Borel space X with a finite positive measure μ. When $\mu(X) = 1$, we call (X, μ) a *standard probability space.* A *standard* action of a discrete group means an essentially free, measure-preserving action on a standard finite measure space.

Definition 1.1. For $i = 1, 2$, let $\alpha_i\colon \Gamma_i \curvearrowright (X_i, \mu_i)$ be a measure-preserving action of a discrete group Γ_i on a standard finite measure space (X_i, μ_i).

(i) The two actions α_1, α_2 are said to be *weakly orbit equivalent (WOE)* if there are Borel subsets $A_1 \subset X_1$, $A_2 \subset X_2$ with $\Gamma_1 A_1 = X_1$, $\Gamma_2 A_2 = X_2$ up to null sets, and a Borel isomorphism $f\colon A_1 \to A_2$ such that
   - the two measures $f_*(\mu_1|_{A_1})$ and $\mu_2|_{A_2}$ are equivalent;
   - $f(\Gamma_1 x \cap A_1) = \Gamma_2 f(x) \cap A_2$ for a.e. $x \in A_1$.

(ii) In (i), if we can take both A_1 and A_2 to have full measure, then α_1 and α_2 are said to be *orbit equivalent (OE)*. In addition, if there is an isomorphism $F\colon \Gamma_1 \to \Gamma_2$ such that for any $g \in \Gamma_1$ and any $x \in A_1$, gx belongs to A_1 and the equation $f(gx) = F(g)f(x)$ holds, then we say that α_1 and α_2 are *conjugate* (via the isomorphism F).

In [K2], several orbit equivalence rigidity results are established for ergodic standard actions of mapping class groups of compact orientable surfaces with higher complexity (see Theorem 3.1). The aim of this note is to construct a family of ergodic standard actions of mapping class groups which are mutually non-WOE. By the rigidity results, this problem reduces to construction of non-conjugate actions. We present several concrete families of generalized Bernoulli actions of mapping class groups which can be classified up to conjugacy, and thus up to WOE (see Theorem 3.8).

Note that a recent result of Bowen [Bo] classifies Bernoulli actions of sofic groups up to conjugacy and gives a classification of Bernoulli actions of mapping class groups up to WOE by combining the rigidity result in [K2].

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2. Generalized Bernoulli actions

Let (X, μ) be a standard probability space, which is *non-trivial*, that is, it does not consist of a single atom. Given a discrete group Γ and a countable Γ-space (i.e., a set on which Γ acts), we define the *generalized Bernoulli action*, called *GBA* briefly, of Γ on the product space $(X, \mu)^K = \prod_K (X, \mu)$ by

$$\gamma(x_k)_{k\in K} = (x_{\gamma^{-1}k})_{k\in K}, \quad \gamma \in \Gamma,\ (x_k)_{k\in K} \in (X, \mu)^K.$$

In this section, we summarize basic properties of GBAs. Lemmas 2.1, 2.2 state sufficient conditions for GBAs to be essentially free and to be ergodic, respectively, which are well-known among experts (see Section 2 in [PV]). Proposition 2.4 is a classification result.

Lemma 2.1. *Let Γ be a discrete group and K a countable Γ-space. Let (X, μ) be a non-trivial standard probability space. Then the GBA $\Gamma \curvearrowright (X, \mu)^K$ is essentially free if each $\gamma \in \Gamma \setminus \{e\}$ moves infinitely many elements of K, that is, for each $\gamma \in \Gamma \setminus \{e\}$, there exist infinitely many $k \in K$ such that $\gamma k \neq k$.*

We say that a measure-preserving action of a group Γ on a measure space is *aperiodic* if any finite index subgroup of Γ acts ergodically on the space.

