

サンプルテスト

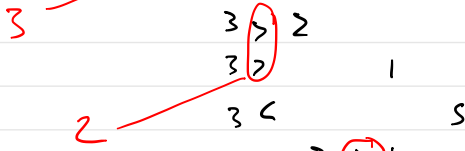
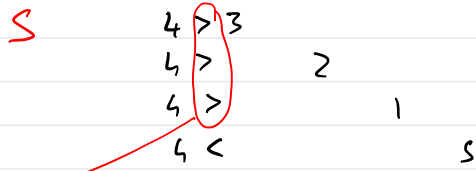
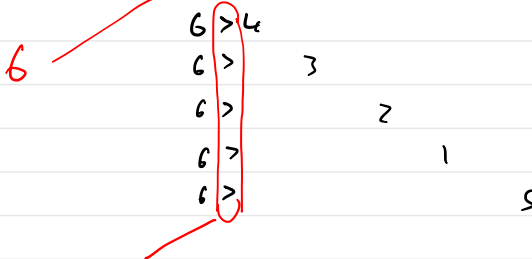
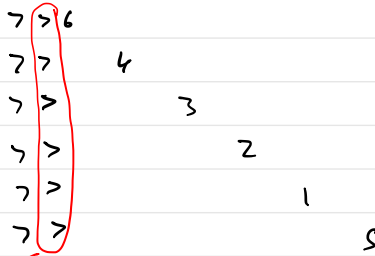
① $\begin{pmatrix} -4 & -8 & 3 & 2 \\ 2 & 6 & 1 & 6 \\ -4 & -4 & 2 & -9 \end{pmatrix}$ の転置行列は

$t \begin{pmatrix} -4 & -8 & 3 & 2 \\ 2 & 6 & 1 & 6 \\ -4 & -4 & 2 & -9 \end{pmatrix} = \begin{pmatrix} -4 & 2 & -4 \\ -8 & 6 & -4 \\ 3 & 1 & 2 \\ 2 & 6 & -9 \end{pmatrix}$

② a) $\text{sgn}(1 \ 2 \ 3) = (-1)^0 = 1$
 $1 < 2$
 $1 < 3$
 $2 < 3$

b) $\text{sgn}(3 \ 4 \ 2 \ 1 \ 5) = (-1)^5 = -1$
 $3 < 4$
 $3 > 2$
 $3 > 1$
 $3 < 5$
 $4 > 2$
 $4 > 1$
 $4 < 5$
 $2 > 1$
 $2 < 5$
 $1 < 5$

c) $\text{sgn}(7643215) = (-1)^{6+5+3+2+1} = (-1)^{17} = -1$



③

$$\begin{pmatrix} -5 & 3 & 1 & -5 \\ 2 & 4 & -5 & -3 \\ 2 & 3 & 0 & -1 \\ 0 & -2 & -2 & -1 \end{pmatrix}$$

第(2,3)小行列は

$$\begin{pmatrix} -5 & 3 & -5 \\ 2 & 3 & -1 \\ 0 & -2 & -1 \end{pmatrix}$$

$$\begin{aligned} \text{第}(2,3)\text{小行列式は} & \underbrace{(-5) \cdot 3 \cdot (-1)} + \underbrace{3 \cdot (-1) \cdot 0} + \underbrace{(-5) \cdot 2 \cdot (-2)} \\ & - \underbrace{(-5) \cdot (-1) \cdot (-2)} - \underbrace{3 \cdot 2 \cdot (-1)} - \underbrace{(-5) \cdot 3 \cdot 0} \end{aligned}$$

$$\begin{aligned} &= 15 + 0 + 20 \\ &- (-10) - (-6) - 0 \end{aligned}$$

$$= 35 + 16 = 51$$

$$\text{第}(2,3)\text{余因子は } (-1)^{2+3} \cdot 51 = -51$$

$$\textcircled{4} \begin{cases} -2x + 2y + 8z - 4w = -10 \\ 4x - 3y - 15z + 7w = 17 \\ 0 \quad -1 \quad -2 \quad +w = a \end{cases}$$

の拡大係数行列は

$$\begin{bmatrix} -2 & 2 & 8 & -4 & -10 \\ 4 & -3 & -15 & 7 & 17 \\ 0 & -1 & -2 & 1 & a \end{bmatrix}$$

$$r_2 + 2r_1 \begin{bmatrix} -2 & 2 & 8 & -4 & -10 \\ 0 & 1 & 1 & -1 & -3 \\ 0 & -1 & -1 & 1 & a \end{bmatrix}$$

$$-\frac{1}{2} r_1 \begin{bmatrix} 1 & -1 & -4 & 2 & 5 \\ 0 & 1 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & a-3 \end{bmatrix}$$

$$r_1 + r_2 \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & a-3 \end{bmatrix}$$