Lemma 2.2. *Let Γ be a discrete group and K a countable Γ-space. Then the following three conditions are equivalent:*

(i) *The GBA $\Gamma \curvearrowright (X, \mu)^K$ is ergodic.*
(ii) *The GBA $\Gamma \curvearrowright (X, \mu)^K$ is aperiodic.*
(iii) *There exist no $k \in K$ such that the orbit Γk is finite.*

Given a group Γ and its subgroup Λ, we define the *left quasi-normalizer $LQN_\Gamma(\Lambda)$* of Λ in Γ by

$$LQN_\Gamma(\Lambda) = \{\gamma \in \Gamma : [\Lambda : \gamma\Lambda\gamma^{-1} \cap \Lambda] < \infty\},$$

which is a subsemigroup of Γ containing Λ. It is easy to check the following.

Lemma 2.3. *Let Γ be a discrete group and Λ a subgroup of Γ. Let Λ act on Γ/Λ so that $\lambda[\gamma] = [\lambda\gamma]$ for $\lambda \in \Lambda$ and $\gamma \in \Gamma$, where $[\gamma]$ denotes the left coset $\gamma\Lambda$ of Λ for $\gamma \in \Gamma$.*

(i) *The equation $LQN_\Gamma(\Lambda) = \Lambda$ holds if and only if each orbit of the action $\Lambda \curvearrowright \Gamma/\Lambda$ other than $\{[e]\}$ consists of infinitely many elements.*
(ii) *Let (X, μ) be a non-trivial standard probability space. If $LQN_\Gamma(\Lambda) = \Lambda$, then the projection*

$$(X, \mu)^{\Gamma/\Lambda} \to (X, \mu), \quad (x_t)_{t\in\Gamma/\Lambda} \mapsto x_{[e]}$$

gives the ergodic decomposition for the GBA $\Lambda \curvearrowright (X, \mu)^{\Gamma/\Lambda}$.

Proposition 2.4. *Let Γ be a discrete group and Λ_i a subgroup of Γ such that $LQN_\Gamma(\Lambda_i) = \Lambda_i$ for $i = 1, 2$. Let (X_1, μ_1) and (X_2, μ_2) be non-trivial standard probability spaces. Then the two GBAs*

$$\alpha_1 \colon \Gamma \curvearrowright (X_1, \mu_1)^{\Gamma/\Lambda_1}, \quad \alpha_2 \colon \Gamma \curvearrowright (X_2, \mu_2)^{\Gamma/\Lambda_2}$$

are conjugate via an isomorphism $F \colon \Gamma \to \Gamma$ if and only if the following two conditions are satisfied:

(i) *There exists $g \in \Gamma$ such that $F(\Lambda_1) = g\Lambda_2 g^{-1}$.*
(ii) *The two probability spaces (X_1, μ_1), (X_2, μ_2) are isomorphic.*

We say that two probability spaces (X, μ), (Y, ν) are *isomorphic* if there exist conull Borel subsets $X' \subset X$, $Y' \subset Y$ and a Borel isomorphism $f\colon X' \to Y'$ such that $f_*\mu = \nu$.

Proof. The proof is essentially given in Theorem 5.4 of [PV] in the framework of operator algebras. We present a proof without operator algebraic arguments. It is easy to see that α_1 and α_2 are conjugate if (i) and (ii) are satisfied. Assume that α_1 and α_2 are conjugate via an isomorphism $F\colon \Gamma \to \Gamma$. Since the action $\Lambda_1 \curvearrowright (X_1, \mu_1)^{\Gamma/\Lambda_1}$ is not ergodic, neither is the action $F(\Lambda_1) \curvearrowright (X_2, \mu_2)^{\Gamma/\Lambda_2}$. Thus, there exists an orbit for the action $F(\Lambda_1) \curvearrowright \Gamma/\Lambda_2$ consisting of only finitely many elements. Take $g \in \Gamma$ so that $g\Lambda_2$ is in the orbit. Then $F(\Lambda_1) \cap g\Lambda_2 g^{-1}$ is of finite index in $F(\Lambda_1)$. Similarly, there exists $h \in \Gamma$ such that $h\Lambda_1 h^{-1} \cap F^{-1}(g\Lambda_2 g^{-1})$ is of finite index in $F^{-1}(g\Lambda_2 g^{-1})$. This implies that $h\Lambda_1 h^{-1} \cap F^{-1}(g\Lambda_2 g^{-1}) \cap \Lambda_1$ is of finite index in $F^{-1}(g\Lambda_2 g^{-1}) \cap \Lambda_1$, which is of finite index in Λ_1. It follows from $LQN_\Gamma(\Lambda_1) = \Lambda_1$ that $h \in \Lambda_1$ and $F(\Lambda_1) \cap g\Lambda_2 g^{-1}$ is of finite index in $g\Lambda_2 g^{-1}$. Therefore, $F(\Lambda_1)$ and $g\Lambda_2 g^{-1}$ have a common finite index subgroup. This implies that $F(\Lambda_1) \subset LQN_\Gamma(g\Lambda_2 g^{-1}) = g\Lambda_2 g^{-1}$. Since Λ_1 and $F^{-1}(g\Lambda_2 g^{-1})$ have a common finite index subgroup, $F^{-1}(g\Lambda_2 g^{-1}) \subset \Lambda_1$ holds similarly. Thus, $F(\Lambda_1) = g\Lambda_2 g^{-1}$. By Lemma 2.3 (ii), (X_1, μ_1) and (X_2, μ_2) are isomorphic. □

Remark 2.1. For a group Γ and a subgroup Λ of Γ, the condition $LQN_\Gamma(\Lambda) = \Lambda$ is the second requirement for Condition $\mathcal{B}$ introduced in Definition 1.4 of [PV].

3. Actions of mapping class groups

In this section, we present families of subgroups of mapping class groups to which Proposition 2.4 can be applied. We always assume a surface to be connected, compact and orientable unless otherwise stated. We write $\kappa(M) = 3g + p - 4$ for a surface $M = M_{g,p}$ of genus g with p boundary components. Let $\Gamma(M)^\diamond$ be the *extended mapping class group* of M, i.e., the group of isotopy classes of all diffeomorphisms of M. We first review some known results.

Let n be a positive integer and M_i a surface with $\kappa(M_i) > 0$ and $M_i \neq M_{1,2}, M_{2,0}$ for $i \in \{1, \ldots, n\}$. Put $G = \Gamma(M_1)^\diamond \times \cdots \times \Gamma(M_n)^\diamond$.

Theorem 3.1 ([K2, Theorem 1.3]). *Let Γ be a finite index subgroup of G. Let (X, μ) and (Y, ν) be standard finite measure spaces. Suppose that Γ admits an aperiodic standard action on (X, μ) and an ergodic standard action on (Y, ν). If the two actions are WOE, then they are conjugate.*

The following describes all isomorphisms between finite index subgroups of G.

Theorem 3.2 ([K1, Corollary 7.3]). *Let Γ be a finite index subgroup of G. Suppose that we have an injective homomorphism $\tau\colon \Gamma \to G$ with the index $[G : \tau(\Gamma)]$ finite. Then we can find a bijection t on the set $\{1, \ldots, n\}$ and an isotopy class g_i of a diffeomorphism $M_{t(i)} \to M_i$ for each $i \in \{1, \ldots, n\}$ such that*

$$\tau(\gamma) = (g_1 \gamma_{t(1)} g_1^{-1}, \ldots, g_n \gamma_{t(n)} g_n^{-1})$$

for any $\gamma = (\gamma_1, \ldots, \gamma_n) \in \Gamma$.