$a \neq 3$ のとき 階数は 3 7、解は 2 変

$a = 3$ のとき 階数は 2 7、解は

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} t + \begin{pmatrix} 2 \\ -3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3s - t + 2 \\ -s + t - 3 \\ s \\ t \end{pmatrix}$$

確認:

$$\begin{aligned} -2(3s - t + 2) + 2(-s + t - 3) + 8s - 4t &= -10 \\ 4(3s - t + 2) - 3(-s + t - 3) - 15s + 7t &= 17 \\ 0 - (-s + t - 3) - s + t &= 3 \end{aligned}$$

⑤

$$\begin{aligned} &\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 4 & 0 & 1 & 0 \\ -3 & 1 & -5 & 0 & 0 & 1 \end{pmatrix} \\ r_2 - 2r_1 &\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & 3 & 0 & 1 \end{pmatrix} \\ r_3 + 3r_1 &\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & 3 & 0 & 1 \end{pmatrix} \\ -r_2 &\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & 3 & 0 & 1 \end{pmatrix} \\ r_3 + r_2 &\begin{pmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{pmatrix} \\ r_1 - 2r_3 &\begin{pmatrix} 1 & 0 & 0 & -1 & 2 & -2 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 4 \\ -3 & 1 & -5 \end{pmatrix} \text{ の逆行列は } \begin{pmatrix} -1 & -2 & -2 \\ 2 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

確認:

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 4 \\ -3 & 1 & -5 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 & -2 \\ 2 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot (-1) + 0 \cdot 2 + 2 \cdot 1 & 1 \cdot (-2) + 0 \cdot (-1) + 2 \cdot 1 & 1 \cdot (-2) + 0 \cdot 0 + 2 \cdot 1 \\ 2 \cdot (-1) + (-1) \cdot 2 + 4 \cdot 1 & 2 \cdot (-2) + (-1) \cdot (-1) + 4 \cdot 1 & 2 \cdot (-2) + (-1) \cdot 0 + 4 \cdot 1 \\ -3 \cdot (-1) + 1 \cdot 2 + (-5) \cdot 1 & -3 \cdot (-2) + 1 \cdot (-1) + (-5) \cdot 1 & -3 \cdot (-2) + 1 \cdot 0 + (-5) \cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

⑥ $\begin{pmatrix} a-1 & 1 & 1 \\ -1 & a-3 & -1 \\ -1 & -1 & a-3 \end{pmatrix}$ が正則 \Leftrightarrow 行列式 $\neq 0$

$$\begin{aligned}
 \text{行列式} &= \underline{(a-1)(a-3)(a-3)} + \underline{1 \cdot (-1) \cdot (-1)} + \underline{1 \cdot (-1) \cdot (-1)} \\
 &\quad - \underline{(a-1) \cdot (-1) \cdot (-1)} - \underline{1 \cdot (-1) \cdot (a-3)} - \underline{1 \cdot (a-3) \cdot (-1)} \\
 &= \underline{(a-1)(a^2-6a+9)} + 1 + 1 \\
 &\quad \underline{-a+1} + \underline{a-3} + \underline{a-3} \\
 &= a^3 - 7a^2 + \underline{15a} - 9 + \underline{a} - 3 \\
 &= a^3 - 7a^2 + 16a - 12 \\
 &= (a-2)^2(a-3)
 \end{aligned}$$

上の行列は正則 \Leftrightarrow $a \neq 2, 3$

⑦

$$-x + y + z = 2$$

$$-x - 3y - z = -1 \quad \text{の解を求めよ。}$$

$$-x - y - 3z = 2$$

$$a = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix} \quad d = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