For a surface M with $\kappa(M) > 0$, let $\Gamma(M)$ be the *mapping class group*, i.e., the group of isotopy classes of all orientation-preserving diffeomorphisms of M. We denote by $V(M)$ the set of all isotopy classes of essential simple closed curves on M, where a simple closed curve on M is said to be *essential* if it is isotopic to neither a point nor a boundary component of M. We denote by $S(M)$ the set of all non-empty finite subsets of $V(M)$ which can be realized disjointly on M. Let $\overline{\mathcal{T}}$ be the Thurston compactification of the Teichmüller space for M, which is a union of the Teichmüller space and the Thurston boundary $\mathcal{PMF}$ for M. It is known that $\Gamma(M)^\diamond$ acts continuously on $\overline{\mathcal{T}}$, and the action of $\Gamma(M)^\diamond$ on $\mathcal{PMF}$ is faithful if $\kappa(M) > 0$ and $M \neq M_{1,2}, M_{2,0}$. The set $S(M)$ is naturally embedded in $\mathcal{PMF}$. We refer to [FLP] or Section 2 in [I2] for more details on $\mathcal{PMF}$. A pseudo-Anosov element of $\Gamma(M)$ admits the following remarkable dynamics on $\overline{\mathcal{T}}$.

Theorem 3.3 ([I2, Theorem 3.5])**.** *Suppose that $\kappa(M) \geq 0$ and $g \in \Gamma(M)$ is pseudo-Anosov. Then there exist two distinct fixed points $F_\pm(g) \in \mathcal{PMF}$ of g such that if K is a compact subset of $\overline{\mathcal{T}} \setminus \{F_-(g)\}$ and U is an open neighborhood of $F_+(g)$ in $\overline{\mathcal{T}}$, then $g^n(K) \subset U$ for all sufficiently large n.*

In this theorem, the two elements $F_\pm(g)$ is called *pseudo-Anosov foliations* for g. The following facts are known:

- $F_\pm(g) \neq \sigma$ for any $\sigma \in S(M)$ and any pseudo-Anosov element $g \in \Gamma(M)$ (see 2.9 in [I2]). In particular, a pseudo-Anosov element fixes no element of $S(M)$.
- The set of all pseudo-Anosov foliations is dense in $\mathcal{PMF}$ (see Section 5, Example 1 in [MP]).

We denote by $\Phi(M)$ the set consisting of the subsets $\{F_\pm(g)\}$ of $\mathcal{PMF}$ for all pseudo-Anosov elements $g \in \Gamma(M)$. The action of $\Gamma(M)^\diamond$ on $\mathcal{PMF}$ makes $\Phi(M)$ a countable $\Gamma(M)^\diamond$-space. The equation $h\{F_\pm(g)\} = \{F_\pm(hgh^{-1})\}$ holds for any pseudo-Anosov element $g \in \Gamma(M)$ and any $h \in \Gamma(M)^\diamond$.

Lemma 3.4. *Let M be a surface with $\kappa(M) > 0$ and $M \neq M_{1,2}, M_{2,0}$. If Γ is a finite index subgroup of $\Gamma(M)^\diamond$ and $\sigma \in S(M)$, then each $\gamma \in \Gamma \setminus \{e\}$ moves infinitely many elements of the Γ-orbit $\Gamma\sigma$.*

Proof. Let $\gamma \in \Gamma$ be an element which moves only finitely many elements of $\Gamma\sigma$. Let $\delta \in \Gamma$ be a pseudo-Anosov element. Then there exists a subsequence $\{n_k\}_k$ of $\mathbb{N}$ such that γ fixes all $\delta^{n_k}\sigma$. Since $\delta^{n_k}\sigma \to F_+(\delta)$ when $k \to \infty$ by Theorem 3.3, γ also fixes $F_+(\delta)$. Similarly, γ fixes $F_-(\delta) = F_+(\delta^{-1})$. Since $F_\pm(\delta_0^n) = F_\pm(\delta_0)$ for any pseudo-Anosov element $\delta_0 \in \Gamma(M)$ and any $n > 0$, γ fixes all pseudo-Anosov foliations in $\mathcal{PMF}$. Thus, γ acts on $\mathcal{PMF}$ by the identity. $\square$

Lemma 3.5. *Let Γ be as in Lemma 3.4. We denote the stabilizer of $\sigma \in S(M)$ in Γ by Γ_σ. Then the following assertions hold:*

(i) *If $\sigma, \tau \in S(M)$ satisfy that $\Gamma_\sigma \cap \Gamma_\tau$ is of finite index in Γ_σ, then $\tau \subset \sigma$.*
(ii) *$LQN_\Gamma(\Gamma_\sigma) = \Gamma_\sigma$ for each $\sigma \in S(M)$.*

The proof of this lemma is given in Proposition 5.1 in [P].

Lemma 3.6. *Let Γ be as in Lemma 3.4 and take $\varphi \in \Phi(M)$. Then each $\gamma \in \Gamma \setminus \{e\}$ moves infinitely many elements of the Γ-orbit $\Gamma\varphi$.*

Proof. Let $\gamma \in \Gamma$ be an element which moves only finitely many elements of $\Gamma\varphi$. Let $\delta \in \Gamma$ be a pseudo-Anosov element and take $\psi \in \Gamma\varphi$ such that $\psi \neq \{F_\pm(\delta)\}$. Then there exists a subsequence $\{n_k\}_k$ of $\mathbb{N}$ such that γ fixes all $\delta^{n_k}\psi$. As in Lemma 3.4, we can prove that γ fixes $F_\pm(\delta)$ and that γ acts on $\mathcal{PMF}$ by the identity. $\square$

Lemma 3.7. *Let Γ be as in Lemma 3.4. We denote the stabilizer of $\varphi \in \Phi(M)$ in Γ by Γ_φ. If $\varphi \in \Phi(M)$ and $\gamma \in \Gamma \setminus \Gamma_\varphi$, then $\gamma\Gamma_\varphi\gamma^{-1} \cap \Gamma_\varphi$ is finite. In particular, $LQN_\Gamma(\Gamma_\varphi) = \Gamma_\varphi$.*

Proof. Take $\varphi \in \Phi(M)$ and $\gamma \in \Gamma$. Choose a pseudo-Anosov element $g \in \Gamma_\varphi$. Since Γ_φ contains the cyclic subgroup generated by g as a finite index one (see [M] or Lemma 5.10 in [I2]), if $\gamma\Gamma_\varphi\gamma^{-1} \cap \Gamma_\varphi$ is infinite, then it contains g^n for some positive integer n. Since $\varphi = \{F_\pm(g)\}$ is the unique fixed point for g^n in $\Phi(M)$ by Theorem 3.3, $\gamma\varphi = \varphi$ holds. □

The following is a classification result for GBAs of mapping class groups.

Theorem 3.8. *Let n be a positive integer and M_i a surface with $\kappa(M_i) > 0$ and $M_i \neq M_{1,2}, M_{2,0}$ for $i \in \{1, \dots, n\}$. Put $G_i = \Gamma(M_i)^\diamond$ and $G = G_1 \times \cdots \times G_n$. Let G act on $\Sigma = \prod_{i=1}^n (S(M_i) \cup \Phi(M_i))$ in the coordinatewise way. Let Γ be a finite index subgroup of G and take $\xi, \zeta \in \Sigma$. Let (X, μ) and (Y, ν) be non-trivial standard probability spaces. Then the two GBAs*

$$\Gamma \curvearrowright (X, \mu)^{\Gamma\xi}, \quad \Gamma \curvearrowright (Y, \nu)^{\Gamma\zeta}$$

are WOE if and only if the following two conditions are satisfied:

(i) *There exist a bijection t on the set $\{1, \dots, n\}$ and an isotopy class g_i of a diffeomorphism $M_{t(i)} \to M_i$ for each $i \in \{1, \dots, n\}$ such that $\pi_g(\Gamma) = \Gamma$ and $g(\Gamma\xi) = \Gamma\zeta$;*

(ii) *The two probability spaces (X, μ), (Y, ν) are isomorphic.*

In the condition (i), note that the isotopy classes $g_1, \dots, g_n$ induce a bijection on Σ, denoted by g. The automorphism π_g of G associated with g is defined by

$$\pi_g(\gamma) = (g_1\gamma_{t(1)}g_1^{-1}, \dots, g_n\gamma_{t(n)}g_n^{-1}), \quad \gamma = (\gamma_1, \dots, \gamma_n) \in G.$$