とするとクラメルの公式によつて

$$x = \frac{|d \ b \ c|}{|a \ b \ c|} = \frac{\begin{vmatrix} 2 & 1 & 1 \\ -1 & -3 & -1 \\ 2 & -1 & -3 \end{vmatrix}}{-12} = \frac{18 + (-2) + 1 - 2 - 3 - (-6)}{-12} = \frac{18}{-12} = -\frac{3}{2}$$

$$\begin{vmatrix} a-1 & 1 & 1 \\ -1 & a-3 & -1 \\ -1 & -1 & a-3 \end{vmatrix} = (a-2)^2(a-3) \Rightarrow |a \ b \ c| = (-2)^2 \cdot (-3) = -12$$

$$y = \frac{|a \ d \ c|}{|a \ b \ c|} = \frac{-1}{-12} \begin{vmatrix} -1 & 2 & 1 \\ -1 & -1 & -1 \\ -1 & 2 & -3 \end{vmatrix} = \frac{-1}{-12} \begin{pmatrix} -3 + 2 + (-2) \\ -2 - 6 - 1 \end{pmatrix} = \frac{-1}{-12} (-12) = 1$$

$$z = \frac{|a \ b \ d|}{|a \ b \ c|} = \frac{-1}{-12} \begin{vmatrix} -1 & 1 & 2 \\ -1 & -3 & -1 \\ -1 & -1 & 2 \end{vmatrix} = \frac{-1}{-12} \begin{pmatrix} 6 + 1 + 2 \\ -(-1) - (-2) - 6 \end{pmatrix} = \frac{-1}{-12} \cdot 6 = \frac{1}{2}$$

確認：

$$\begin{aligned}
 -\left(\frac{-3}{2}\right) + 1 + \left(\frac{-1}{2}\right) &= 2 && \checkmark \\
 -\left(\frac{-3}{2}\right) - 3 - \left(\frac{-1}{2}\right) &= -1 && \checkmark \\
 -\left(\frac{-3}{2}\right) - 1 - \left(\frac{-1}{2}\right) &= 2 && \checkmark
 \end{aligned}$$

⑧

$2 \cdot 2 \cdot 2 \cdot \dots = 0$

$$\begin{pmatrix}
 -3 & 1 & \cdot & \cdot & \cdot \\
 \cdot & 2 & 3 & 4 & \cdot \\
 \cdot & 2 & 3 & \cdot & \cdot \\
 \cdot & 2 & 3 & \cdot & \cdot \\
 -3 & -3 & \cdot & \cdot & \cdot \\
 \cdot & \cdot & 2 & \cdot & \cdot \\
 \cdot & \cdot & 4 & \cdot & \cdot \\
 \cdot & 2 & \cdot & \cdot & \cdot
 \end{pmatrix}$$

$$= (-1)^{2+5} (-1)^{2+4} (-1)^{2+4} (-1)^{2+4} (-1)^{5+4} \begin{pmatrix} -3 & 1 & \cdot & \cdot \\ -3 & -3 & \cdot & \cdot \\ \cdot & \cdot & 2 & -1 \\ \cdot & \cdot & 4 & -3 \end{pmatrix}$$

$$= (-1)(1)(1)(1)(-1) \begin{vmatrix} -3 & 1 \\ -3 & -3 \end{vmatrix} \begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix}$$

$$= \left((-3)(-3) - (-3)(1) \right) \left(2 \cdot (-3) - (-1) \cdot 4 \right)$$

$$= (9 + 3) (-6 + 4)$$

$$= 12 \cdot (-2)$$

$$= -24$$

9

$$\begin{pmatrix} 2 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & -1 & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & -1 & \\ -1 & & & & & & 2 \end{pmatrix}_{7 \times 7}$$

$$= 8$$

3重対角行列 T^n

$$D_n = a D_{n-1} - b c D_{n-2}$$

$$D_0 = 1 \quad D_1 = 0 \quad t \geq 2$$

$$T = \begin{pmatrix} a & & \\ & c & \\ & & c \end{pmatrix} \quad a=2, b=c=1$$

$$D_{-1} = 0$$

$$D_0 = 1$$

$$D_1 = 2 D_0 - D_{-1} = 2$$

$$D_2 = 2 D_1 - D_0 = 4 - 1 = 3$$

$$D_3 = 2 D_2 - D_1 = 6 - 2 = 4$$

$$D_4 = 2 D_3 - D_2 = 8 - 3 = 5$$

$$D_5 = 2 D_4 - D_3 = 10 - 4 = 6$$

$$D_6 = 2 D_5 - D_4 = 12 - 5 = 7$$

$$D_7 = 2 D_6 - D_5 = 14 - 6 = 8$$

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$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 6 & 3 & 7 & 8 \\ 6^2 & 3^2 & 7^2 & 8^2 \\ 6^3 & 3^3 & 7^3 & 8^3 \end{vmatrix} = (3-6)(7-6)(8-6)(7-3)(8-3)(8-7)$$

$$= (-3)(1)(2)(4)(5)(1)$$

$$= -120$$

$$\prod_{1 \leq i < j \leq n} (x_j - x_i) \quad t \geq 2$$