Proof. The "if" part is clear. Note that the two Γ-spaces, $\Gamma\xi$ and Γ/Γ_ξ, are naturally identified for each $\xi \in \Sigma$, where Γ_ξ is the stabilizer of ξ in Γ. Then the GBA $\Gamma \curvearrowright (X, \mu)^{\Gamma\xi}$ is essentially free by Lemmas 2.1, 3.4 and 3.6. Lemmas 3.5 and 3.7 imply that $LQN_\Gamma(\Gamma_\xi) = \Gamma_\xi$ and $LQN_\Gamma(\Gamma_\zeta) = \Gamma_\zeta$. Proposition 2.4 and Theorem 3.2 show that if the two actions $\Gamma \curvearrowright (X, \mu)^{\Gamma\xi}$, $\Gamma \curvearrowright (Y, \nu)^{\Gamma\zeta}$ are conjugate, then the conditions (i), (ii) are both satisfied. By Lemma 2.2 and Lemma 2.3 (i), the actions $\Gamma \curvearrowright (X, \mu)^{\Gamma\xi}$, $\Gamma \curvearrowright (Y, \nu)^{\Gamma\zeta}$ are both aperiodic. Theorem 3.1 then completes the proof. □

Corollary 3.9. *Let n be a positive integer and M_i a surface with $\kappa(M_i) > 0$ for $i \in \{1, \dots, n\}$. Let Γ be a finite index subgroup of $\Gamma(M_1)^\diamond \times \cdots \times \Gamma(M_n)^\diamond$. Then Γ admits continuously many ergodic standard actions which are mutually non-WOE.*

Note that Epstein [E] proves that any non-amenable groups admit continuously many ergodic standard actions which are mutually non-OE.

Remark 3.1. Let Λ be a discrete group and N a finite normal subgroup of Λ. Let $p\colon \Lambda \to \Lambda/N$ be the quotient map. Suppose that there exists a homomorphism $f\colon \Lambda \to F$ into a finite group F such that f is injective on N. Put $\Lambda_1 = \ker f$. Then Λ_1 and $p(\Lambda_1)$ are finite index subgroups of Γ and Γ/N, respectively. If $p(\Lambda_1) \simeq \Lambda_1$ admits two non-WOE ergodic standard actions on (X, μ) and (Y, ν), then the two actions, $X \uparrow_{\Lambda_1}^{\Lambda}$ and $Y \uparrow_{\Lambda_1}^{\Lambda}$, of Λ induced from the actions of Λ_1 are ergodic, standard and mutually non-WOE. Induced actions are defined as follows: Given an action $\Lambda_1 \curvearrowright Z$, let $\Lambda_1 \times \Lambda$ act on $Z \times \Lambda$ by

$$(\lambda_1, \lambda)(z, \lambda_0) = (\lambda_1 z, \lambda_1 \lambda_0 \lambda^{-1}), \quad \lambda_1 \in \Lambda_1,\ \lambda, \lambda_0 \in \Lambda,\ z \in Z.$$

The induced action $Z \uparrow_{\Lambda_1}^{\Lambda}$ is defined to be the action $\Lambda \curvearrowright (Z \times \Lambda)/(\Lambda_1 \times \{e\})$.

Proof of Corollary 3.9. It is known that the quotient group of $\Gamma(M_{1,2})^\diamond$ (resp. $\Gamma(M_{2,0})^\diamond$) by its center, which is generated by the hyperelliptic involution, is isomorphic to a finite index subgroup of $\Gamma(M_{0,5})^\diamond$ (resp. $\Gamma(M_{0,6})^\diamond$) (see [Bi] and [L]). It is also known that $\Gamma(M)^\diamond$ is residually finite for any surface M with $\kappa(M) \geq 0$ (see [BL], [G], [I1]). The corollary follows from Theorem 3.8 and Remark 3.1 since there exist continuously many standard probability spaces which are mutually non-isomorphic. □

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